$\pi\pi$ and πK scatterings in resummed Chiral Perturbation Theory

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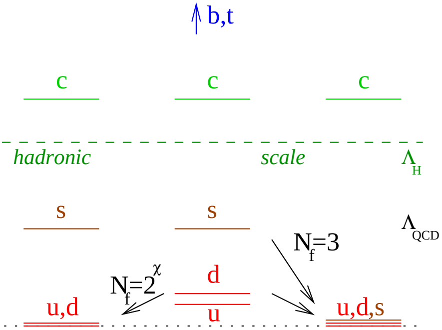
Three chiral limits of interest



$m_u, m_d \longrightarrow 0$

$N_f = 3 \quad : \quad m_s \longrightarrow 0$

$N_f = 2 \quad : \quad m_s$ physical

$N_f = 2^{\mathrm{lat}} \quad : \quad$ no dynamical s

Two versions of χPT	$N_f = 2$:	π only d.o.f	(few param. & processes)
	$N_f = 3$:	π, K, η d.o.f	(more param. & processes)

From 2 to 3 massless flavours

$$\Sigma(2; m_s) = \lim_{m_u, m_d \to 0} -\langle 0|\bar{u}u|0\rangle \qquad \left\{ \begin{array}{rcl} \Sigma(3) & = & \Sigma(2; 0) \\ \Sigma(2) & = & \Sigma(2; m_s^{\rm phys}) \\ \Sigma(2^{\rm lat}) & = & \Sigma(2; \infty) \end{array} \right.$$

- Theory: $\Sigma(3) \leq \Sigma(2)$ but relative size ? $\qquad X(N_f) = \frac{2m\Sigma(N_f)}{F_\pi^2 M_\pi^2}$
- Experimentally : $\Sigma(2) \leftrightarrow \pi$ physics $\qquad \Sigma(3) \leftrightarrow K, \eta$ physics

Three possibilities $\left\{ \begin{array}{l} \Sigma(3) \simeq \Sigma(2) \text{ and large (common lore)} \\ \Sigma(3) \simeq \Sigma(2) \text{ and small} \\ \Sigma(3) \text{ small and } \Sigma(2) \text{ large} \end{array} \right.$

SDG, L. Girlanda, J. Stern, JHEP 0001:041,2000

SDG, N.Fuchs, L.Girlanda, J.Stern, Eur.Phys.J.C34:201-227,2004

Consequences for three-flavour chiral series

$$F_\pi^2 M_\pi^2 = 2m\Sigma(3) + 64m(m_s+2m)B_0^2\Delta L_6 + 64m^2 B_0^2 \Delta L_8 + O(m_q^2)$$

- $B_0 = -\lim_{m_u,m_d,m_s\to 0}\langle \bar{u}u\rangle / F_\pi^2 \qquad m = m_u = m_d$
- $\Delta L_8 = L_8(M_\rho) + 0.20\cdot 10^{-3} = O(p^4)\ \mathrm{LEC} + \chi\log$
- $\Delta L_6 = L_6(M_\rho) + 0.26\cdot 10^{-3} = O(p^4)\ \mathrm{LEC} + \chi\log$

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L_6 contribution dangerous here

- Enhanced by m_s, related to $\langle(\bar{u}u)(\bar{s}s)\rangle\ldots$
- …and "guestimated" assuming Zweig rule in scalar sector

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Possible numerical competition between $O(p^2)$ and $O(p^4)$
$2mB_0 = M_\pi^2 + \dots$ not good approx when treating chiral series

$\Longrightarrow$ Resummed Chiral Perturbation Theory

SDG, hep-ph/0703154

One-loop resummed χPT (1)

All Green functions in generating functional at one loop $Z = Z_t + Z_u + Z_A$
"Bare" expansion in terms of chiral couplings $F_0, B_0, L_i \ldots$ with LO masses

$$\overset{\circ}{M}{}^2_\pi = Y(3) M^2_\pi, \ \overset{\circ}{M}{}^2_K = \frac{r+1}{2} Y(3) M^2_\pi \qquad r = \frac{m_s}{m}, Y(3) = \frac{2mB_0}{M^2_\pi}$$

Where $\overset{\circ}{M}{}^2_P \to M^2_P$ in bare expansion ? Only if physically supported !

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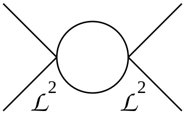
- Z_A purely topological, no chiral couplings

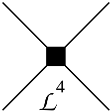


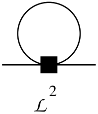
- Z_u one-loop graphs with two $O(p^2)$ vertices

Unitarity cuts at $(\overset{\circ}{M}_P + \overset{\circ}{M}_Q)^2$
converging to $(M_P + M_Q)^2$
when higher orders into account

$\Longrightarrow$ Replace $\overset{\circ}{M}{}^2_P \to M^2_P$ everywhere in Z_u

One-loop resummed χPT (2)

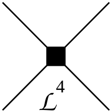


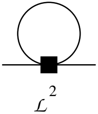


Z_t tree and tadpole graphs

- $O(p^2)$ and $O(p^4)$ tree graphs : chiral couplings
- tadpoles : factors of log modified by higher orders
  but no physical expectation, so just a change in the log for simplicity

$$\frac{\overset{\circ}{M}{}_P^2}{32\pi^2}\log\frac{\overset{\circ}{M}{}_P^2}{\mu^2} \quad\to\quad \frac{\overset{\circ}{M}{}_P^2}{32\pi^2}\log\frac{M_P^2}{\mu^2} \qquad \text{not} \qquad \frac{M_P^2}{32\pi^2}\log\frac{M_P^2}{\mu^2}\;!$$





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- Good observables : correlators of axial/vector currents and derivatives
- Bare expansion at one loop in terms of chiral couplings
- Reexpression in terms of physical masses : Z_u and log-part in Z_t
- Higher-order contributions introduced through NNLO remainders
- Algebraic use of resulting relations (no perturbative reexpression)

Differences with the usual treatment

- particular subset of observables
- distinction between physical meson masses and $O(p^2)$ truncation
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- ratios cannot be expanded (perturbatively) with neglect of remainders

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- r not fixed from M_K^2/M_π^2, but free parameter

$$r_1 = 2\frac{F_K M_K}{F_\pi M_\pi} - 1 \sim 8 \leq r \leq r_2 = 2\frac{F_K^2 M_K^2}{F_\pi^2 M_\pi^2} - 1 \sim 36\,.$$

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- L_5 not determined from F_K/F_π

Why is it a resummation ?

$$X(3) = \frac{2m\Sigma(3)}{F_\pi^2 M_\pi^2}\,, \quad Z(3) = \frac{F^2(3)}{F_\pi^2}\,, \quad r = \frac{m_s}{m}$$

$$\begin{aligned} F_\pi^2 &= F_\pi^2 Z(3) + 8(r+2)Y(3)M_\pi^2 \Delta L_4 + 8Y(3)M_\pi^2 \Delta L_5 + F_\pi^2 e_\pi \\ F_\pi^2 M_\pi^2 &= F_\pi^2 M_\pi^2 X(3) + 16 Y^2(3) M_\pi^4 [(r+2)\Delta L_6 + \Delta L_8] + F_\pi^2 M_\pi^2 d_\pi \end{aligned}$$

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$$\begin{aligned} Y(3) &= \frac{2mB_0}{M_\pi^2} = \frac{2[1-\epsilon(r)-d]}{[1-\eta(r)-e] + \sqrt{[1-\eta(r)-e]^2 + k \times [2\Delta L_6 - \Delta L_4]}} \\ k &\simeq 32(r+2)\frac{M_\pi^2}{F_\pi^2} \qquad\qquad e, d \leftrightarrow e_{\pi,K}, d_{\pi,K} \end{aligned}$$

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- If small vacuum fluctuations : $k\times[2\Delta L_6 - \Delta L_4] \simeq 0$ and $Y(3)\simeq 1$
  $\Longrightarrow$ usual (iterative and perturbative) treatment of chiral series
- But $k \simeq 1900$: $\Delta L_6, \Delta L_4 = O(10^{-3})$ yields shift of $Y(3)$ from 1,
  $\Longrightarrow$ resummation of $k\times[2\Delta L_6 - \Delta L_4]$ needed

$\pi\pi$ and πK scattering : ReχPT (1)

- $\pi\pi$ scattering

$$\begin{aligned} &A(\pi^a(p_1) + \pi^b(p_2) \to \pi^c(p_3) + \pi^d(p_4)) \\ &\qquad = \delta^{ab}\delta^{cd} A(s,t,u) + \delta^{ac}\delta^{bd} A(t,u,s) + \delta^{ad}\delta^{bc} A(u,t,s) \end{aligned}$$

A symmetric under $t-u$ exchange

- πK scattering with two isospin s-channel $I = 3/2$ and $I = 1/2$:

$$A(\pi^a(p_1) + K^i(p_2) \to \pi^b(p_3) + K^j(p_4)) = F^I_{\pi K}(s,t,u)$$

$$\begin{aligned} B(s,t,u) &= \frac{2}{3} F^{3/2}_{\pi K}(s,t,u) + \frac{1}{3} F^{1/2}_{\pi K}(s,t,u)\,, \\ C(s,t,u) &= -\frac{1}{3} F^{3/2}_{\pi K}(s,t,u) + \frac{1}{3} F^{1/2}_{\pi K}(s,t,u)\,. \end{aligned}$$

B, C symmetric/antisymmetric under $s-u$ exchange

$\pi\pi$ and πK scattering : ReχPT (2)

Good observables : $F_\pi^4 A$, $F_\pi^2 F_K^2 B$ and $F_\pi^2 F_K^2 C$

- One-loop bare expansions of these quantities
- Reexpress $O(p^4)$ LECs L_4, L_5, L_6, L_8 in terms of r, $X(3)$ and $Z(3)$, and NNLO remainders related to π and K masses and decay constants
- Add a polynomial modeling higher-order contributions

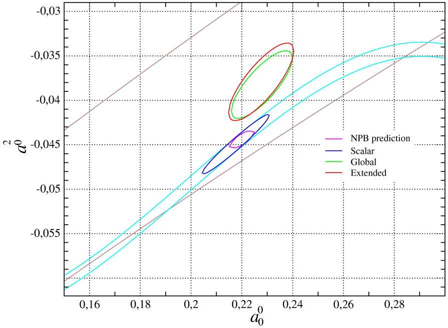
$$\begin{aligned} F_\pi^4 A^{Re\chi PT} &= F_\pi^4 A^{LO+NLO} + F_\pi^2 (s_A - M_\pi^2) a_1 + F_\pi^2 (s - s_A) a_2 \\ &\quad + (s - s_A)^2 a_3 + [(t - t_A)^2 + (u - u_A)^2] a_4 \end{aligned}$$

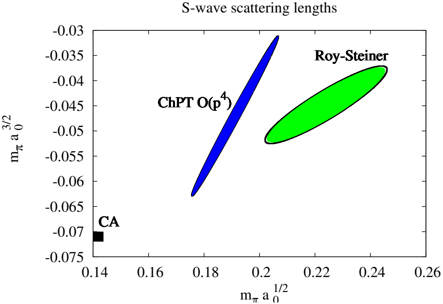
$(s_A, t_A, u_A) = (4/3 M_\pi^2, 4/3 M_\pi^2, 4/3 M_\pi^2)$ around which we perform the expansion of the NNLO polynomial

$\Longrightarrow$ReχPT expansion of $\pi\pi$ scattering amplitude at one loop
Same procedure for πK scattering amplitudes

- Data on $\pi\pi$ and πK scattering
- Extrapolation down to unphysical region where χPT converges well

$\Longrightarrow$ Dispersion relations - Roy and Roy-Steiner equations





B. Ananthanarayan et al. Phys.Rept.353:207,2001
SDG, N.Fuchs, L.Girlanda, J.Stern, Eur.Phys.J.C24:469,2002
P. Buettiker, SDG and B. Moussallam, Eur.Phys.J.C33:409,2004

Frequentist framework (Rfit)

- Data from dispersion relations (s, t in unphysical region)

$$V^T = [A(s_1,t_1),\ldots A(s_n,t_n), B(s'_1,t'_1),\ldots B(s'_n,t'_n),\ldots]$$

- Theory parameters $T_n = r, X(3), Z(3), L_1^r, L_2^r, L_3$ + NNLO remainders

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$$\chi^2(T_n) = -2\log\mathcal{L}(T_n) = -2\log[\mathcal{L}_{\rm th}(T_n)\mathcal{L}_{\rm exp}(T_n)]$$

- Experiment: C covariance matrix, (dispersive) values V_{exp}

$$\mathcal{L}_{\rm exp}(T_n) \propto \exp\left(-\frac{1}{2}(V_{th} - V_{exp})^T C^{-1}(V_{th} - V_{exp})\right) / \sqrt{\det C}\,.$$

- Theory : ranges ($X(3), Z(3) \geq 0$, dimensional args for remainders)

$\mathcal{L}_{\rm th}(T_n) = 1$ if all parameters in the range, $= 0$ otherwise

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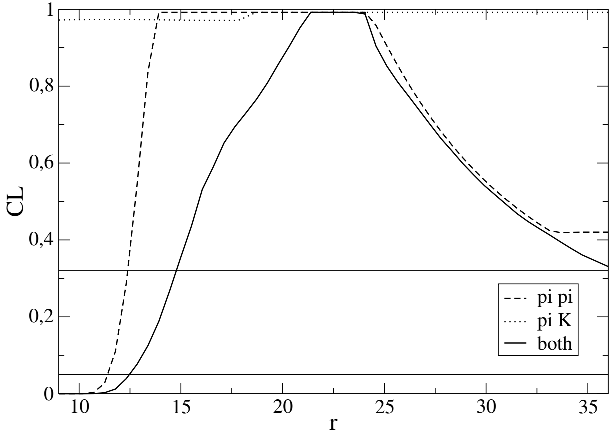
Confidence level: $\mathcal{P}(t_i) = {\rm Prob}[\chi^2_{{\rm min;fix}i}(t_i) - \chi^2_{\rm min;all}, 1]$

$\Longrightarrow$1 for best fit, test agreement between data and theo parameters

Results for $CL(r)$



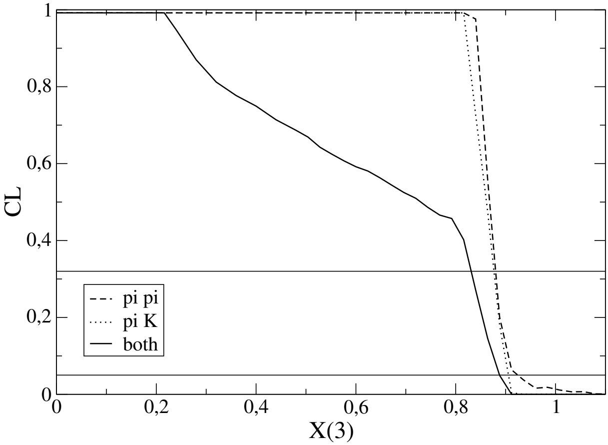
$$r = \frac{m_s}{m}$$

(free, not fixed by M_K^2/M_π^2)

For $\pi\pi$ scattering, good agreement with previous work (lower bound on r)

SDG, N.Fuchs, L.Girlanda, J.Stern, Eur.Phys.J.C34:201-227,2004

Results for $CL(X(3))$



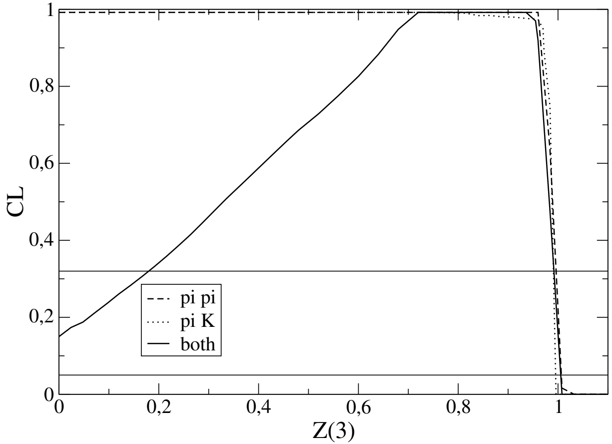
$$X(3) = \frac{2m\Sigma(3)}{F_\pi^2 M_\pi^2}$$

(free, not fixed close to 1 by Zweig rule)

$$X(3) \leq X(2) = 0.81 \pm 0.08, \text{ fixed by } \pi\pi \text{ data}$$

SDG, N.Fuchs, L.Girlanda, J.Stern, Eur.Phys.J.C24:469-483,2002

Results for $CL(Z(3))$



$$Z(3) = \frac{F^2(3)}{F_\pi^2}$$

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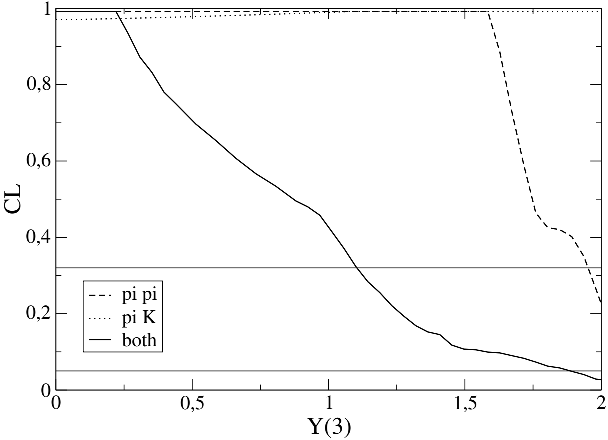
Results for $CL(Y(3))$



$$Y(3) = \frac{2mB_0}{M_\pi^2}$$

(free, not fixed close to 1 by Zweig rule)

Conclusions

$N_f = 2$ and $N_f = 3$ massless flavours

- Possible strong suppression of chiral order parameters
- related to Zweig-rule violating $O(p^4)$ constants L_4 and L_6
- leading to instabilities in chiral expansions : $A_{LO} \simeq A_{NLO}$

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Application to dispersive information on $\pi\pi$ and πK scattering

- $\pi\pi$ scattering constrains essentially $r = m_s/m$
- πK scattering provides few constraints
- Combination is much more powerful

  $r \geq 14.8\,,\ X(3) \leq 0.83\,,\ Y(3) \leq 1.1\,,\ 0.18 \leq Z(3) \leq 1\ \ [68\%\ \mathrm{CL}]$

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- even when perturbative treatment of χPT fails

Application to dispersive information on $\pi\pi$ and πK scattering

- $\pi\pi$ scattering constrains essentially $r = m_s/m$
- πK scattering provides few constraints
- Combination is much more powerful

$$r \geq 14.8\,,\ X(3) \leq 0.83\,,\ Y(3) \leq 1.1\,,\ 0.18 \leq Z(3) \leq 1\ \ [68\%\ \mathrm{CL}]$$

New data on $\pi\pi$ (NA48) and πK (Babar, Belle) awaited eagerly

Backup

Combination of $\pi\pi$ and πK

- $\pi\pi$ constrains r
- πK alone yields no constrains
- combination very powerful

CL computed as minimum over all allowed values for theo param
$\Longrightarrow$ if underdetermination (like in πK) yields CL flat $= 1$

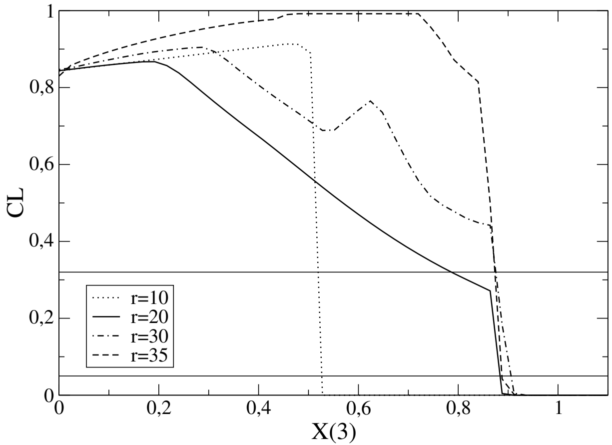
How is the underdetermination lifted ?

- L_1, L_2, L_3 set to the absolute minimum of the χ^2 for $\pi\pi + \pi K$

$$L_1^r(M_\rho) = -0.31\cdot 10^{-3}\,, L_2^r(M_\rho) = 2.12\cdot 10^{-3}\,, L_3^r(M_\rho) = -0.64\cdot 10^{-3}\,.$$

- r set to four different values $r = 10, 20, 30, 35$
- compute CL for πK data alone with these values fixed

Combination of $\pi\pi$ and πK (2)

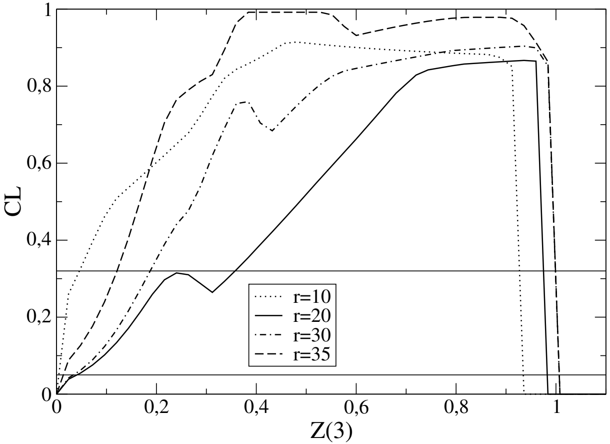


$$X(3) = \frac{2m\Sigma(3)}{F_\pi^2 M_\pi^2}$$

with L_i fixed and 4 values of r

- CL for πK alone max (enveloppe) of all curves
- CL for $\pi K + \pi\pi$ curve for $r \simeq 20$

Combination of $\pi\pi$ and πK (3)

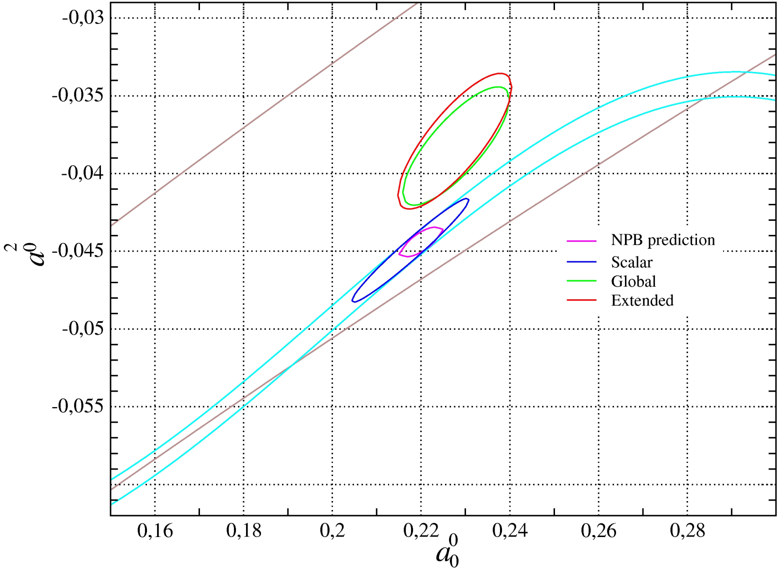


$$Z(3) = \frac{F(3)^2}{F_\pi^2}$$

with L_i fixed and 4 values of r

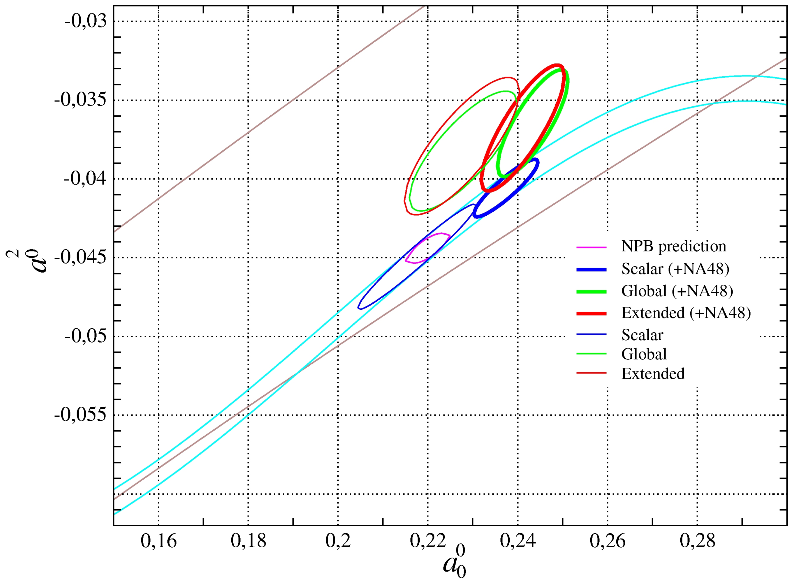
- CL for πK alone max (enveloppe) of all curves
- CL for $\pi K + \pi\pi$ curve for $r \simeq 20$

$\pi\pi$ data (1)



-0,03
-0,035
-0,04
-0,045
-0,05
-0,055
0,16
0,18
0,2
0,22
0,24
0,26
0,28
a_0^0
a_0^2
NPB prediction
Scalar
Global
Extended

$\pi\pi$ data (2)



-0,03
-0,035
-0,04
-0,045
-0,05
-0,055
a_0^2
0,16
0,18
0,2
0,22
0,24
0,26
0,28
a_0^0
NPB prediction
Scalar (+NA48)
Global (+NA48)
Extended (+NA48)
Scalar
Global
Extended