Lattice QCD with dynamical fermions

Luigi Del Debbio

luigi.del.debbio@ed.ac.uk

University of Edinburgh

Recent progress in lattice QCD

Major advances in simulations of fermions on the lattice

- algorithmic developments
  [Hasenbusch 01, Luscher03f, Urbach et al 05, Clark & Kennedy 06]
- program efficiency & faster computers
  [PC clusters, APE, QCDOC $\longrightarrow$ BG]

Numerical studies can now access the chiral regime of QCD, at large volumes and small lattice spacing, with different discretizations.

- taming systematic errors
- quantitative tool for non–perturbative QFT
- revised cost estimates in more detail...

New scope for lattice QCD

Cost formula for 100 cnfg: eg $N_f = 2$, $O(a)$ improved, Wilson quarks, $2L \times L$ lattice

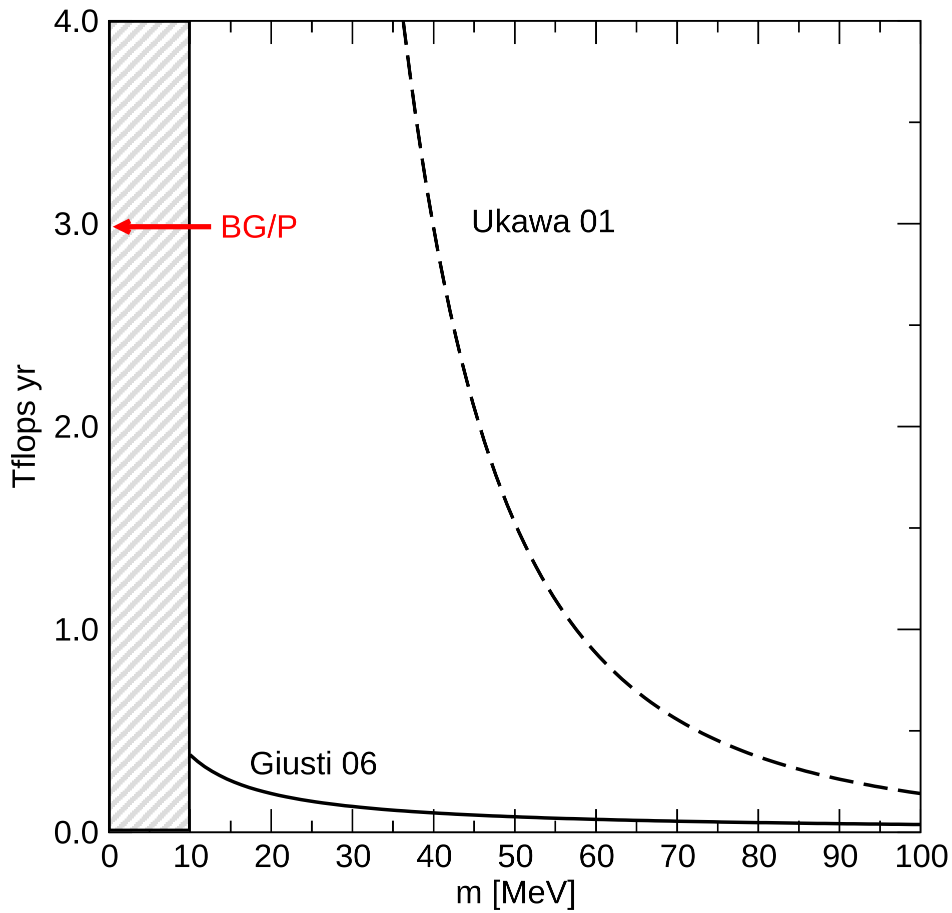
Ukawa lat01

$$5 \left[\frac{20\ \mathrm{MeV}}{m}\right]^3 \left[\frac{L}{5\ \mathrm{fm}}\right]^5 \left[\frac{0.1\ \mathrm{fm}}{a}\right]^7$$

Giusti lat06

$$0.05 \left[\frac{20\ \mathrm{MeV}}{m}\right]^1 \left[\frac{L}{5\ \mathrm{fm}}\right]^5 \left[\frac{0.1\ \mathrm{fm}}{a}\right]^6$$

64×32^3 lattice, $L \approx 2.5\ \mathrm{fm}$, $a \approx 0.09\ \mathrm{fm}$

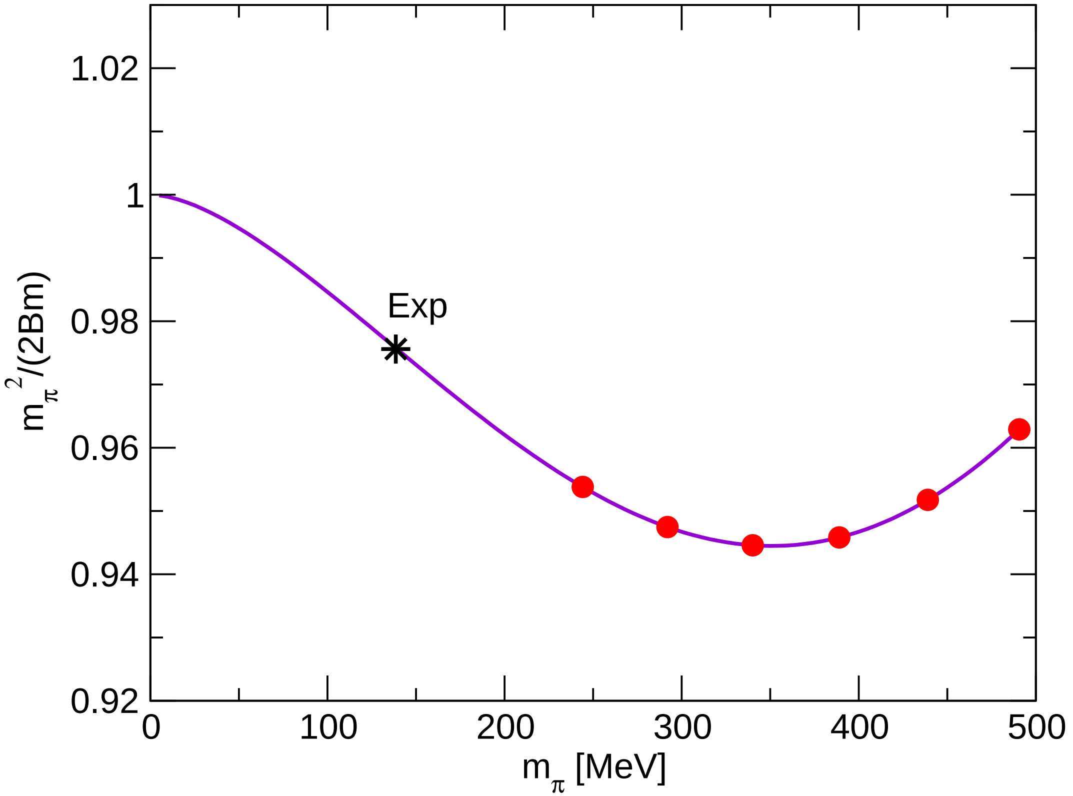


Chiral regime of (full) QCD

- SU(2) chiral perturbation theory predicts:

$$m_\pi^2 = M^2 \left[1 + \frac{M^2}{32\pi^2 F^2} \log\left(M^2/\Lambda_3^2\right) + \ldots \right], \qquad M^2 = 2Bm$$

$$f_\pi = F - \frac{M^2}{16\pi^2 F^2} \log\left(M^2/\Lambda_4^2\right) + \ldots$$

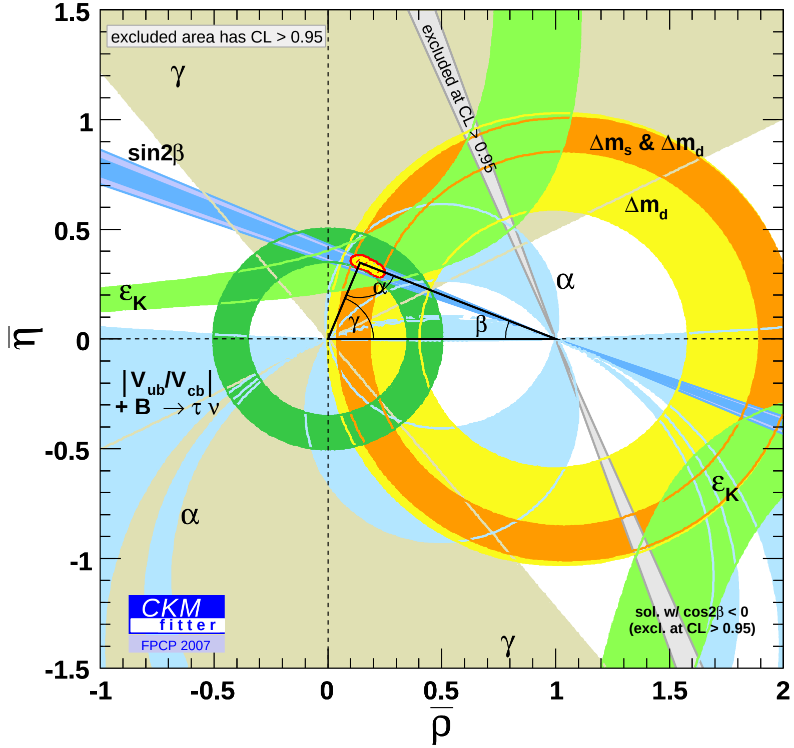


LEC determined from phenomenological studies

small masses and $\approx 1\%$ precision

- $\rho \to \pi\pi$ decays
- flavour–singlet phenomenology

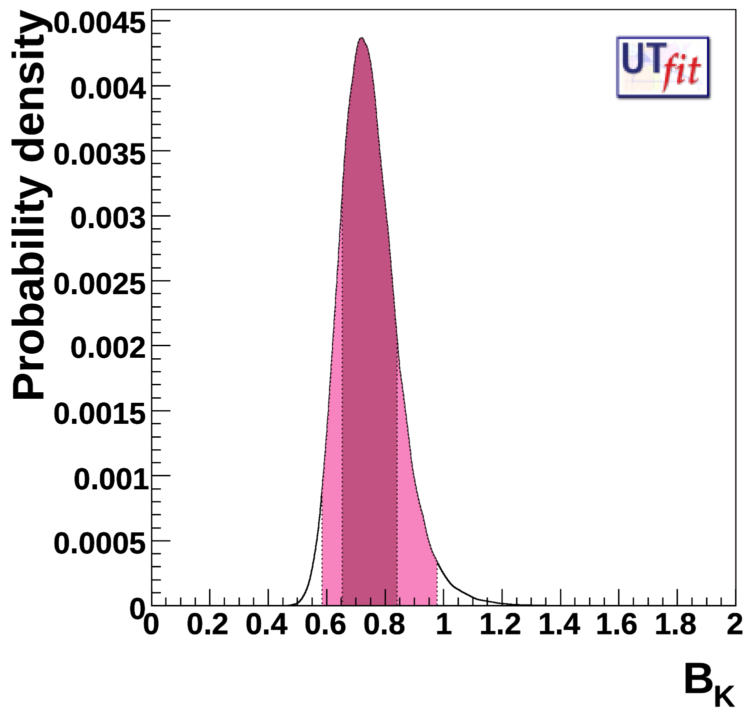
UT phenomenology



$$\hat{B}_K = 0.75 \pm 0.09$$

Lattice input [CKMfitter,UTfit]:

- K_{l3} form factor [Zanotti, EPS07]
- semileptonic B decays form factors
- f_B, B_B, ξ for B mixing
  [Papinutto, EPS07]
- B_K for kaon mixing



Non Perturbative New Physics?

Growing interest for BSM strong dynamics:

- five–dimensional gauge theories

  [Knechtli, EPS07]

- novel models of technicolor

  [Sannino et al 04f]

- orientifold planar equivalence

  [Armoni, Shifman, Veneziano 04f]

- susy on the lattice

  [eg Giedt, lat06]

Dynamical fermions play a crucial role in (almost all) these models.

Explore space of theories as N_c, N_f, and the fermion representation is varied.

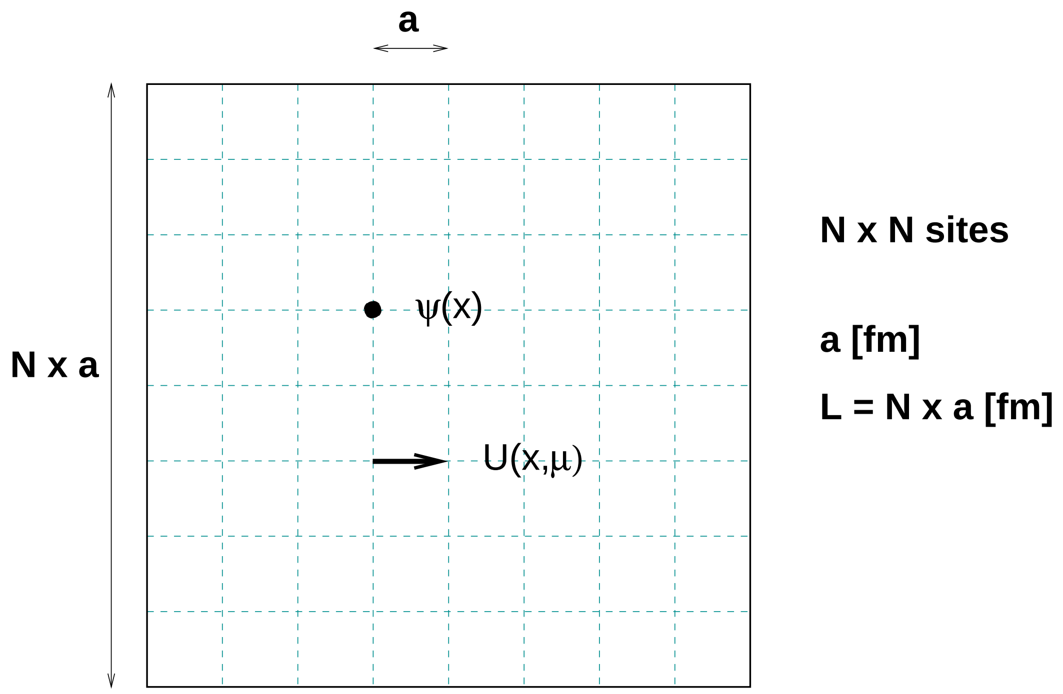
When all are one...

Different actions for lattice QCD:

- precision computations
- affordable cost
- clean theoretical grounds

Control of systematics:

- light masses: $m_\pi \approx 300\ \mathrm{MeV}$
- large volume: $m_\pi L \geq 3$, ie $L \geq 2 \div 3\ \mathrm{fm}$
- lattice artefacts: $O(a)$ improved theories
- NP renormalization
- sampling of topology
- U(1)$_A$ sector



$$Z = \int dU\, d\psi\, d\bar{\psi}\ \exp\left[-S(U,\psi,\bar{\psi})\right]$$
$$= \int dU\, d\phi\, d\phi^*\ \exp\left[-S_G - |D^{-1}\phi|^2\right]$$

Lattice Dirac operator

For massless fermions:

$$S_F = a^4 \sum_{x,y} \bar{\psi}(x)\, D(x,y)\, \psi(y)$$

Naive continuum limit suggests:

(a) $D(x,y)$ is local

(b) $D(p) = i\gamma_\mu p_\mu + O(ap^2)$

(c) $D(p)$ invertible for $p \neq 0$

(d) $\{\gamma_5, D\} = 0$

Nielsen-Ninomiya thm: (a)–(d) can not hold simultaneously

$\hookrightarrow$ **doublers:** anomaly–cancelling spectrum

Wilson: explicit breaking of chiral symmetry

Staggered: left with four species (“tastes”), rooting [Sharpe lat06, Kronfeld lat07]

Ginsparg–Wilson: relax (d) $\longrightarrow \{\gamma_5, D\} = aD\gamma_5 D$ [Ginsparg & Wilson 82]

Wilson fermions

$$D = \frac{1}{2}\sum_\mu \left\{\gamma_\mu\left(\nabla_\mu + \nabla^*_\mu\right) - \nabla^*_\mu \nabla_\mu\right\} + m$$

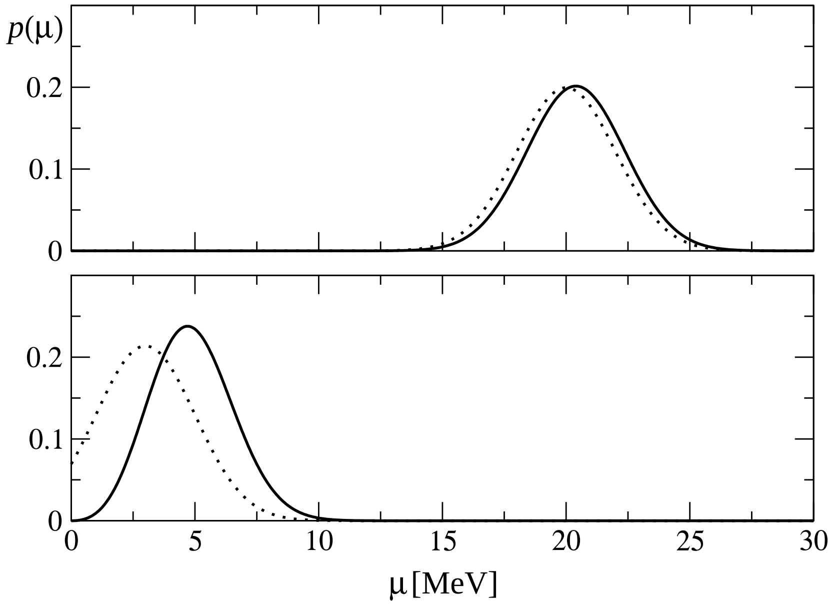
- doublers removed by adding **irrelevant operator**
- explicit breaking of axial symmetry at $m = 0$
- NP renormalization [ALPHA]
- NP $O(a)$ improvement [ALPHA]
- $N_f = 2$ CERN–TOV, QCDSF (UKQCD, CP–PACS)
- $N_f = 2 + 1$ PACS–CS

breaking of axial symmetry $\Longrightarrow$ non–trivial approach to the chiral limit...

Eigenvalues of the Dirac operator

- continuum theory: $\{\gamma_5, D\} = 0$
  $\Longrightarrow$ the massive Dirac operator is protected from small e.v.
- Wilson fermions break chiral symmetry
  $\Longrightarrow$ no protection for $D_m = D_w + m_0$
- for particular gauge fields: **exceptionally small e.v.**
  $\Longrightarrow$ instabilities / more expensive computations
- CERN-TOV observed smooth runs: can NOT be a property of the algorithm
  distribution of the **spectral gap** is a property of D_m
- scaling as $a \to 0$
  $m \to 0$
  $V \to \infty$
- analytical control: PQChPt/RMT, $O(a^2)$ effects, finite volume [Sharpe 06]

Gap distribution - 1



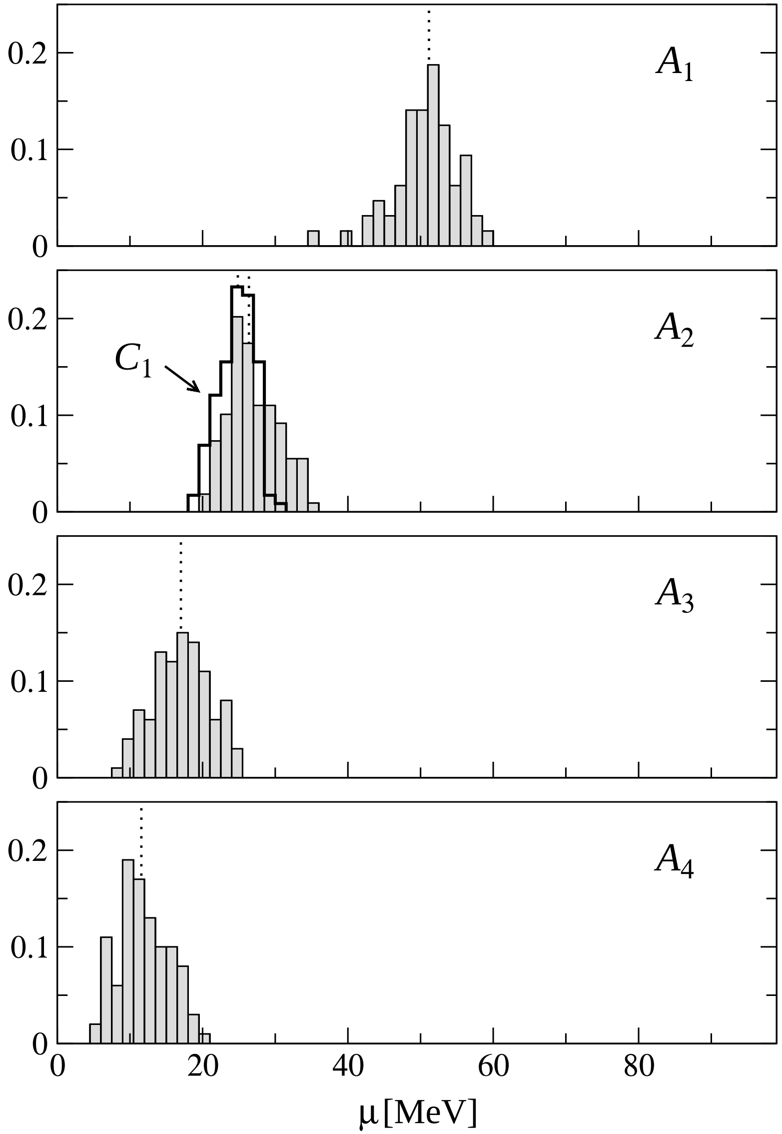
- μ lowest eigenvalue of D_m
- integration instabilities / reversibility
- ergodicity problem
- sampling of observables: $p(\mu)/\mu^2$

difficult regime for simulations:

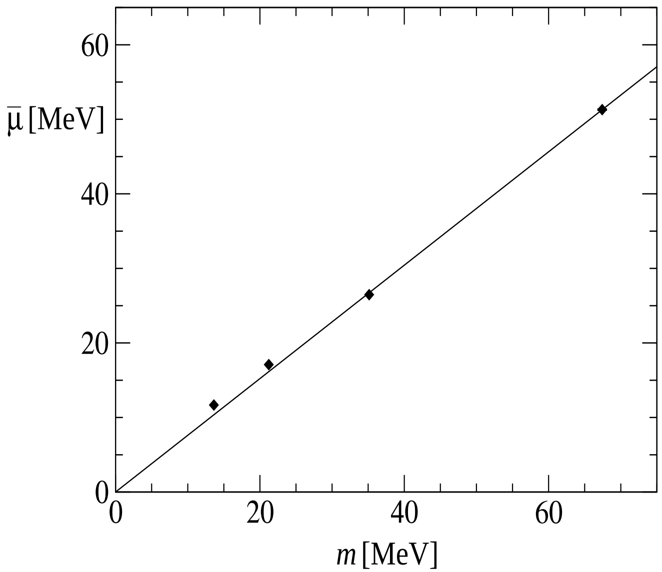
$$\boxed{\begin{array}{l} m \longrightarrow 0 \\ a, V \ \text{ fixed} \end{array}}$$

$\hookrightarrow$ is there a safe regime?

Gap distribution - 2



- stability is related to the width of the distribution
- $\bar{\mu}$ shifts linearly with m
- width independent of the mass
- scaling with a and V: $\sigma \propto aV^{-1/2}$



[Idd et al 05]

Safety distance

To avoid instabilities:

$$\bar{\mu} \geq 3\sigma$$

which yields:

$$m \geq \frac{3a}{Z_A\sqrt{V}}$$

Introducing $B = m_\pi^2/2m$ and lattice with geometry $2L \times L^3$:

$$(m_\pi L)^2 \geq 3\sqrt{2}B/Z_A a$$

From our numerical study:

$$m_\pi L \geq \begin{cases} 2.8 & \text{at } a = 0.07 \text{ fm}, \\ 2.3 & \text{at } a = 0.05 \text{ fm} \end{cases}$$

$\hookrightarrow$ chiral symmetry and continuum limit entangled

tmQCD

$$D = \frac{1}{2}\sum_{\mu}\left\{\gamma_\mu\left(\nabla_\mu + \nabla^*_\mu\right) - \nabla^*_\mu\nabla_\mu\right\} + (m + i\gamma_5\tau_3\mu)$$

Tune $m = m_{\rm crit}(\mu)$ to achieve *maximal twist*:

$$m_{\rm pcac} = \frac{\sum_{\mathbf{x}}\langle\partial_0 A_0^a(t,\mathbf{x})P^a(0)\rangle}{\sum_{\mathbf{x}}\langle P^a(t,\mathbf{x})P^a(0)\rangle} = 0$$

- breaking of vector symmetry and parity - $\pi^{\pm},\pi^0$ splitting
- NP renormalization
- at maximal twist: NP $O(a)$ improvement automatic for parity–even operators
- tuning only one parameter to achieve maximal twist
- $N_f = 2$ ETMC [Urbach lat07]

Ginsparg–Wilson fermions

- Ginsparg-Wilson relation (82)

$$\boxed{\{\gamma_5, D\} = \bar{a} D \gamma_5 D}$$

- Neuberger overlap operator (97)

$$\begin{aligned} D &= \frac{1}{\bar{a}} \left[1 + \gamma_5\, \epsilon(H_\mathrm{w})\right] \\ H_\mathrm{w} &= \gamma_5 \left[a D_\mathrm{w} - (1+s)\right] \end{aligned}$$

  where $\bar{a} = a/(1+s)$, and $s \in [0,2]$

- lattice chiral symmetry [Lüscher 98]

$$\begin{aligned} \delta\psi &= \hat{\gamma}_5 \psi, \qquad \hat{\gamma}_5 = \gamma_5 (1 - \bar{a} D) \\ \delta\bar{\psi} &= \bar{\psi}\gamma_5 \end{aligned}$$

- anomaly and index theorem on the lattice [Hasenfratz et al 97]

$$a^4 Q(x) = \frac{\bar{a}}{2} \mathrm{Tr}\; \left[\gamma_5 D(x,x)\right]$$

Domain-wall fermions

[Shamir & Furman 93]

- Introduce extra dimension L_s
- Dirac operator: $D_{x,s;x',s'} = \delta_{s,s'} D^{||}_{x,y}(-M) + \delta_{x,y} D^{\perp}_{s,s'}(m)$
- at finite L_s, partial decoupling of chiral modes:

$$\partial_\mu A^a_\mu = 2mJ^a_5 + 2J^a_{5q}$$

- for $m = 0$, $L_s \to \infty$: chiral symmetry restored
- residual chiral symmetry breaking parameterized by the **residual mass**
- (almost) automatic $O(a)$ improvement

- extra computational cost
- chiral limit is **decoupled** from the continuum one
- AWI on the lattice
- $N_f = 2 + 1$ RBC/UKQCD collaboration [Boyle lat07]

First results

[lat07]

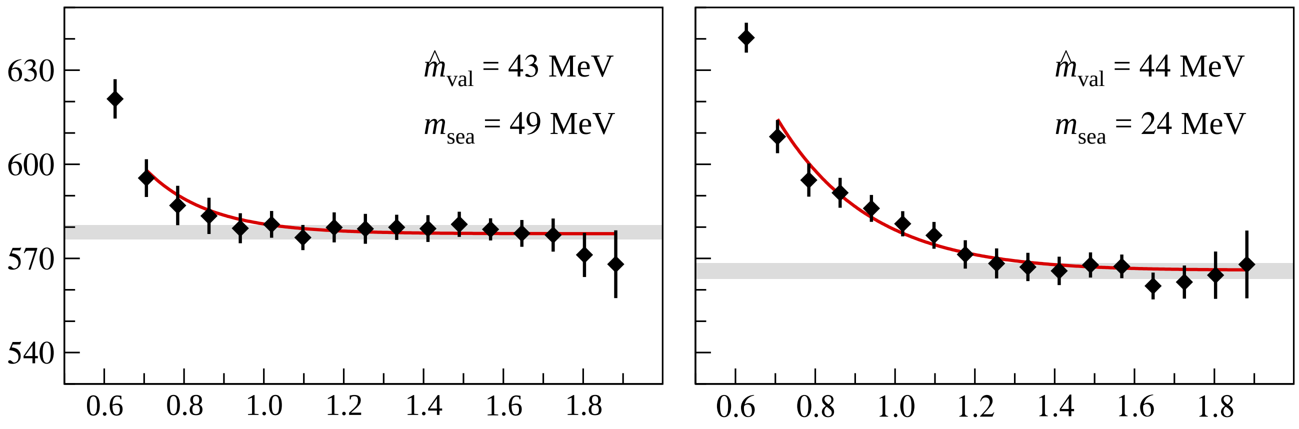
Several collaborations worldwide:

- ALPHA $N_f = 2$, $O(a)$ improved Wilson
- CERN-TOV $N_f = 2$, $O(a)$ improved Wilson
- JLQCD $N_f = 2$, overlap
- QCDSF $N_f = 2$, $O(a)$ improved Wilson, baryon physics
- ETMC $N_f = 2$, tmQCD

- BGR $N_f = 2 + 1$, perfect action
- HPQCD, MILC $N_f = 2 + 1$, staggered
- PACS-CS $N_f = 2 + 1$ $O(a)$, improved Wilson
- NPLQCD mixed action
- RBC/UKQCD $N_f = 2 + 1$, Domain Wall Fermions

Effects of dynamical fermions

CERN-TOV, $a = 0.078$ fm, $[M_\mathrm{V}/M_\mathrm{PS}]$, $L \approx 2$ fm [Idd et al 05]



Pseudoscalar two-pt function - coupling to multi-particle states:

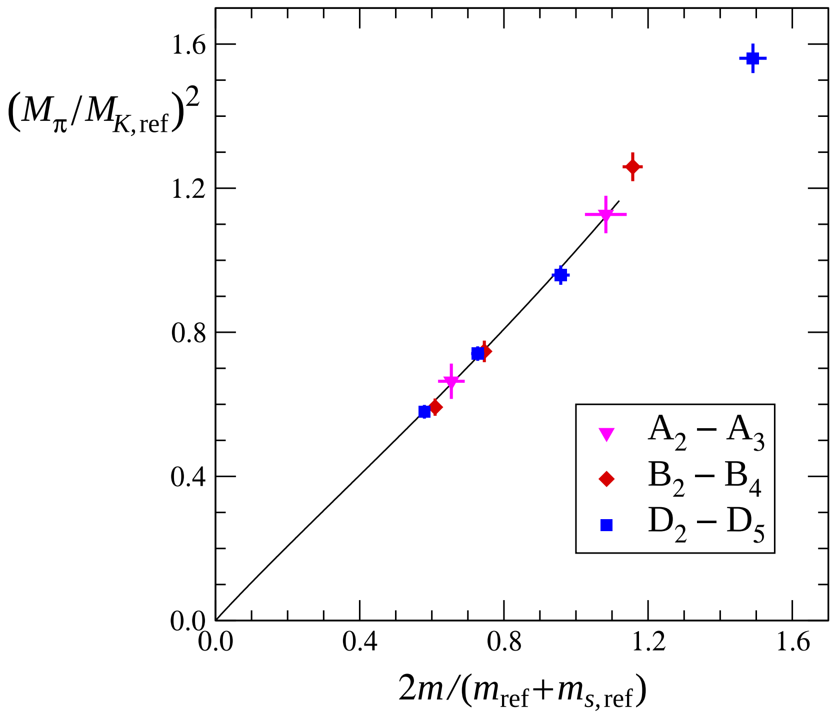
$$f_\mathrm{PP}(t) = c_0 e^{-M_0 t} + c_1 e^{-M_1 t} + \ldots$$

In the full theory: $M_1 = M_0 + 2M_\pi$, ie. three meson state at rest

$$M_\mathrm{eff}(t) = -\frac{d}{dt}\log f_2(t) = M_0 + C\, e^{-(M_1 - M_0)t} + \ldots$$

Chiral extrapolations

[Idd et al. 06]



$$x = \frac{2m}{m_{\mathrm{ref}} + m_{s,\mathrm{ref}}}, \quad C = \frac{M^2_{K,\mathrm{ref}}}{32^2_\pi F^2_{K,\mathrm{ref}}}$$

$$\hat{F} = \frac{F}{F_{K,\mathrm{ref}}}, \quad \hat{B} = \frac{m_{\mathrm{ref}} + m_{s,\mathrm{ref}}}{M^2_{K,\mathrm{ref}}} B$$

$$\hat{l}_3 = \log(\Lambda_3^2 / M^2_{K,\mathrm{ref}})$$

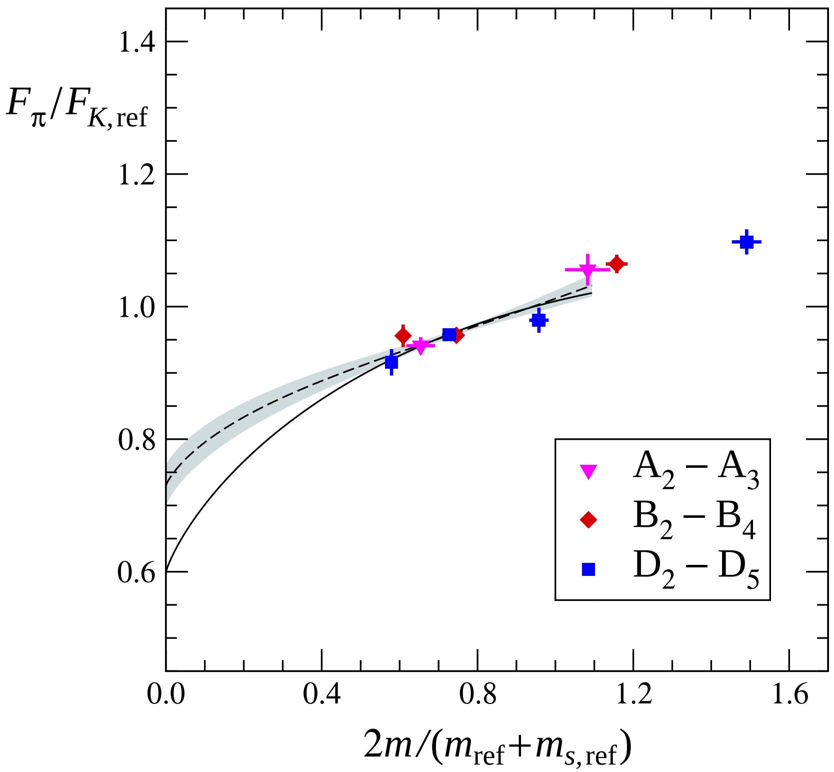
$$\frac{M_\pi^2}{M^2_{K,\mathrm{ref}}} = \hat{B}x + C\frac{\hat{B}^2 x^2}{\hat{F}^2}\left[\log(\hat{B}x) - \hat{l}_3\right]$$

$$\hat{B} = 1.11(6)(3), \quad \hat{l}_3 = 0.5(5)(1)$$

lightest pion: $m_\pi \approx 377\ \mathrm{MeV}$

Chiral extrapolations

[Idd et al. 06]



$$x = \frac{2m}{m_{\mathrm{ref}} + m_{s,\mathrm{ref}}}, \quad C = \frac{M_{K,\mathrm{ref}}^2}{32_\pi^2 F_{K,\mathrm{ref}}^2}$$

$$\hat{F} = \frac{F}{F_{K,\mathrm{ref}}}, \quad \hat{B} = \frac{m_{\mathrm{ref}} + m_{s,\mathrm{ref}}}{M_{K,\mathrm{ref}}^2} B$$

$$\hat{l}_4 = \log(\Lambda_4^2/M_{K,\mathrm{ref}}^2)$$

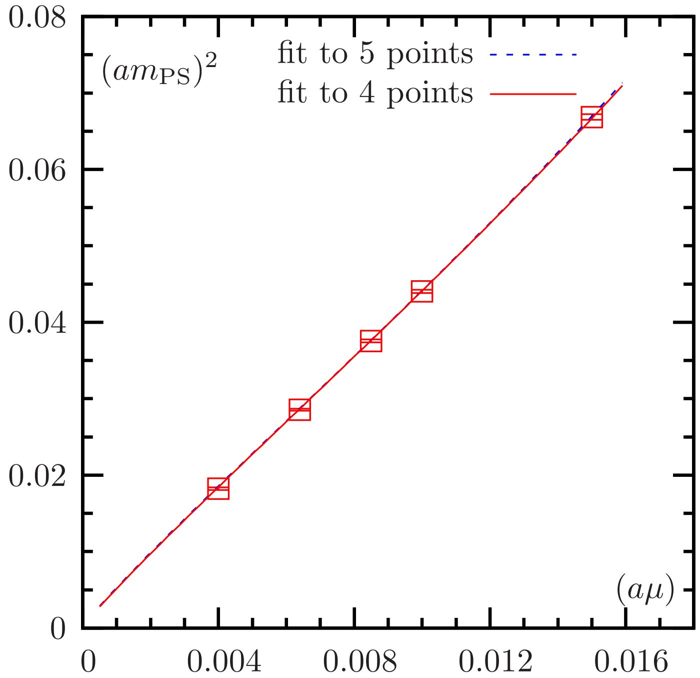
$$\frac{F_\pi}{F_{K,\mathrm{ref}}} = \hat{F} - 2C\frac{\hat{B}x}{\hat{F}}\left[\log(\hat{B}x) - \hat{l}_4\right]$$

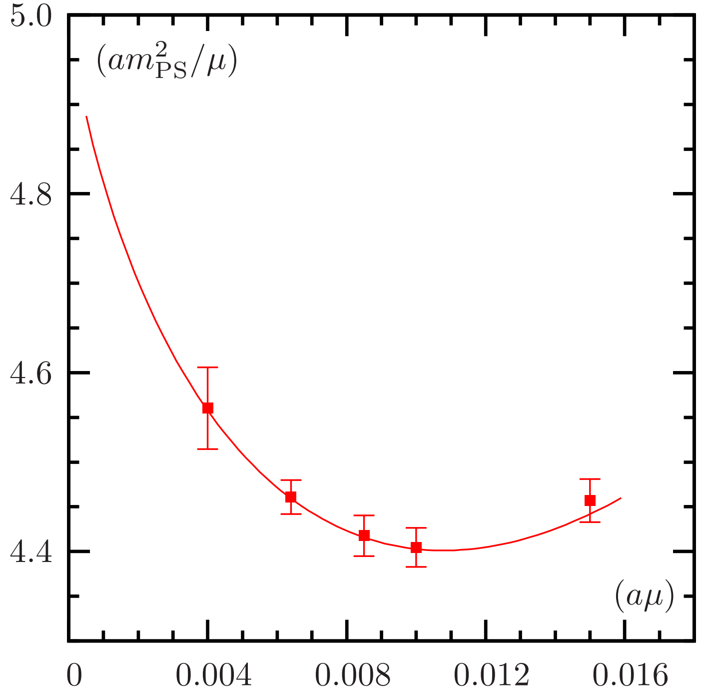
$$\hat{F} = 0.60(4), \quad \hat{l}_4 = 1.6(1)$$

Chiral extrapolations

ETMC, $a = 0.087$ fm, [chiral fit], $L \geq 2$ fm

[Boucaud et al 07]

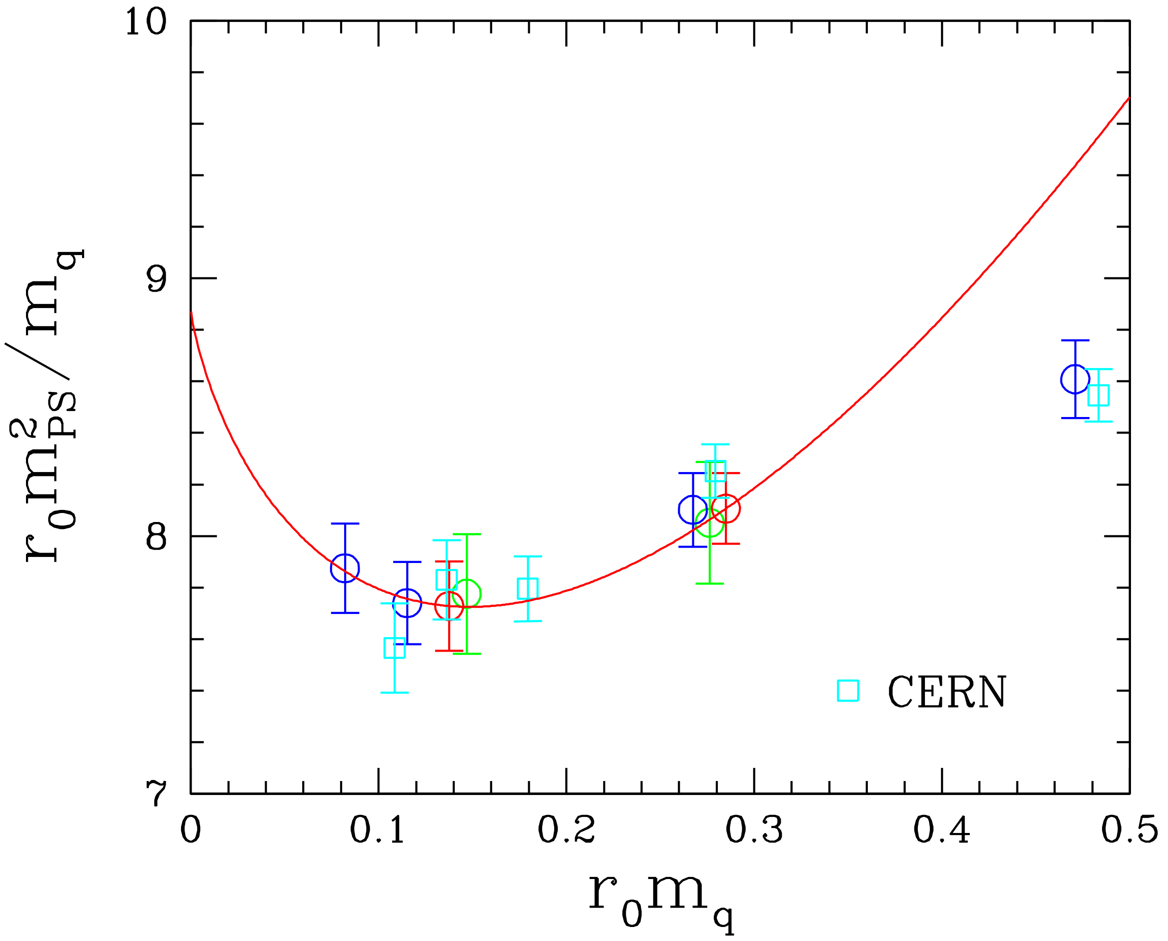




combined chiral fit to m_π and f_π, lightest pion $m_\pi \approx 309$ MeV

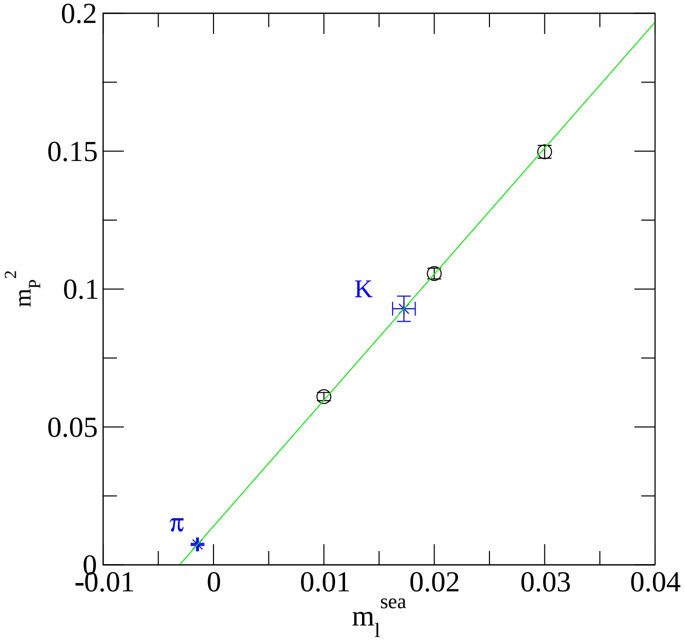
Chiral extrapolations

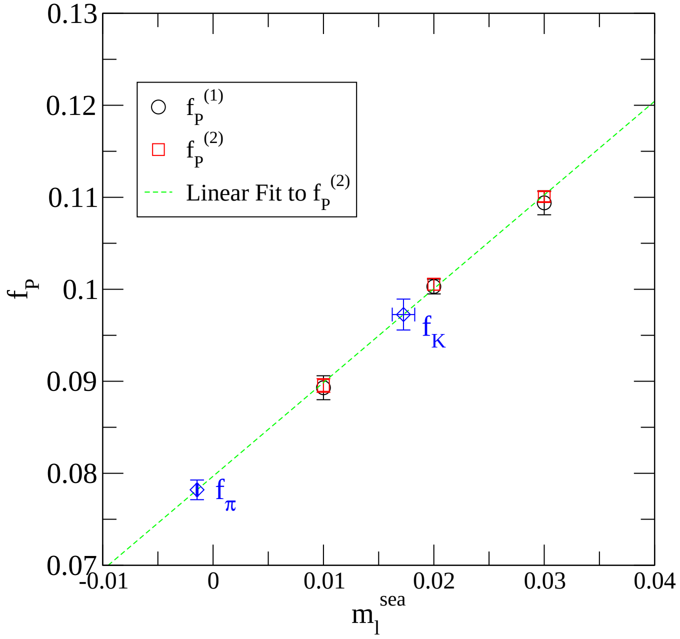
QCDSF, $a = 0.07\ \mathrm{fm}$, [chiral fit], $L \geq 2\ \mathrm{fm}$, lightest pion $m_\pi \approx 313\ \mathrm{MeV}$



Chiral extrapolations

RBC/UKQCD, $a = 0.123$ fm, $[M_\mathrm{V}/M_\mathrm{PS}]$, $L \geq 2$ fm



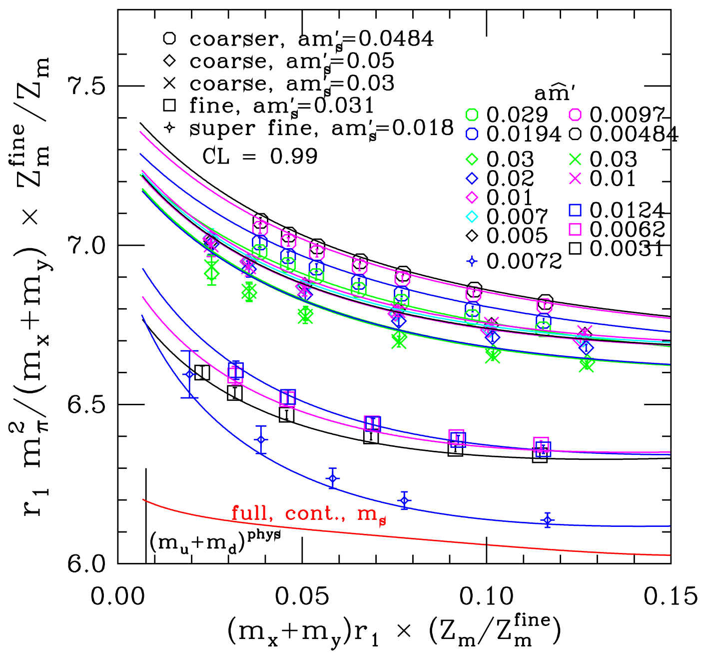


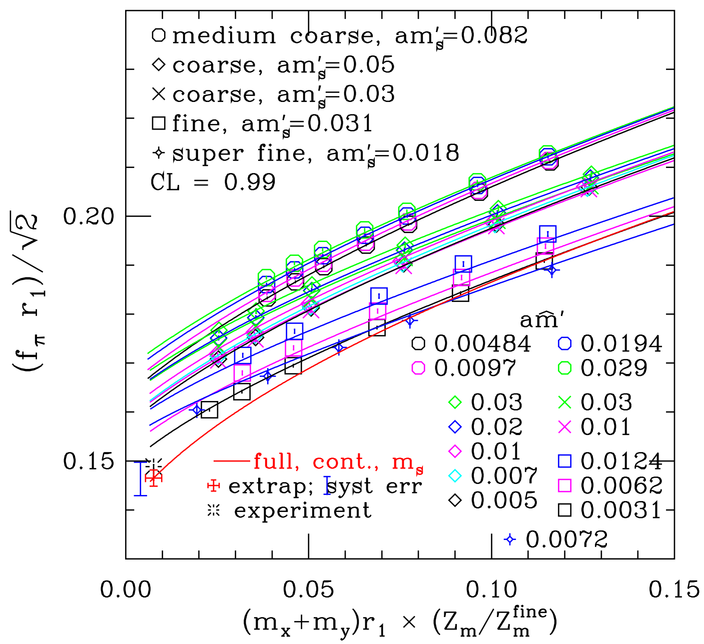
16^3 lattice, lightest pion $m_\pi \approx 390$ MeV

new data on a larger volume and lighter mass available soon

Chiral extrapolations

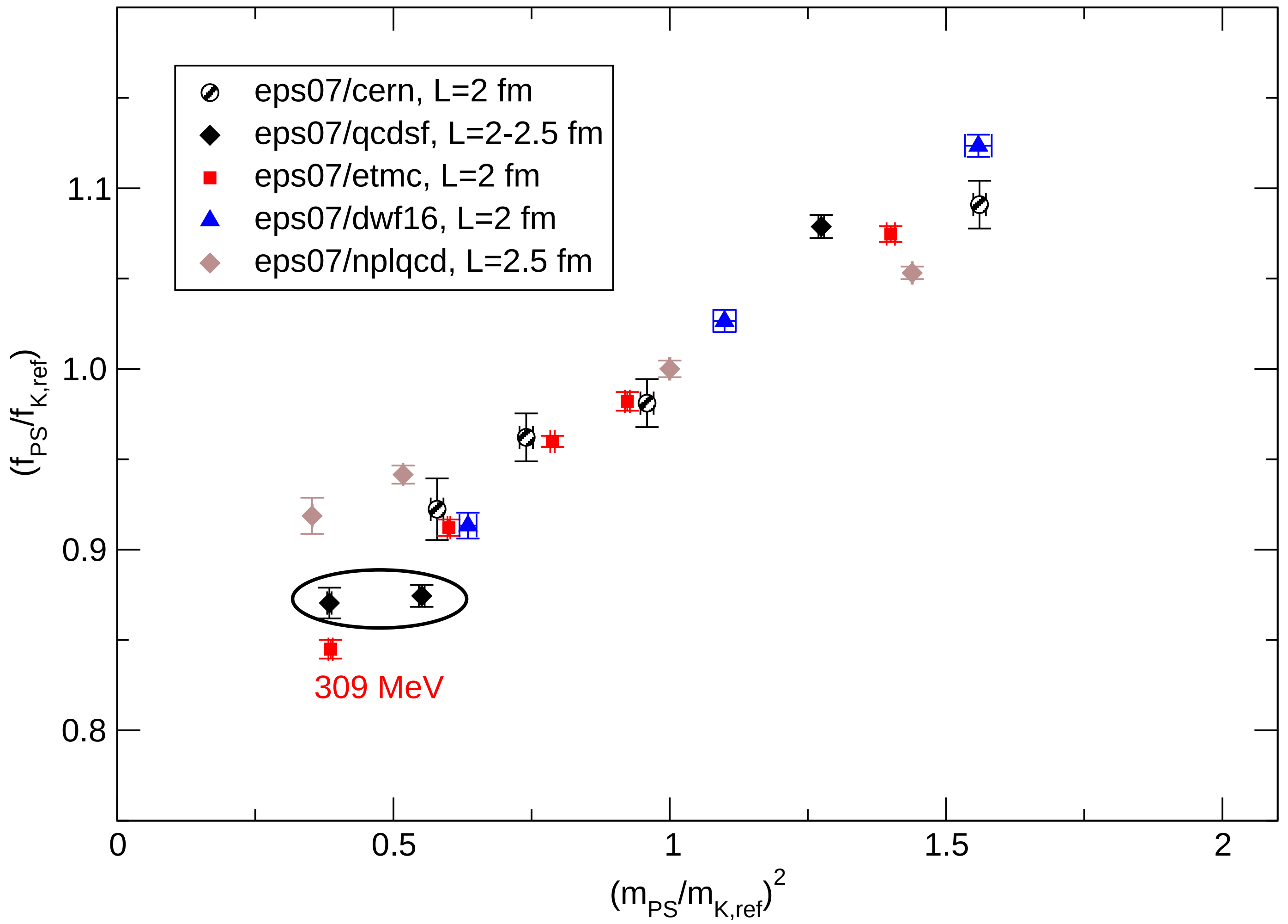
MILC, $a = 0.06 \div 0.015$ fm, $L = 2.4 \div 3.4$ fm





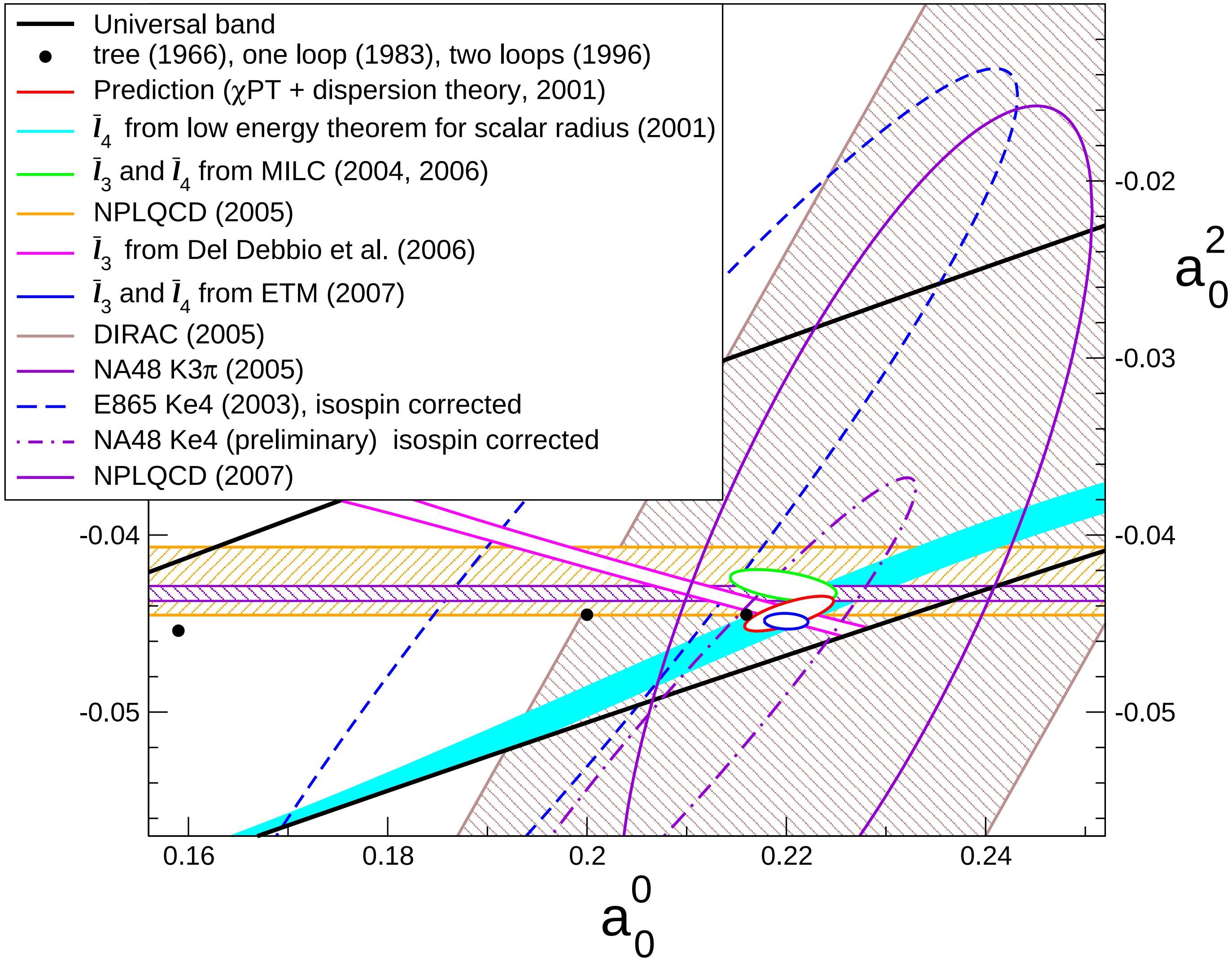
26–parameter combined fit to NLO (S)ChPT using the lighter masses [Sugar lat06]

Chiral extrapolations - summary



precision data at light quark masses is within reach!! see [lat07] for new data

Chiral extrapolations - impact



[Leutwyler Moriond07]

Kaon mixing – DWF

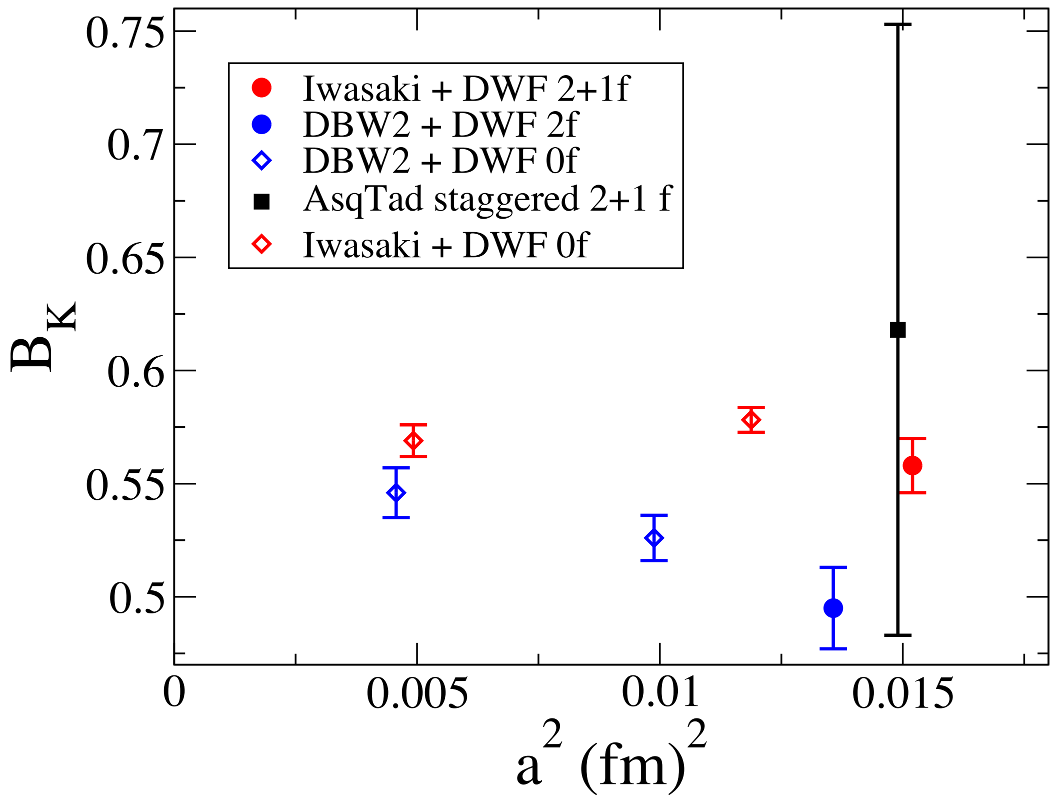
non–perturbative contribution encoded in:

$$B_K = \frac{\langle K^0|\mathcal{O}_{VV+AA}|\bar{K}^0\rangle}{\frac{8}{3} f_K^2 M_K^2}$$

RI-MOM multiplicative renormalization:

$$B_K^{\overline{\mathrm{MS}}} = Z_{B_K}^{\overline{\mathrm{MS}}} B_K^{\mathrm{lat}}$$

no chirality mixing, $O(a^2)$ lattice artefacts



cfr. Wilson fermions:

$$\mathcal{O}_{VV+AA}^R = Z(g_0)\left[\mathcal{O}_{VV+AA} + \sum_i c_i(g_0)\mathcal{O}_i\right]^{\mathrm{lat}}$$

$$\boxed{\hat{B}_K = 0.77(2)(4)}$$

[Antonio et al 07]

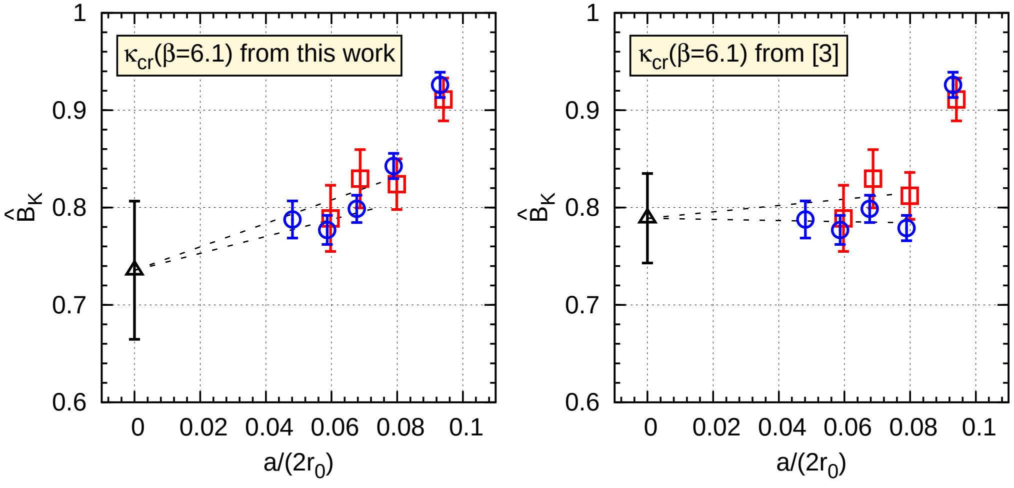
$$[\hat{B}_K = 0.79(2)(9)]$$

Kaon mixing – tmQCD

$$\boxed{\langle K^0|\mathcal{O}_{VA+AV}|\bar{K}^0\rangle_{\rm tmQCD} = i\,\langle K^0|\mathcal{O}_{VA+AV}|\bar{K}^0\rangle_{\rm QCD}}$$

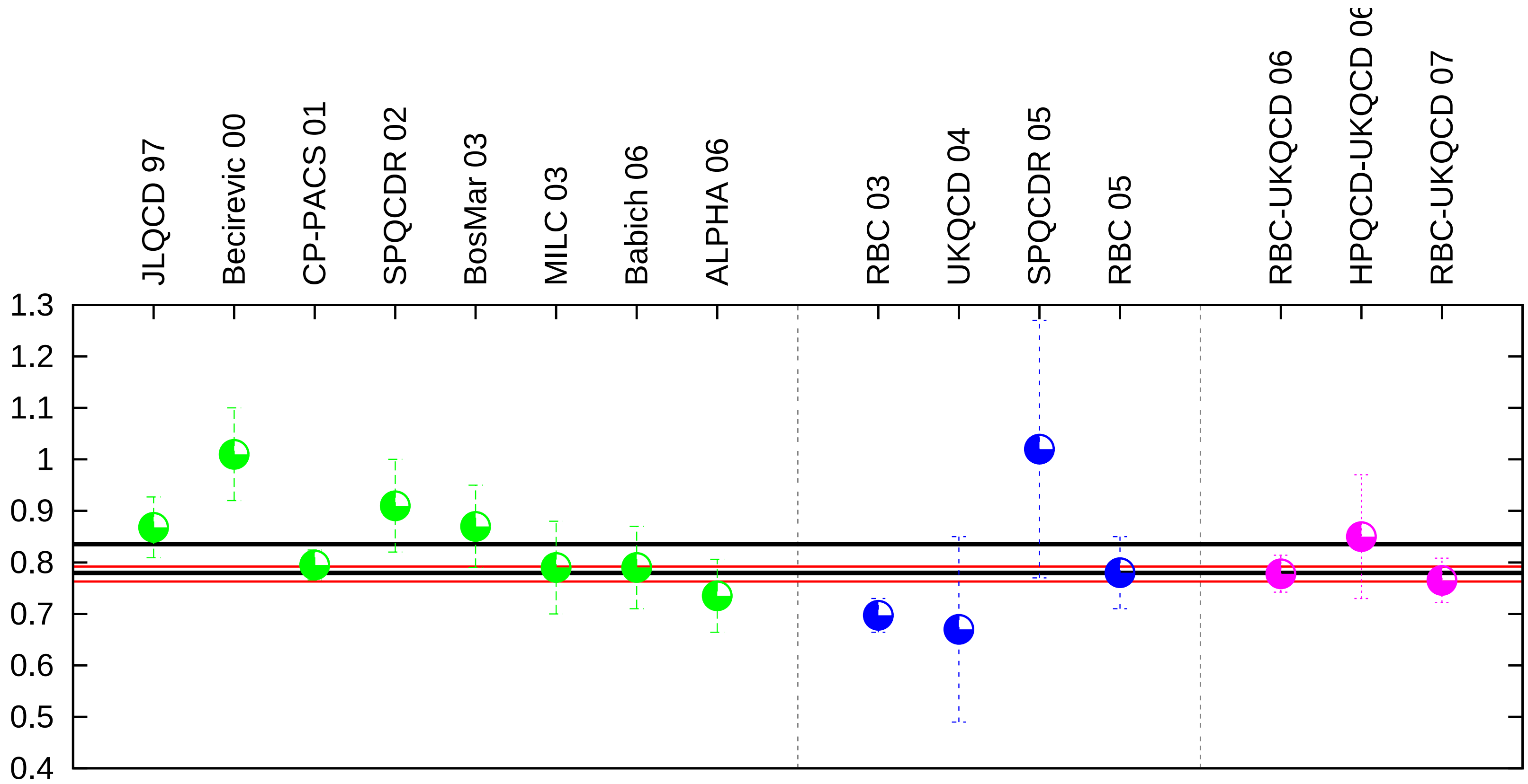
- $\mathcal{O}_{VA+AV}$ protected from mixing
- WME is not improved $\Rightarrow O(a)$ artefacts
- tuning of the mixing angle: $m_{\rm crit}$
- quenched result!!

$$\boxed{\hat{B}_K = 0.74(7)}$$



[Dimopoulos et al 07]

Kaon mixing – summary



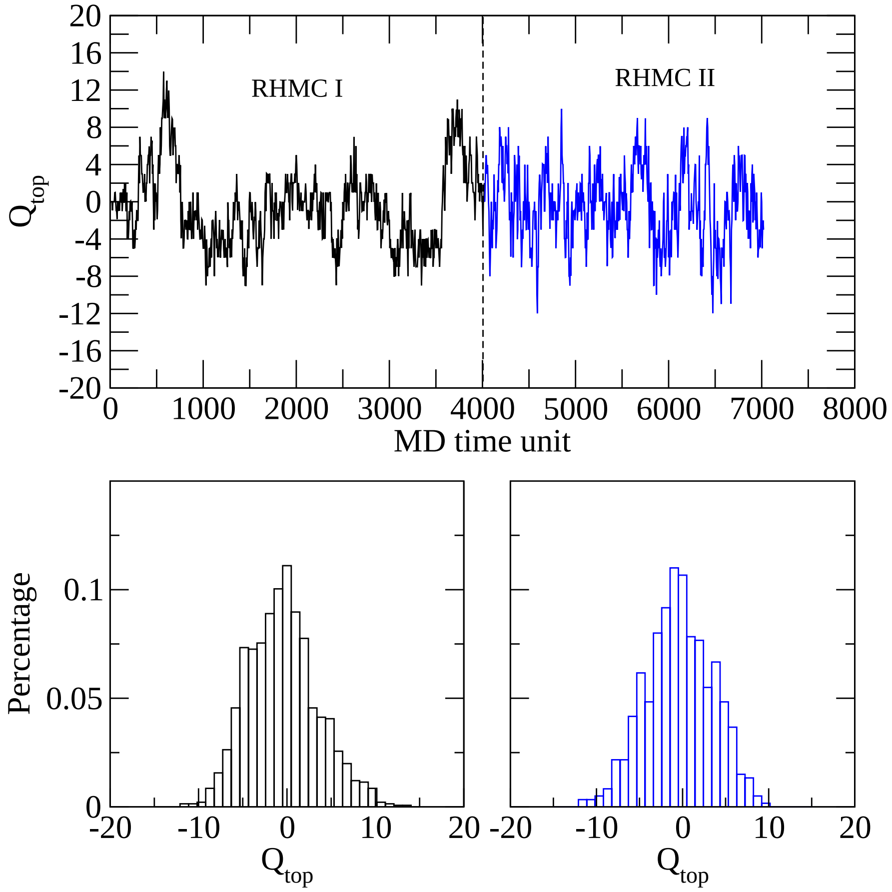
updated from [Tantalo CKM06]

Theta dependence

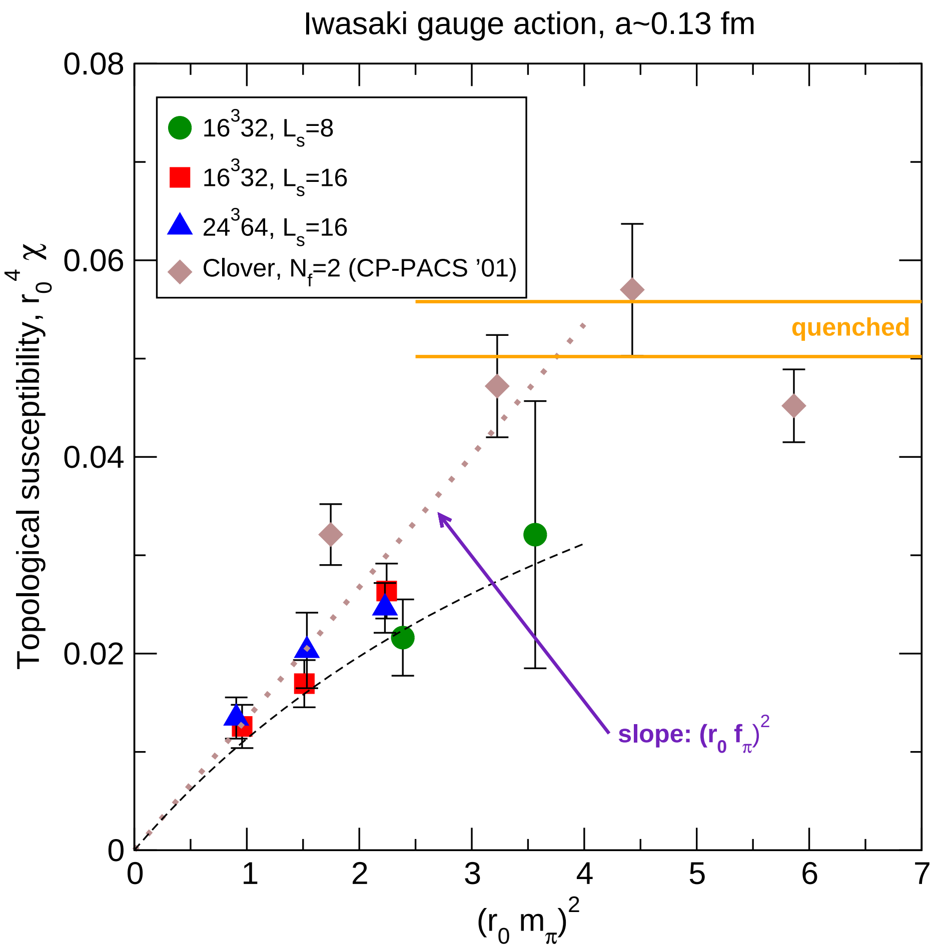
$$\begin{aligned} Z(\theta) &= \int [dA] \exp\left[- \left(\int \frac{1}{4g^2} G^2 + i\,\theta\nu \right) \right], \qquad \nu = \int Q(x) = \frac{1}{32\pi^2} \int G\tilde{G} \\ &= \exp\left[-V F(\theta) \right] \\ f(\theta) &= r_0^4\, F(\theta) \\ &= \frac{1}{2} (r_0^4 \chi)\, \theta^2 \left[1 + b_2 \theta^2 + \ldots \right] \end{aligned}$$

- complex weight in Euclidean path integral
- anomaly connects the flavour and gauge sectors
- θ dependence cancels for $m = 0$: $\chi = f_\pi^2 m_\pi^2 + \ldots$
- axial Ward Identities: WV formula, U(1)$_A$ sector phenomenology
- lattice AWIs using GW... [Luscher 98, Giusti et al 02f]
- ...if fermionic estimators are used; heuristic studies with bosonic estimators
  [Teper, Di Giacomo et al 90f, Idd et al 02f, Giusti et al 07, RBC/UKQCD 06f, ...]

Topology sampling



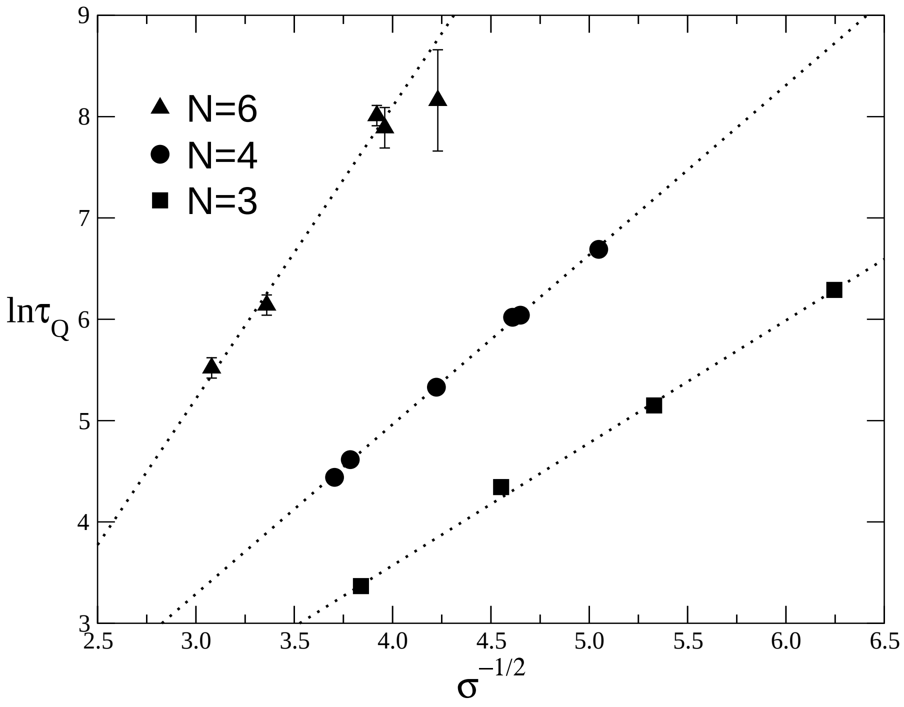
[Allton et al 07]



[Hart, ldd]

Fixed topology

Severe critical slowing down



[Idd et al 02]

- work at fixed topology – violates cluster property

  $$\boxed{\langle O\rangle_Q = \langle O\rangle_\theta + O(1/V)}$$

  exact formula can be worked out for each correlator at fixed Q

  [Aoki et al 07]

- improve the tunneling of the HMC
  still needs to be explored in detail – acceptance

  [Golterman & Shamir 07]

Conclusions

- lattice QCD has entered the era of dynamical simulations
- many interesting effects of light sea quark will be seen
- taming of systematics is crucial for precision physics

- faster machines will be available
- next two/three years: lighter masses, bigger volumes
- study of the continuum limit
- precision to impact on SM phenomenology