Phase space constraints and statistical jet studies in heavy-ion collisions

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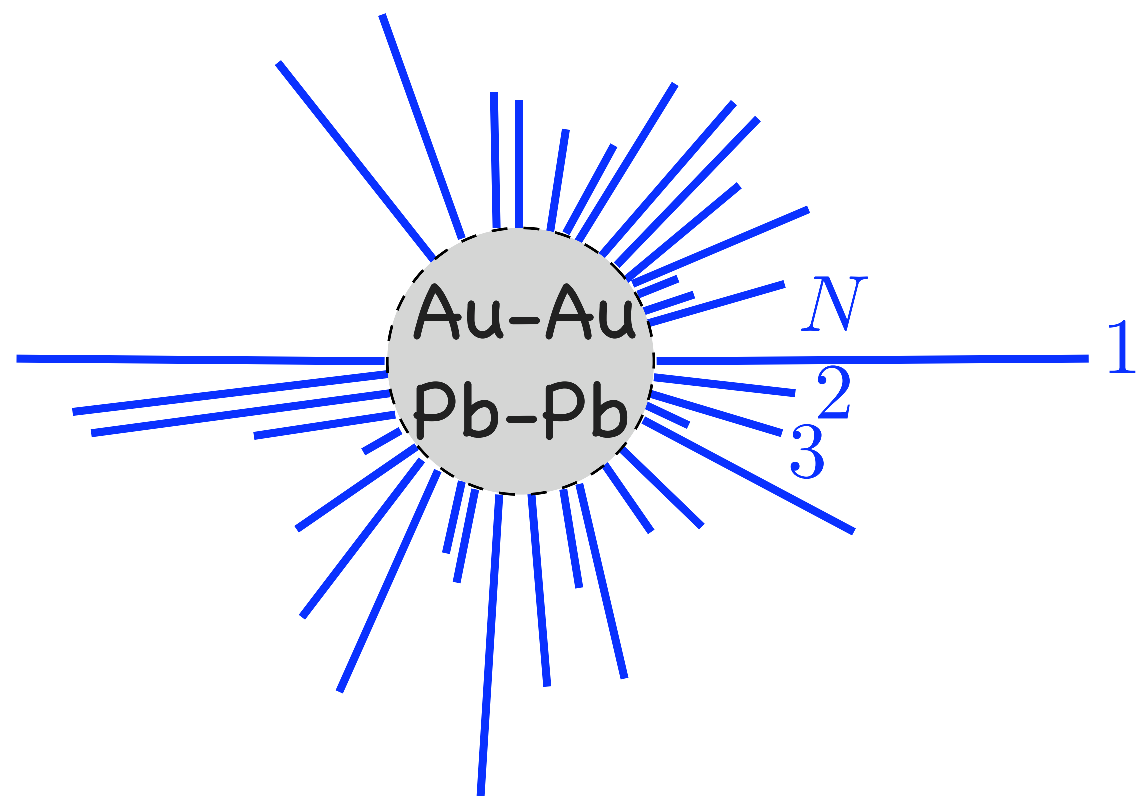
A well-defined mathematical problem...

Consider N particles constrained by (total) momentum conservation:

for instance, in the center-of-mass frame of the colliding nuclei, the N particles emitted in a Au-Au collision satisfy $\mathbf{p}_1 + \mathbf{p}_2 + \cdots + \mathbf{p}_N = \mathbf{0}$.



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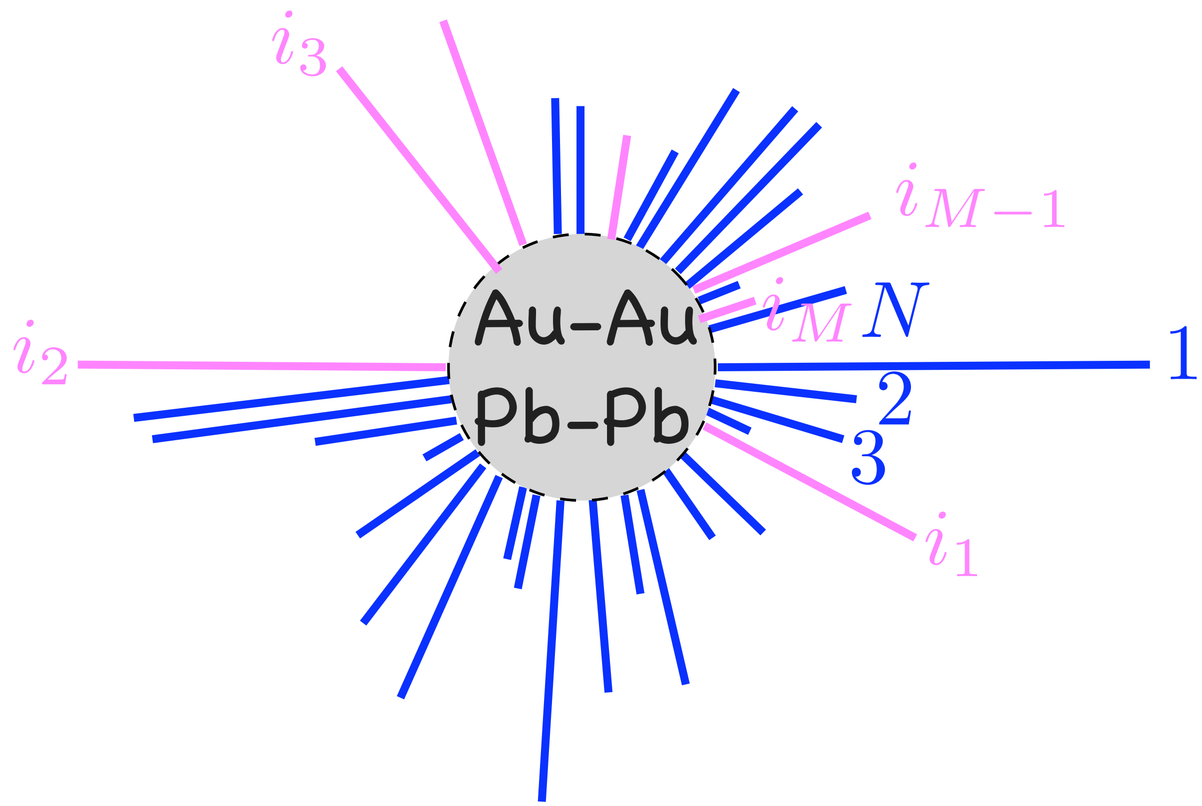

What is the correlation between M arbitrary particles induced by the momentum-conservation constraint?

An old idea...

PHYSICAL REVIEW D VOLUME 6, NUMBER 11 1 DECEMBER 1972

Azimuthal Correlations of High-Energy Collision Products

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(Received 24 July 1972)

Experimental distributions of azimuthal angles between particles produced in pp and pd collisions at 28 GeV/c and K^-p collisions at 9 GeV/c are presented and studied.

The study of two-particle correlations is a natural step beyond the investigation of single-particle distributions.[1,2] Such a study could be very useful in clarifying our understanding of multiple-particle production in high-energy collisions.

In this paper we concentrate on azimuthal correlations, that is, distributions $d\sigma/d\phi_{ij}$ where ϕ_{ij} is the angle between transverse momenta $\vec{k}_i$ and $\vec{k}_j$ of two final-state particles.

The main goal of our study is to identify the correlations which arise simply from momentum conservation and the experimentally observed damping of transverse momenta.

An old idea...

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Azimuthal Correlations of High-Energy Collision Products

II. MOMENTUM-CONSERVATION CONSTRAINT

We consider the azimuthal distribution $d\sigma^n/d\phi$ in a general reaction with n particles in the final state. Transverse-momentum conservation imposes some constraints on this distribution. Denoting the transverse momentum of the ith particle by $\vec{k}_i$, we see that transverse momentum conservation gives the condition

$$\sum_i k_i^{\,2} + \sum_{i\neq j} \vec{k}_i \cdot \vec{k}_j = 0 \ .$$

Upon averaging over all particles, we find $n\langle k_i^{\,2}\rangle + n(n-1)\langle \vec{k}_i \cdot \vec{k}_j\rangle = 0$, which suggests that $\langle \cos\phi\rangle \approx -1/(n-1)$ and that a distribution $d\sigma^n/d\phi$ might be expected to peak at $\phi = \pi$, the peak becoming less pronounced as n increases.

$$\frac{d\sigma^n}{d\phi} \equiv \sum_{i\neq j} \frac{d\sigma^n}{d\phi_{ij}}$$

Total momentum conservation and statistical studies of jets

- A few useful definitions and properties
  - probability distributions, cumulants, generating functions...

- Multiparticle correlation induced by total momentum conservation
  - a general, model-independent calculation

Eur. Phys. J. C **30** (2003) 381

- Focus on two- and three-particle correlations due to total momentum conservation: looking for a "minimally-biased reference" for jet studies

Phys. Rev. C **75** (2007) 021904(R); PoS (LHC07) 013

Multiparticle distributions & cumulants

- M-particle probability distribution $f(\mathbf{p}_{i_1}, \ldots, \mathbf{p}_{i_M})$:

probability that particles $\{i_1, i_2, \ldots, i_M\}$ have momenta $\mathbf{p}_{i_1}, \mathbf{p}_{i_2}, \ldots, \mathbf{p}_{i_M}$ irrespective of the momenta of the $N-M$ other particles.

☞ normalized to unity: $f(\{\mathbf{p}_{i_k}\}) = \mathcal{O}(1), \ \forall M$

A useful mathematical tool:

Generating function of the probability distribution:

$$G(x_1, \ldots, x_N) = 1 + x_1\, f(\mathbf{p}_1) + x_2\, f(\mathbf{p}_2) + \ldots + x_1 x_2\, f(\mathbf{p}_1, \mathbf{p}_2) + \ldots$$

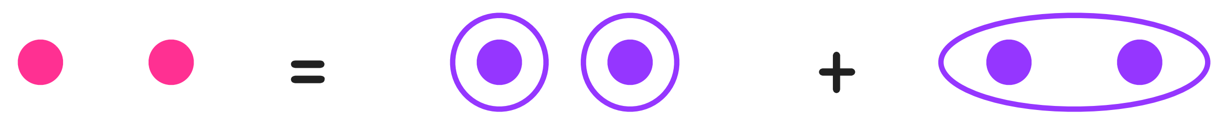
$x_1, \ldots, x_N$ auxiliary (complex) variables

Independent particles: $f(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_N) = f(\mathbf{p}_1)\, f(\mathbf{p}_2) \cdots f(\mathbf{p}_N)$

Multiparticle distributions & cumulants

- M-particle cumulant of the probability distribution $f_c(\mathbf{p}_{i_1}, \ldots, \mathbf{p}_{i_M})$: connected part of the probability distribution, responsible for the "correlations" (= deviations from statistical independence)

$$f(\mathbf{p}_1, \mathbf{p}_2) = f_c(\mathbf{p}_1)\, f_c(\mathbf{p}_2) + f_c(\mathbf{p}_1, \mathbf{p}_2)$$



(note: $f(\mathbf{p}) = f_c(\mathbf{p})$ ...)

At the three-particle level:

In the following, I shall also use "reduced cumulants"

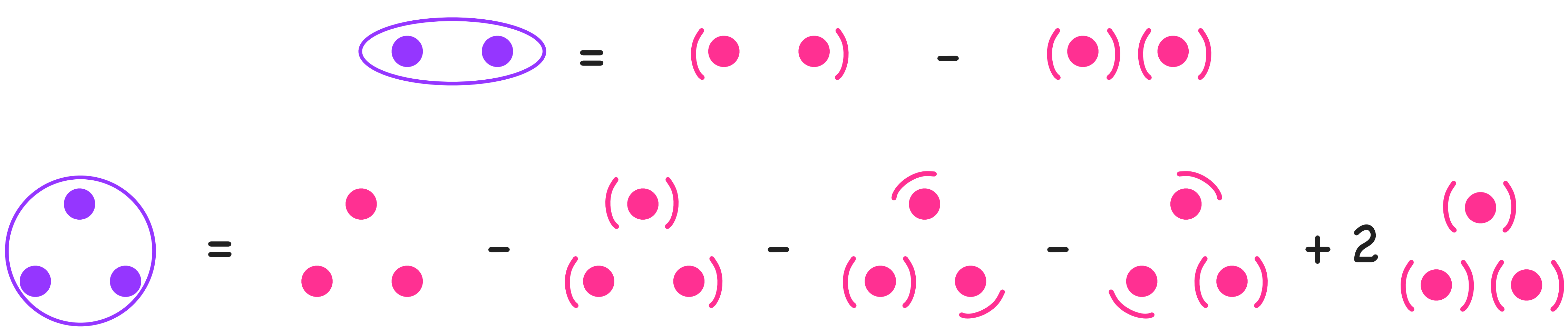
$$\bar{f}_c(\mathbf{p}_1, \ldots, \mathbf{p}_M) \equiv \frac{f_c(\mathbf{p}_1, \ldots, \mathbf{p}_M)}{f(\mathbf{p}_1) \cdots f(\mathbf{p}_M)}$$

Multiparticle distributions & cumulants

Generating function of the cumulants:

$$\ln G(x_1,\ldots,x_N) = x_1\, f_c(\mathbf{p}_1) + x_2\, f_c(\mathbf{p}_2) + \ldots + x_1 x_2\, f_c(\mathbf{p}_1,\mathbf{p}_2) + \ldots$$

automatically performs the inversions:



and so on...

One can show that for a system made of independent sub-systems (or with short-range correlations only), the cumulants scale like

$$f_c(\mathbf{p}_{i_1},\ldots,\mathbf{p}_{i_M}) = \mathcal{O}\left(\frac{1}{N^{M-1}}\right)$$

Multiparticle distributions & cumulants induced by total momentum conservation

Total momentum conservation and M-particle distribution

In the presence of the constraint from total momentum conservation, the M-particle probability distribution reads:

$$f(\mathbf{p}_1,\ldots,\mathbf{p}_M) \equiv \frac{\left(\prod_{j=1}^{M} F(\mathbf{p}_j)\right)\int \delta^D(\mathbf{p}_1+\cdots+\mathbf{p}_N)\prod_{j=M+1}^{N}\left[F(\mathbf{p}_j)\,\mathrm{d}^D\mathbf{p}_j\right]}{\int \delta^D(\mathbf{p}_1+\cdots+\mathbf{p}_N)\prod_{j=1}^{N}\left[F(\mathbf{p}_j)\,\mathrm{d}^D\mathbf{p}_j\right]}$$

which one then inserts in the generating function...

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in the absence of constraint

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M-independent denominator $\equiv 1/\mathcal{C}_D$

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M-independent denominator $\equiv 1/\mathcal{C}_D$

$$\int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D}\prod_{j=1}^{N} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}_j}$$

which one then inserts in the generating function...

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Generating function

Introducing the notation $\langle g(\mathbf{p})\rangle \equiv \int g(\mathbf{p})F(\mathbf{p})\,\mathrm{d}^D\mathbf{p}$, one finds:

$$G(x_1,\ldots,x_N) = \mathcal{C}_D \int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D} \langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle^N \exp\left(\sum_{j=1}^N x_j F(\mathbf{p}_j)\frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}_j}}{\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle}\right)$$

$$= \mathcal{C}_D \int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D} \exp\left[N \underbrace{\left(\ln\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle + \sum_{j=1}^N \frac{\bar{x}_j}{N}\frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}_j}}{\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{p}}\rangle}\right)}_{\mathcal{F}(\mathbf{k})}\right]$$

One can show (using a saddle-point method) that

$$G(x_1,\ldots,x_N) \propto \mathrm{e}^{N\mathcal{F}(\mathbf{k}_0)}\left(1+\sum_{q>l}\frac{x^l}{N^q}\right)$$

saddle-point

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saddle-point

Cumulants

The generating function of cumulants thus reads

$$\ln G(\bar{x}_1,\ldots,\bar{x}_N) = \underbrace{\ln \mathcal{C}_D}_{\text{independent of } \bar{x}} + \underbrace{N\mathcal{F}(\mathbf{k}_0)}_{\text{function of } \frac{\bar{x}}{N}} + \ln\left(\text{function of } \frac{\bar{x}^l}{N^{q\geq l}}\right)$$

$\mathcal{F}$ only depends on $\dfrac{\bar{x}}{N}$

$\mathbf{k}_0$ function of $\dfrac{\bar{x}}{N}$ (solution of $\mathcal{F}'(\mathbf{k}_0)=0$)

Hence the (scaled) cumulants:

$$\bar{f}_c(\mathbf{p}_{i_1},\ldots,\mathbf{p}_{i_M}) = \begin{array}{c}\text{coef. of } \bar{x}_{i_1}\cdots\bar{x}_{i_M} \\ \text{in } N\mathcal{F}(\mathbf{k}_0)\end{array} + \mathcal{O}\left(\frac{1}{N^M}\right) = \mathcal{O}\left(\frac{1}{N^{M-1}}\right)$$

The cumulants arising from total momentum conservation follow the same scaling behaviour as those from short-range correlations!

N.B. 2003

Computing the first cumulants

- The saddle-point $\mathbf{k}_0$ is given by $\boxed{\mathcal{F}'(\mathbf{k}_0) = 0,}$ i.e.

$$\left(\sum_{j=1}^{N} \frac{\bar{x}_j}{N} \frac{\mathrm{e}^{\mathrm{i}\mathbf{k}_0\cdot\mathbf{p}_j}}{\langle \mathrm{e}^{\mathrm{i}\mathbf{k}_0\cdot\mathbf{p}} \rangle} - 1 \right) \langle \mathbf{p}\, \mathrm{e}^{\mathrm{i}\mathbf{k}_0\cdot\mathbf{p}} \rangle = \sum_{j=1}^{N} \frac{\bar{x}_j}{N} \mathbf{p}_j \mathrm{e}^{\mathrm{i}\mathbf{k}_0\cdot\mathbf{p}_j}$$

- The cumulants are given by $\boxed{\ln G(\bar{x}_1, \ldots, \bar{x}_N) = N\mathcal{F}(\mathbf{k}_0)}$

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To lowest order*, $\mathrm{i}\mathbf{k}_0 = -\dfrac{D}{\langle \mathbf{p}^2\rangle} \displaystyle\sum_{j=1}^{N} \frac{\bar{x}_j}{N} \mathbf{p}_j$, hence

$$\mathcal{F}(\mathbf{k}_0) = \sum_{j=1}^{N} \frac{\bar{x}_j}{N} - \frac{D}{2\langle \mathbf{p}^2\rangle} \left(\sum_{j=1}^{N} \frac{\bar{x}_j}{N} \mathbf{p}_j\right)^2$$

which gives $\bar{f}_c(\mathbf{p}_1, \mathbf{p}_2) = -\dfrac{D\,\mathbf{p}_1\cdot\mathbf{p}_2}{N\langle \mathbf{p}^2\rangle}$, of order $\mathcal{O}\left(\dfrac{1}{N}\right)$ as expected

* assuming $F(\mathbf{p})$ isotropic, so that $\langle \mathbf{p}\rangle = 0$ and $\langle(\mathbf{k}_0\cdot\mathbf{p})^2\rangle = \mathbf{k}_0^2\langle \mathbf{p}^2\rangle/D$

Computing the first cumulants

Going to the next order in $\dfrac{\bar{x}}{N}$, the generating function $\ln G(\bar{x}_1,\ldots,\bar{x}_N)$ yields the 3-particle cumulant:

$$\bar{f}_c(\mathbf{p}_1,\mathbf{p}_2) = -\frac{D\,\mathbf{p}_1\cdot\mathbf{p}_2}{N\langle\mathbf{p}^2\rangle}$$

Back-to-back correlation, larger for particles with larger momenta

$$\begin{aligned}\bar{f}_c(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3) &= -\frac{D}{N^2\langle\mathbf{p}^2\rangle}(\mathbf{p}_1\cdot\mathbf{p}_2+\mathbf{p}_1\cdot\mathbf{p}_3+\mathbf{p}_2\cdot\mathbf{p}_3)\\ &\quad+\frac{D^2}{N^2\langle\mathbf{p}^2\rangle^2}\left[(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_3)+(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{p}_2\cdot\mathbf{p}_3)\right.\\ &\quad\left.+(\mathbf{p}_1\cdot\mathbf{p}_3)(\mathbf{p}_2\cdot\mathbf{p}_3)\right]\end{aligned}$$

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Want to relax the "isotropic emission" assumption? (take D = 3)

$$\bar{f}_c(\mathbf{p}_1,\mathbf{p}_2) = -\frac{p_{1,x}p_{2,x}}{N\langle p_x^2\rangle}-\frac{p_{1,y}p_{2,y}}{N\langle p_y^2\rangle}-\frac{p_{1,z}p_{2,z}}{N\langle p_z^2\rangle}$$

x,y,z principal axes of the $\langle\mathbf{p}\otimes\mathbf{p}\rangle$ tensor

N.B. 2003, Chajęcki & Lisa 2006

Total momentum conservation induces correlations between any number of final-state particles

So what?

Multiparticle probability distributions

Let me use more precise definitions:

- Marginal M-particle probability distribution $f(\mathbf{p}_{i_1},\ldots,\mathbf{p}_{i_M})$: probability that particles $\{i_1, i_2, \ldots, i_M\}$ have momenta $\mathbf{p}_{i_1}, \mathbf{p}_{i_2}, \ldots, \mathbf{p}_{i_M}$ irrespective of the momenta of the $N-M$ other particles.

- Conditional M-particle probability distribution: probability that particles $\{i_1, i_2, \ldots, i_M\}$ have momenta $\mathbf{p}_{i_1}, \mathbf{p}_{i_2}, \ldots, \mathbf{p}_{i_M}$ provided the momenta of the $N-M$ other particles take definite values

☞ $f(\mathbf{p}_{i_1},\ldots,\mathbf{p}_{i_M} \,|\, \mathbf{p}_{i_{M+1}},\ldots,\mathbf{p}_{i_N})$

which can be integrated over $\mathbf{p}_{i_{M+1}}, \ldots, \mathbf{p}_{i_N}$:

$$f(\mathbf{p}_{i_1},\ldots,\mathbf{p}_{i_M}) = \int f(\mathbf{p}_{i_1},\ldots,\mathbf{p}_{i_M} \,|\, \mathbf{p}_{i_{M+1}},\ldots,\mathbf{p}_{i_N})\, \mathrm{d}\mathbf{p}_{i_{M+1}}\ldots\mathrm{d}\mathbf{p}_{i_N}$$

⚠ In the presence of correlations (= non-vanishing cumulants), marginal and conditional probabilities differ!

Two-particle correlation due to total (transverse) momentum conservation

In an event with $N \gg 1$ particles in the final state, the conservation of total momentum induces a correlation between 2 arbitrary outgoing particles, so that the two-particle probability distribution reads:

$$f(\mathbf{p}_1, \mathbf{p}_2) = f(\mathbf{p}_1) f(\mathbf{p}_2) \left(1 - \frac{p_{1,x}p_{2,x}}{N\langle p_x^2 \rangle} - \frac{p_{1,y}p_{2,y}}{N\langle p_y^2 \rangle} - \frac{p_{1,z}p_{2,z}}{N\langle p_z^2 \rangle} \right)$$

Considering for the sake of simplicity central nucleus-nucleus collisions (isotropic particle emission: $\langle p_x^2 \rangle = \langle p_y^2 \rangle = \langle p_T^2 \rangle / 2$) and neglecting the longitudinal z-term (because $\langle p_z^2 \rangle \gg \langle p_x^2 \rangle, \langle p_y^2 \rangle$; additionally, we can focus on particles emitted close to mid-rapidity), this yields

$$f(\mathbf{p}_{T1}, \mathbf{p}_{T2}) = f(\mathbf{p}_{T1}) f(\mathbf{p}_{T2}) \left(1 - \frac{2\, p_{T1} p_{T2} \cos(\varphi_2 - \varphi_1)}{N\langle p_T^2 \rangle} \right)$$

i.e. $$f(\mathbf{p}_{T2} | \mathbf{p}_{T1}) \equiv \frac{f(\mathbf{p}_{T2}, \mathbf{p}_{T1})}{f(\mathbf{p}_{T1})} = f(\mathbf{p}_{T2}) \left(1 - \frac{2\, p_{T1} p_{T2} \cos(\varphi_2 - \varphi_1)}{N\langle p_T^2 \rangle} \right)$$

$$\neq f(\mathbf{p}_{T2})$$

Two-particle correlation due to total transverse momentum conservation

Thus, given a first "trigger" particle with transverse momentum $\mathbf{p}_{T1}$, then the conditional probability to find a second "associated" particle with transverse momentum $\mathbf{p}_{T2}$ is NOT given by the (marginal) single-particle probability distribution (nor by a "minimum bias" version).

For instance, even if the emission is a priori isotropic, the probability for $\mathbf{p}_{T2}$ is larger "away" (in azimuth) from $\mathbf{p}_{T1}$.

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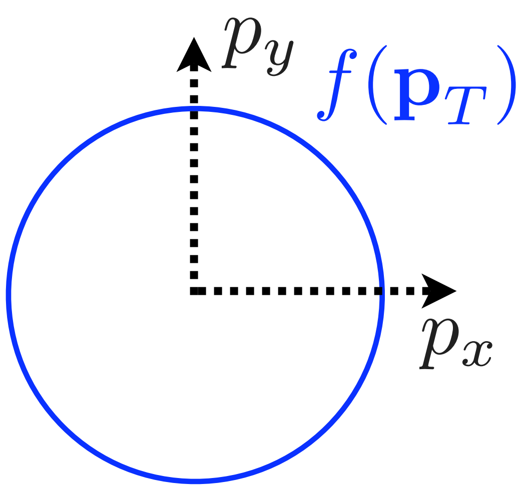

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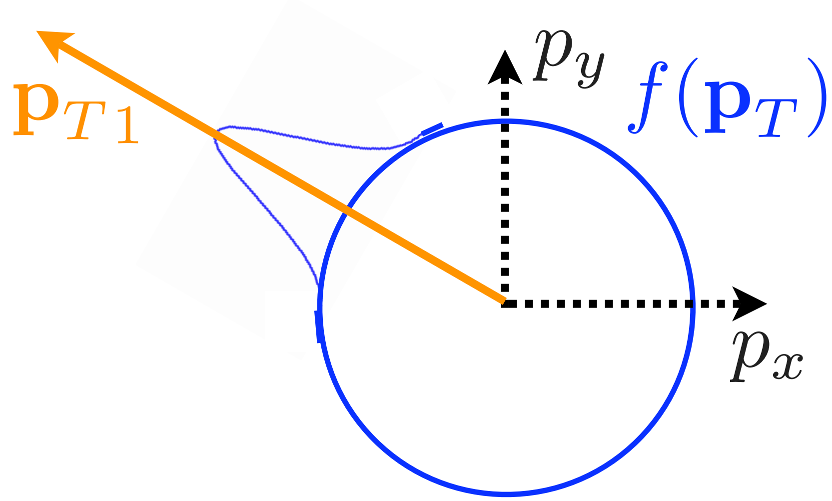

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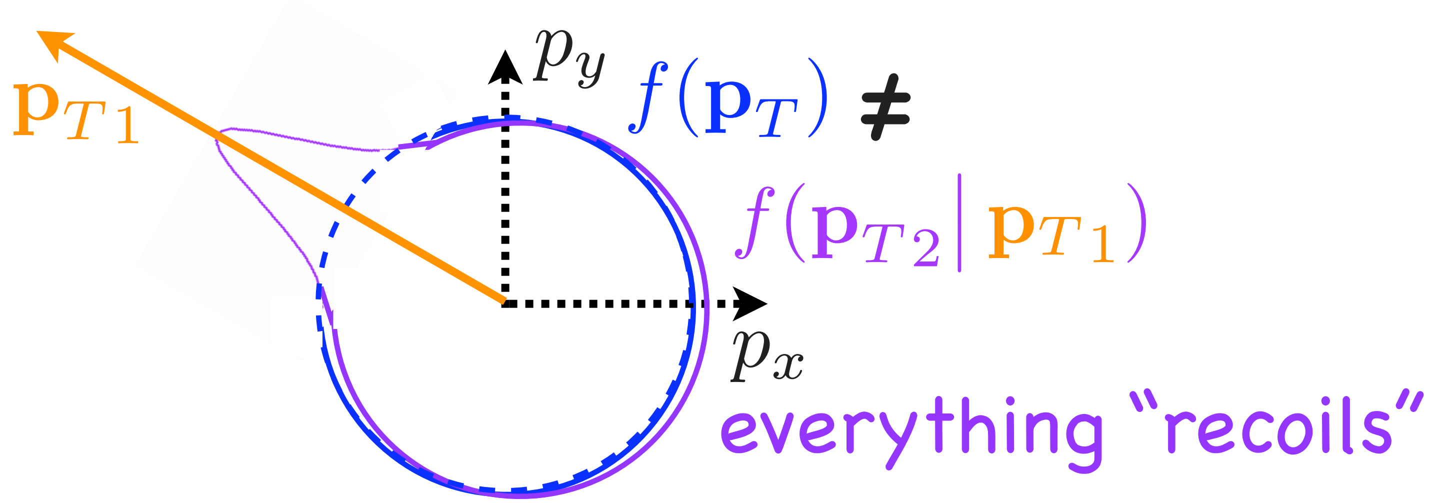

One cannot speak of "a jet + an (uncorrelated) background event"!

Two-particle correlation due to total transverse momentum conservation

Because of global (transverse)-momentum conservation, the probability distribution of particles "associated" to a "trigger" differs from the single-particle probability distribution:

$$f(\mathbf{p}_{T2}|\,\mathbf{p}_{T1}) = f(\mathbf{p}_{T2})\left(1 - \frac{2\,p_{T1}p_{T2}\cos(\varphi_2-\varphi_1)}{N\langle p_T^2\rangle}\right)$$

The difference increases with both p_{T1} and p_{T2}, and decreases with increasing number of final state particles N.

Similarly, at the three-particle level:

$$\begin{aligned} f(\mathbf{p}_{T2},\mathbf{p}_{T3}|\,\mathbf{p}_{T1}) &= f(\mathbf{p}_{T2})f(\mathbf{p}_{T3}) \\ &\quad\times\Big[1+\bar{f}_c(\mathbf{p}_{T2},\mathbf{p}_{T3})+\bar{f}_c(\mathbf{p}_{T1},\mathbf{p}_{T3})+\bar{f}_c(\mathbf{p}_{T1},\mathbf{p}_{T2}) \\ &\qquad+\bar{f}_c(\mathbf{p}_{T1},\mathbf{p}_{T2},\mathbf{p}_{T3})\Big] \\ &\neq f(\mathbf{p}_{T2})f(\mathbf{p}_{T3}) \end{aligned}$$

Conditional and marginal probability distributions are different...

Because of global (transverse)-momentum conservation, the probability distribution of particles "associated" to a "trigger" differs from the single-particle probability distribution.

As a consequence, the average transverse momentum of associated particles restricted to an angular sector away from the trigger is always larger than the average transverse momentum of the whole event:

$$\langle p_T\rangle_{\rm assoc.} = \int_{\pi-\theta}^{\pi+\theta} \frac{{\rm d}(\varphi_2-\varphi_1)}{2\theta} \int {\rm d}p_{T\,2} f\big(\mathbf{p}_{T\,2} \big| \, \mathbf{p}_{T\,1}\big) = \langle p_T\rangle_{\rm all} + \frac{2p_{T\,1}}{N}\frac{\sin\theta}{\theta}$$

(note that the difference between $\langle p_T\rangle_{\rm assoc.}$ and $\langle p_T\rangle_{\rm all}$ depends on the trigger-particle transverse momentum $p_{T\,1}$).

But this does not reflect any dynamics!

Conditional and marginal probability distributions are different...

In the case of an anisotropic transverse emission of particles, characterized by $\bar{v}_2 \equiv \dfrac{\langle p_x^2 - p_y^2\rangle}{\langle p_x^2 + p_y^2\rangle}$, one finds

$$\begin{aligned} f(\mathbf{p}_{T\,2}|\,\mathbf{p}_{T\,1}) &= f(\mathbf{p}_{T\,2})\left[1 - \frac{2}{N\langle p_T^2\rangle}\left(\frac{p_{1,x}p_{2,x}}{1+\bar{v}_2} + \frac{p_{1,y}p_{2,y}}{1-\bar{v}_2}\right)\right] \\ &= f(\mathbf{p}_{T\,2})\left[1 - \frac{2p_{T\,1}p_{T\,2}}{N\langle p_T^2\rangle}(\cos(\varphi_1-\varphi_2) - \bar{v}_2\cos(\varphi_1+\varphi_2))\right] \end{aligned}$$

The size of the "bump" away from the trigger ($\varphi_1-\varphi_2=\pi$) is larger for out-of-plane ($\varphi_1 = \dfrac{\pi}{2}$ mod π) than for in-plane ($\varphi_1 = 0$ mod π) trigger particles.

But this does not reflect any dynamics!

Total momentum conservation and statistical studies of jets

Total momentum conservation induces correlations between the particles emitted in a collision.

These correlations can be computed... and their value can be estimated if one "knows" the total emitted multiplicity N and the mean square momentum $\langle \mathbf{p}^2 \rangle$.

☞ can be treated as parameters

Should we care? Yes!*

* undoubtedly at SPS, most probably at RHIC, possibly (in the end, certainly) at LHC

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The main goal of our study is to identify the correlations which arise simply from momentum conservation in order to be in position to assess quantitatively genuine dynamical effects, not something trivial, through correlation studies...

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