Unification of Flavor, CP, and Modular Symmetries

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TUM

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SFB 1258 | Neutrinos Dark Matter Messengers

Unification of Flavor, CP, and Modular Symmetries

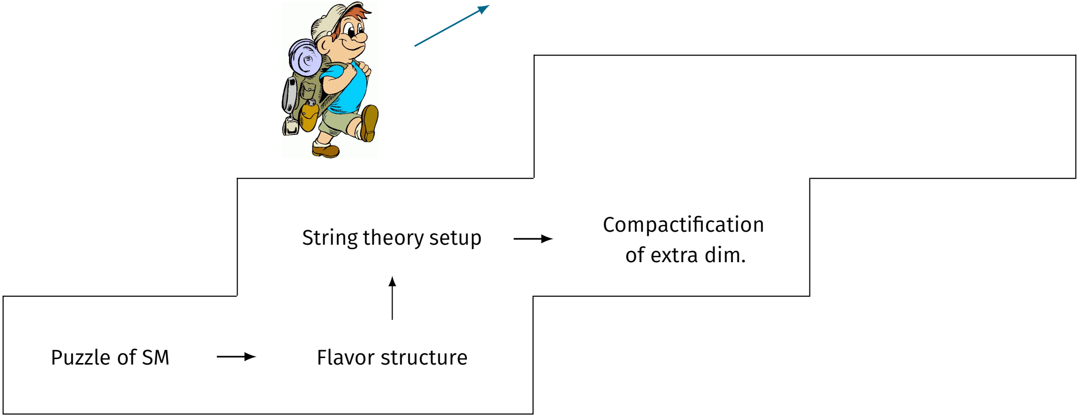
based on: A. B. , H.P. Nilles, A. Trautner, P.K.S. Vaudrevange – 1901.03251 (PLB)

Outline:

Motivation

Puzzle of SM $\longrightarrow$ Flavor structure

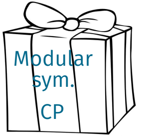
Motivation



Puzzle of SM
Flavor structure
String theory setup
Compactification
of extra dim.

Motivation

Renewed interest in modular symmetries:
Feruglio and many more



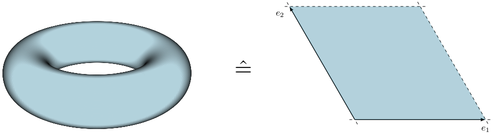
Orbifolds → Aim:
Flavor symmetry
from an orbifold

String theory setup → Compactification
of extra dim.

Puzzle of SM → Flavor structure

Orbifold (Pictures)

Torus: $\mathbb{T}^2$



Orbifold (Pictures)

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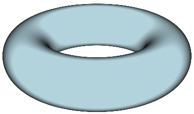

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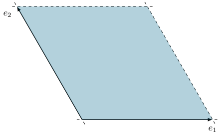
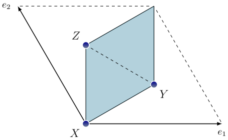


$\longrightarrow$

Orbifold: $\mathbb{T}^2/\mathbb{Z}_3$



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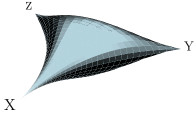

Orbifold (Pictures)

Torus: $\mathbb{T}^2$

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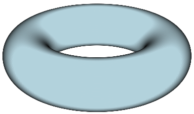

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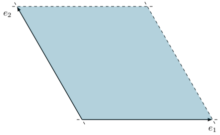

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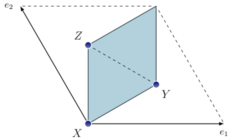

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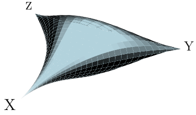
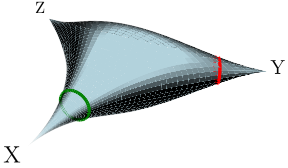
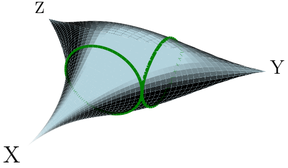


Twisted strings



Winded strings



Orbifold (Math)

Symmetry of $\mathbb{R}^d$ $\longrightarrow$ $\boxed{\text{Poincaré group}}$

Symmetry of orbifold $\longrightarrow$

Orbifold (Math)

Symmetry of $\mathbb{R}^d$ $\longrightarrow$ **Poincaré group**

$\big\downarrow$ discretize

Symmetry of orbifold $\longrightarrow$ **Space group**

- Discrete translations $\vec{t}$ → Torus
- Discrete rotations P → Orbifold
- The space group is a discrete version of the Poincaré group

ORBIFOLD (MATH)

Symmetry of $\mathbb{R}^d$ $\longrightarrow$ Poincaré group

$\downarrow$ discretize

Symmetry of orbifold $\longrightarrow$ Space group

Orbifold (Math)

Symmetry of $\mathbb{R}^d$ $\longrightarrow$ Poincaré group

$\downarrow$ discretize

Symmetry of orbifold $\longrightarrow$ Space group

$\downarrow$ strings

Narain space group

- Narain construction accounts for left and right mover
- $(d + d)$ dimensional
- The Narain space group is a stringy version of the space group

Orbifold (Math)

$$
\begin{tikzcd}
\text{Symmetry of } \mathbb{R}^d \arrow[r] & \boxed{\text{Poincaré group}} \arrow[d, "\text{discretize}"] & \\
\text{Symmetry of orbifold} \arrow[r] & \boxed{\text{Space group}} \arrow[d, "\text{strings}"] & \\
& \boxed{\text{Narain space group}} \arrow[r] & \text{Symmetry of string momenta}
\end{tikzcd}
$$

Orbifold (Math)

Symmetry of $\mathbb{R}^d$ → Poincaré group

↓ discretize

Symmetry of orbifold → Space group

↓ strings

Narain space group → Symmetry of string momenta

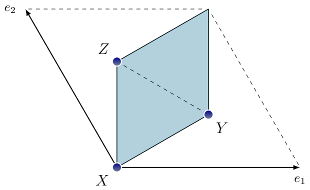
↓

Symmetry among string states

Flavor Symmetry of Orbifolds

Flavor Symmetry of Orbifolds

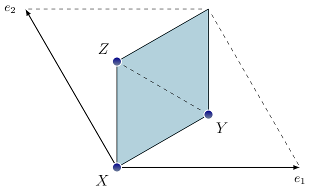
Traditional approach



Flavor Symmetry of Orbifolds

Traditional approach

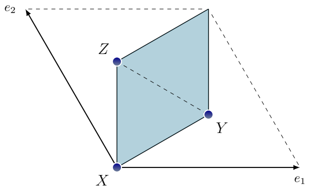
Geometrical Symmetries S_3



Flavor Symmetry of Orbifolds

Traditional approach

Geometrical Symmetries	S_3
String Selection Rules	$\mathbb{Z}_3 \times \mathbb{Z}_3$



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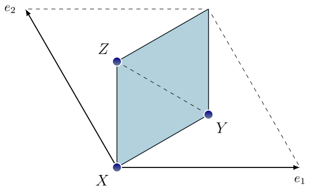

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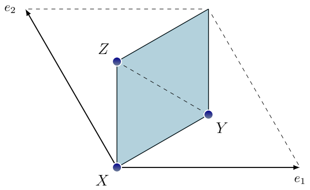

[T. Kobayashi et al.: hep-ph/0611020]

Flavor Symmetry of Orbifolds

New approach

1. Explicitly calculate the automorphisms of the Narain space group
2. Derive how string states transform under these symmetry operations

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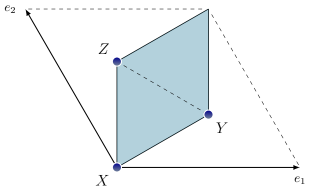
Traditional approach

Geometrical Symmetries S_3

String Selection Rules $\mathbb{Z}_3 \times \mathbb{Z}_3$

$\Delta(54)$

"Traditional Flavor Symmetry"



[T. Kobayashi et al.: hep-ph/0611020]

Narain Orbifold

Narain space group. The Narain space group can be represented by augmented matrices:

$$\left(\begin{array}{cc|c} \vartheta_{\mathrm{R}} & & t_{\mathrm{R}} \\ & \vartheta_{\mathrm{L}} & t_{\mathrm{L}} \\ \hline \multicolumn{2}{c|}{0} & 1 \end{array}\right)$$

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Narain lattice. The Narain space group acts on momenta that lie in a Narain lattice:

$$\begin{pmatrix} p_{\mathrm{R}} \\ p_{\mathrm{L}} \end{pmatrix} = \frac{e^{-T}}{\sqrt{2}} \begin{pmatrix} G-B & -\mathbb{1} \\ G+B & \mathbb{1} \end{pmatrix} \begin{pmatrix} \omega \\ n \end{pmatrix}, \qquad G = \frac{r}{2}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}, \quad B = b\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Momenta are parametrized by strings winding and Kaluza-Klein quantum numbers ω and n.

[K. S. Narain et al.: Asymmetric Orbifolds], [S. Groot Nibbelink, P. Vaudrevange: 1703.05323]

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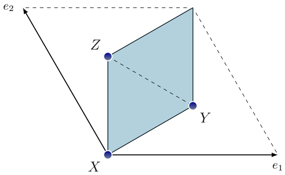
As explicit example ... choose the $\mathbb{T}^2/\mathbb{Z}_3$ orbifold with all Wilson lines turned off.



[K. S. Narain et al.: Asymmetric Orbifolds], [S. Groot Nibbelink, P. Vaudrevange: 1703.05323]

Automorphisms

Form of the automorphisms. Demand the automorphisms to be of the same form as the space group, i.e.

$$h = \left(\begin{array}{c|c} \mathrm{GL}(2d,\mathbb{R}) & \begin{matrix} t_\mathrm{R} \\ t_\mathrm{L} \end{matrix} \\ \hline 0 & 1 \end{array}\right)$$

Further conditions.

0. Automorphism of Narain space group, i.e. $G \xmapsto{h} G$
1. Preserve the Narain metric
2. Leave p_L^2 and p_R^2 invarinat

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Results.

Translation in KK number	Translation in winding number	180° rotation
$n = \frac{1}{3}\begin{pmatrix}1\\1\end{pmatrix}$	$\omega = \frac{1}{3}\begin{pmatrix}1\\2\end{pmatrix}$	$\vartheta = -\mathbb{1}_4$

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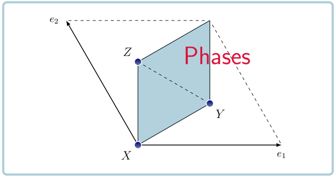
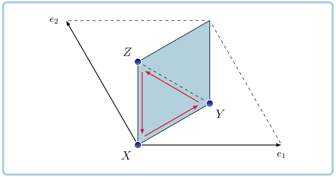
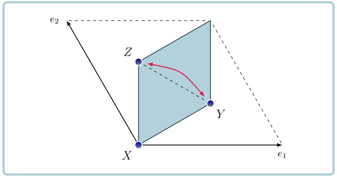
Automorphisms







[AB, H. P. Nilles, A. Trautner, P. Vaudrevange: 19xx.xxxx]

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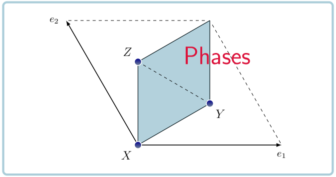
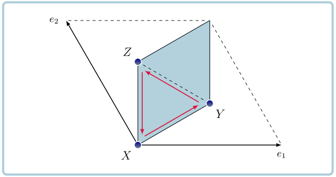
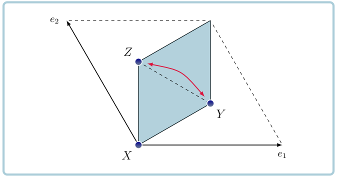
Automorphisms

String selection rules

Geometrical S_3







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Automorphisms

Flavor symmetry $\Delta(54)$

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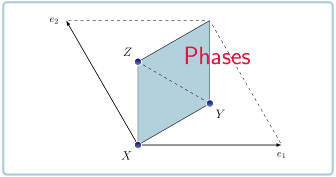

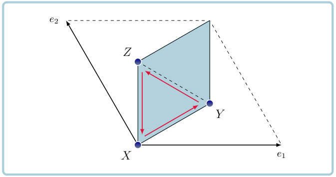

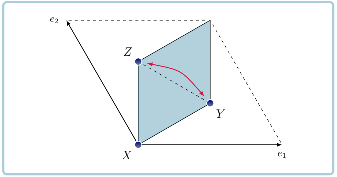

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Automorphisms

Flavor symmetry $\Delta(54)$

✔ Reproduced traditional result

String selection rules

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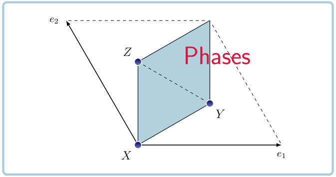

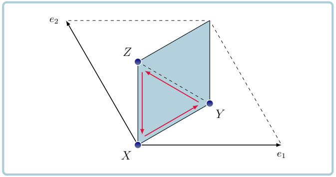

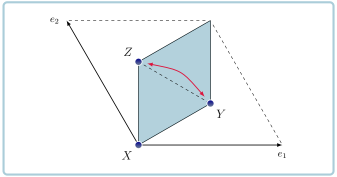

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Automorphisms

$\boxed{\text{Flavor symmetry } \Delta(54)}$

✔ Reproduced traditional result

However: There are even more automorphisms!

$\rightarrow$ Identify those as modular transformations

Translation in KK number	Translation in winding number	180° rotation
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Modular Transformations

Modular Transformations. The modular transformations of the Torus:

$$\Big[(\mathrm{SL}(2,\mathbb{Z})_\rho \times \mathrm{SL}(2,\mathbb{Z})_\tau) \rtimes (\mathbb{Z}_2 \times \mathbb{Z}_2) \Big] \Big/ \mathbb{Z}_2$$

Modular Transformations

Modular Transformations. The modular transformations of the Torus $/\,\mathbb{Z}_3$ Narain Orbifold:

$\rho = \mathrm{e}^{2\pi \mathrm{i}/3}$

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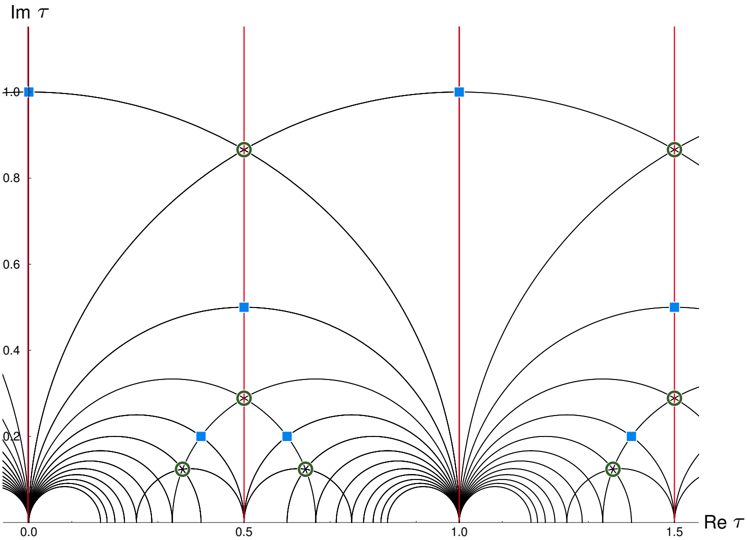
Conditions.

0. Automorphism of Narain space group, i.e. $G \overset{h}{\longmapsto} G$
1. Preserve the Narain metric
2. Leave p_{L}^2 and p_{R}^2 invarinat $\quad\Leftrightarrow\quad$ Leave moduli invariant

Modular transformations fulfill these conditions at their fixed points in moduli space!

Flavor Symmetry of $\mathbb{T}^2/\mathbb{Z}_3$ Orbifold



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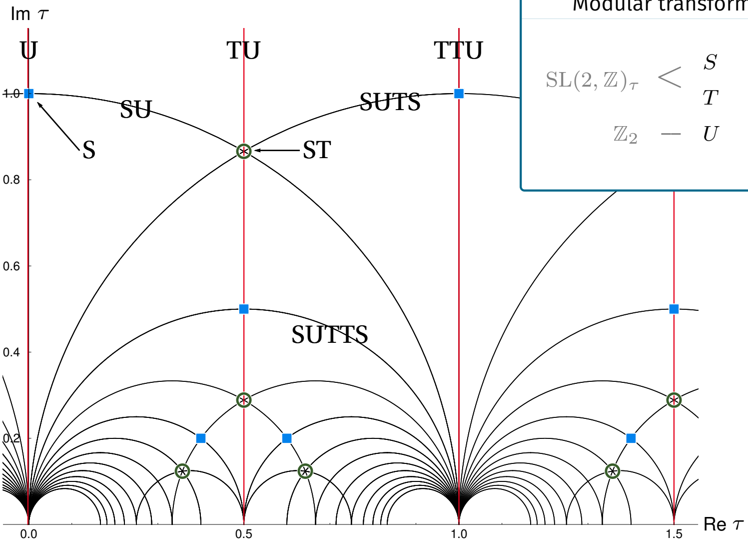
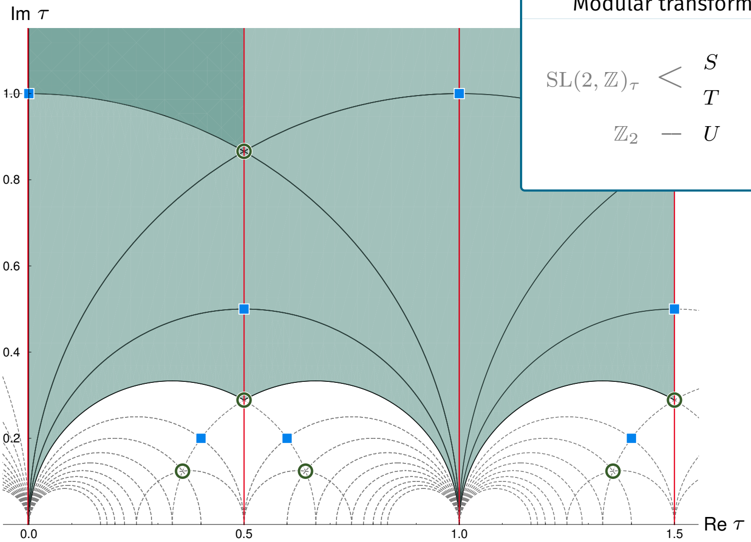


Modular transformations

$$\mathrm{SL}(2,\mathbb{Z})_\tau < \begin{array}{ll} S & \tau \mapsto -\frac{1}{\tau} \\ T & \tau \mapsto \tau + 1 \end{array}$$

$$\mathbb{Z}_2 \quad - \quad U \quad \tau \mapsto -\overline{\tau}$$

Flavor Symmetry of $\mathbb{T}^2/\mathbb{Z}_3$ Orbifold



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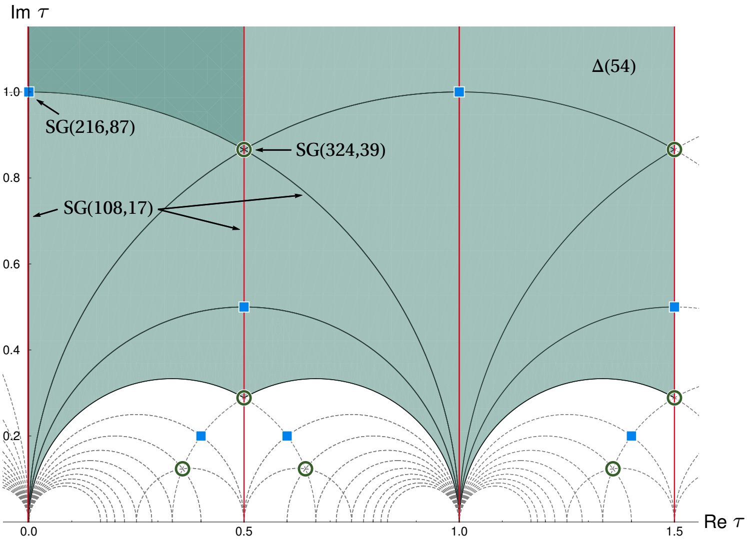

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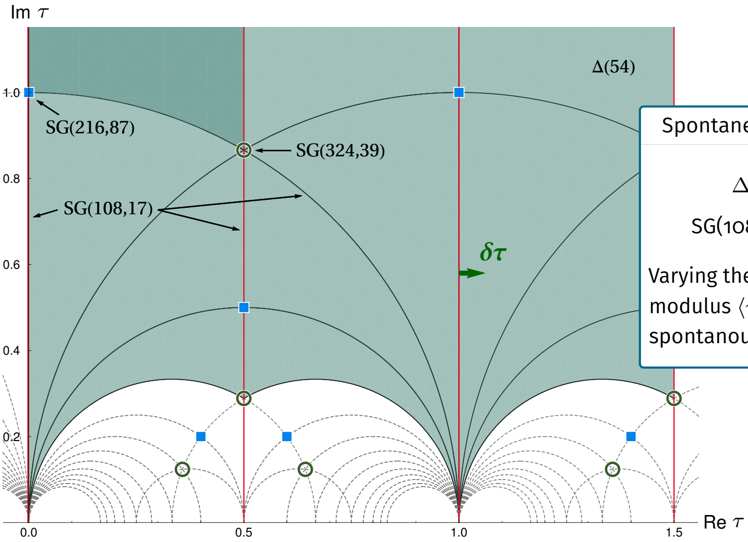

Spontaneous CP breaking

$\Delta(54)$: CP ✗

SG(108,17): CP ✓

Varying the VEV of the Kähler modulus $\langle\tau\rangle$ by $\delta\tau$ breaks CP spontaneously

Conclusions

- Designed a generic method to find flavor symmetries of orbifolds
- Traditional flavor symmetry is enhanced by modular transformations (including CP)
- However, not all modular transformations can appear as flavor symmetries
- The concept of local flavor symmetries allows different flavor groups for different sectors of the theory
- Next step: Calculate flavor symmetries of 6-dim Orbifolds