To Positivity and Beyond, where Higgs-Dilaton Inflation has never gone before

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arXiv:1905.08816

PASCOS 2019
1July 2019, Manchester, UK

Higgs-Dilaton inflation

Higgs inflation [Bezrukov & Shaposhnikov `07]
can be incorporated into a larger framework

$$S = \int d^4x\sqrt{|g|}\left(-\frac{1}{2}(2\xi_h\varphi^\dagger\varphi + \xi_\chi\chi^2)R + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\varphi,\chi)\right) + S_{\mathrm{SM}}(\lambda \to 0)$$

$$V(\varphi,\chi) = \lambda\left(\varphi^\dagger\varphi - \frac{\alpha}{2\lambda}\chi^2\right)^2 + \cancel{\beta\chi^4}$$

Dilaton χ is a singlet under the SM group

All scales are generated by SSB of global scale invariance

$$M_P \equiv \xi_h\langle\phi\rangle^2 + \xi_\chi\langle\chi\rangle^2 \propto \xi_\chi\langle\chi\rangle^2 \qquad\qquad \langle\varphi^\dagger\varphi\rangle = \frac{\alpha}{2\lambda}\langle\chi\rangle^2$$

M. Shaposhnikov, D. Zenhausern, Phys.Lett. B671 (2009) 187-192
J. García-Bellido, JR, M. Shaposhnikov, D. Zenhausern, Phys.Rev. D84 (2011) 123504
Bezrukov, Karananas, Rubio and Shaposhnikov, Phys.Rev. D87 (2013) 096001
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Higgs-Dilaton inflation

$$S = \int d^4x\sqrt{|g|}\left(-\frac{1}{2}(2\xi_h\varphi^\dagger\varphi + \xi_\chi\chi^2)R + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\varphi,\chi)\right) + S_{\mathrm{SM}}(\lambda \to 0)$$

$$V(\varphi,\chi) = \lambda\left(\varphi^\dagger\varphi - \frac{\alpha}{2\lambda}\chi^2\right)^2 + \beta\chi^4$$

Regularization scheme cannot be uniquely fixed and
predictions of the model are sensitive to the details of the UV completion

Bezrukov, Karananas, Rubio and Shaposhnikov, Phys.Rev. D87 (2013) 096001

How can we examine Higgs-Dilaton inflation as an EFT?

Amplitudes' Positivity

Adams, Arkani-Hamed, Dubovsky,
Nicolis and Rattazzi [hep-th/0602178].

- UV completion of an EFT:
  - Unitary
  - Causal
  - Local
  - Lorentz invariant

Amplitudes' Positivity

Consider an amplitude $a, b \to a, b$

$$\mathscr{A}_{ab}(s) = \mathscr{M}_{ab}(s, t = 0)$$ (forward limit)

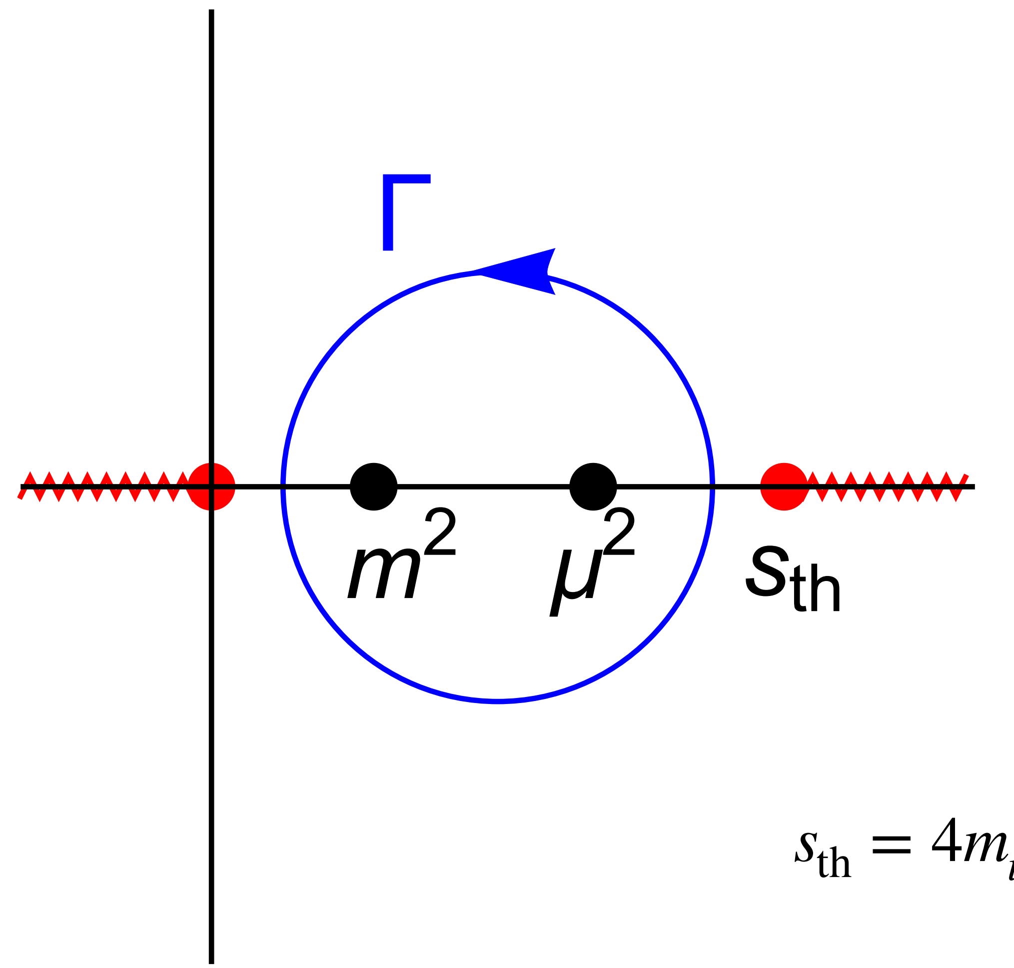
Construct

$$\Sigma_{\rm IR}^{ab} = \frac{1}{2\pi i}\oint_\Gamma ds\,\frac{\mathscr{A}^{ab}(s)}{(s-\mu^2)^3},$$

Adams, Arkani-Hamed, Dubovsky, Nicolis and Rattazzi [hep-th/0602178]

Amplitudes' Positivity

$$\Sigma^{ab}_{\mathrm{IR}} = \frac{1}{2\pi i}\oint_{\Gamma} ds\, \frac{\mathcal{A}^{ab}(s)}{(s-\mu^2)^3}$$



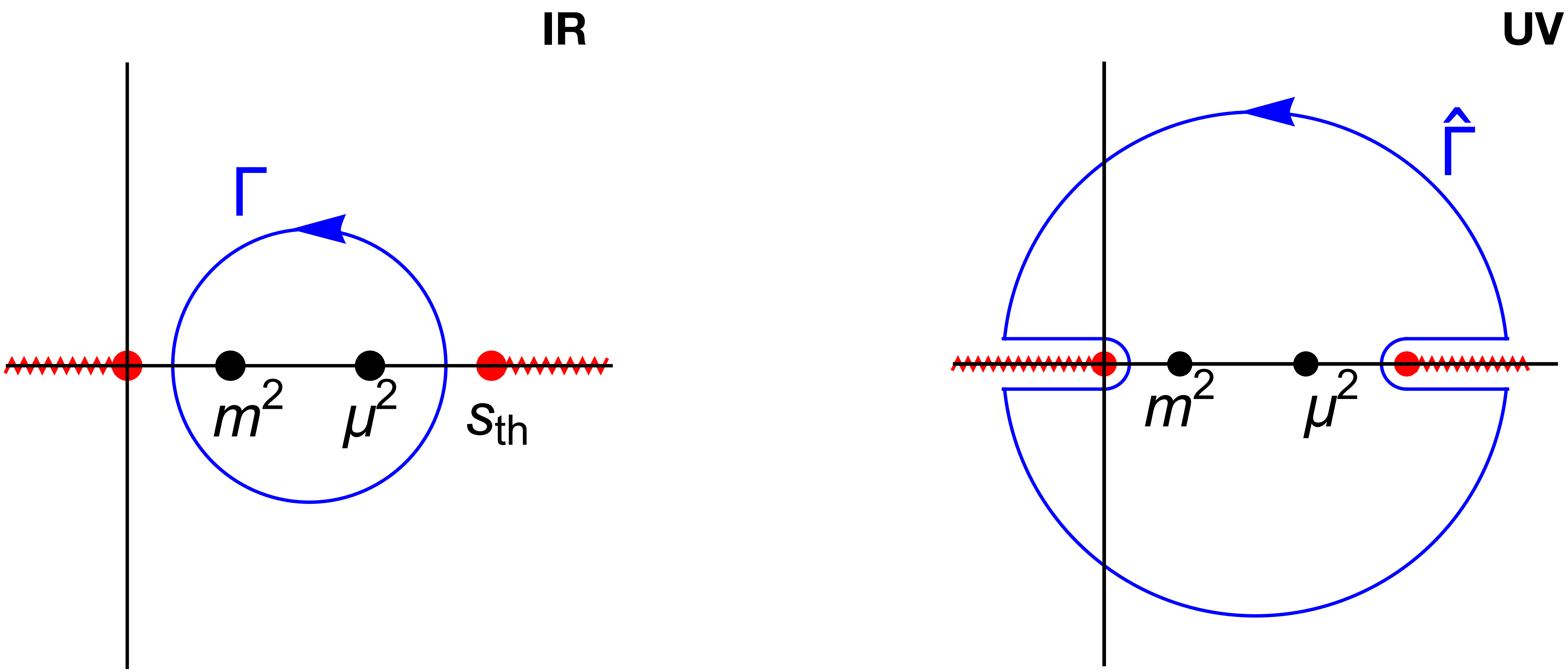
Froissart-Martin bound

$$\lim_{s\to\infty} |\mathcal{A}(s)| < \text{constant} \cdot s(\log s)^2$$

$$s_{\mathrm{th}} = 4m_i^2$$

$$\Sigma^{ab}_{\mathrm{IR}} = \sum_{\text{poles}} \mathrm{Res}\left(\frac{\mathcal{A}^{ab}(\mathrm{s})}{(\mathrm{s}-\mu^2)^3}\right).$$

Amplitudes' Positivity



$$\Sigma_{\mathrm{IR}}^{ab} = \frac{1}{2\pi i}\oint_{\Gamma} ds\, \frac{\mathcal{A}^{ab}(s)}{(s-\mu^2)^3},$$

Crossing symmetry

$$\Sigma_{\mathrm{IR}}^{ab} = \int_{s_{\mathrm{th}}^2}^{\infty} \frac{ds}{\pi}\left(\frac{\mathrm{Im}\mathcal{A}^{\mathrm{ab}}(\mathrm{s})}{(s-\mu^2)^3} + \frac{\mathrm{Im}\mathcal{A}_{\times}^{\mathrm{ab}}(\mathrm{s})}{(\mu^2 - u(s))^3}\right)$$

$$\mathcal{A}_{\times}^{ab}(s) = \mathcal{A}^{ab}(u(s))$$

Amplitudes' Positivity

assuming that UV complete theory is unitary, we can use the optical theorem:

$$\mathrm{Im}\mathscr{A}^{\mathrm{ab}}(\mathrm{s}) = \mathrm{s}\sqrt{-\mathrm{u}(\mathrm{s})/\mathrm{s}}\ \sigma^{\mathrm{ab}}(\mathrm{s}) \boxed{> 0}$$

For

$$\Sigma^{ab}_{\mathrm{IR}} = \int_{s^2_{\mathrm{th}}}^{\infty} \frac{ds}{\pi} \left(\frac{\mathrm{Im}\mathscr{A}^{\mathrm{ab}}(\mathrm{s})}{(s-\mu^2)^3} + \frac{\mathrm{Im}\mathscr{A}^{\mathrm{ab}}_{\times}(\mathrm{s})}{(\mu^2 - u(s))^3} \right)$$

We get the **positivity bound**:

$$\boxed{\Sigma^{ab}_{\mathrm{IR}} > 0}$$

Adams, Arkani-Hamed, Dubovsky, Nicolis and Rattazzi [hep-th/0602178].

Beyond Positivity

Bellazzini, Riva, Serra and Sgarlata, 1710.02539

The total cross section: $\sigma^{ab}(s) = \sum_X \sigma^{ab \to X}$

$$\Sigma_{\mathrm{IR}}^{ab} = \sum_X \int_{s_{\mathrm{th}}^2}^{\infty} \frac{ds}{\pi} \sqrt{-u(s)s} \left(\frac{\sigma^{ab\to X}(s)}{(s-\mu^2)^3} + \frac{\sigma_{\times}^{ab\to X}(s)}{(\mu^2 - u(s))^3} \right)$$

Every term in the sum is **strictly positive.**
We can consider only the states present in the EFT:

$$\boxed{\Sigma_{\mathrm{IR}}^{ab} > \sum_{X_{\mathrm{EFT}}} \int_{E_{\mathrm{IR}}}^{E_{\mathrm{UV}}} \frac{ds}{\pi} \sqrt{-u(s)s} \left(\frac{\sigma^{ab\to X}(s)}{(s-\mu^2)^3} + \frac{\sigma_{\times}^{ab\to X}(s)}{(\mu^2 - u(s))^3} \right)}$$

Beyond Positivity

Bellazzini, Riva, Serra and Sgarlata, 1710.02539

$$\boxed{\Sigma_{\mathrm{IR}}^{ab} > \sum_{X_{\mathrm{EFT}}} \int_{E_{\mathrm{IR}}}^{E_{\mathrm{UV}}} \frac{ds}{\pi} \sqrt{-u(s)s} \left(\frac{\sigma^{ab\to X}(s)}{(s-\mu^2)^3} + \frac{\sigma_{\times}^{ab\to X}(s)}{(\mu^2-u(s))^3} \right)}$$

$$\Sigma_{\mathrm{IR}}^{ab} = \frac{1}{2\pi i} \oint_{\Gamma} ds \, \frac{\mathscr{A}^{ab}(s)}{(s-\mu^2)^3}$$

is larger than a quantity that can be computed *within the EFT*, without knowledge of a UV completion.

Higgs-Dilaton inflation

$$S = \int d^4x\sqrt{|g|}\left(-\frac{1}{2}(2\xi_h\varphi^\dagger\varphi + \xi_\chi\chi^2)R + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\varphi,\chi)\right) + S_{\mathrm{SM}}(\lambda \to 0)$$

$$V(\varphi,\chi) = \lambda\left(\varphi^\dagger\varphi - \frac{\alpha}{2\lambda}\chi^2\right)^2 + \beta\chi^4$$

In the limit $\beta = 0$ and $\alpha \lll 1$
both Higgs φ and dilaton χ can be considered massless

The values of the non-minimal couplings are constrained by CMB observations

$$\xi_h \sim 10^3 - 10^5, \qquad \xi_\chi \lesssim 10^{-3}\,.$$

Change of variables

in the Einstein frame:

$$S = \int d^4x\sqrt{g|}\left(-\frac{M_P^2}{2}R + \frac{1}{2}K - U\right)$$

We change variables.
The kinetic term inherits non-renormalizability

$$K = \frac{1}{2}\left(1 + \frac{\xi_\chi - \xi_h}{M_P^2(1+6\xi_\chi)}\phi^2\right)\partial_\mu\varrho\partial^\mu\varrho + \frac{1}{2}\left(1 + \frac{2\xi_h + 6\xi_h^2 - \xi_\chi}{M_P^2(1+6\xi_\chi)}\phi^2 + \mathcal{O}(\phi^4)\right)\partial_\mu\phi\partial^\mu\phi$$

$$U = \frac{\lambda}{4}\phi^4$$

For our purposes both frames are equivalent,
see the talk by Mario Herrero-Valea for a general statement

Cut-off scales

We can readily read the cut-off scale

$$K = \frac{1}{2}\left(1 + \boxed{\frac{\xi_\chi - \xi_h}{M_P^2(1+6\xi_\chi)}}\phi^2\right)\partial_\mu\varrho\partial^\mu\varrho + \frac{1}{2}\left(1 + \boxed{\frac{2\xi_h + 6\xi_h^2 - \xi_\chi}{M_P^2(1+6\xi_\chi)}}\phi^2 + \mathcal{O}(\phi^4)\right)\partial_\mu\phi\partial^\mu\phi$$

$$\Lambda = \min\left\{ M_P\sqrt{\left|\frac{1+6\xi_\chi}{\xi_\chi - \xi_h}\right|}, M_P\sqrt{\left|\frac{1+6\xi_\chi}{2\xi_h + 6\xi_h^2 - \xi_\chi}\right|}, M_P\sqrt{\frac{\xi_\chi(1+6\xi_\chi)}{6\xi_h^2}} \right\}$$

$W_L W_L \to W_L W_L$

Bezrukov, Karananas, Rubio and Shaposhnikov, arXiv:1212.4148

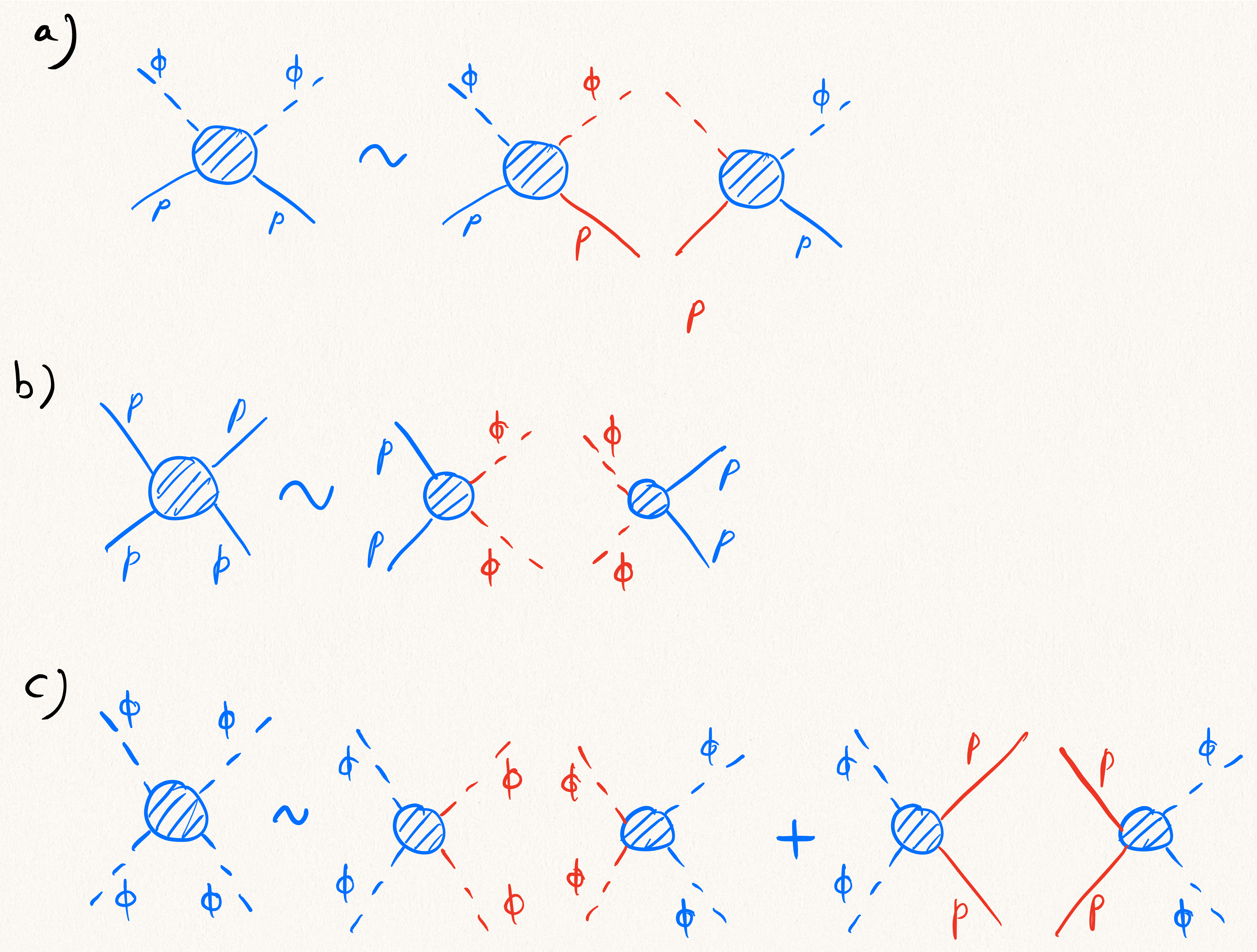
Higher derivative operators in the EFT

The kinetic term K contains non-renormalizable operators in the scalar sector. They are inherited from the non-minimal couplings to gravity in the original action.

First higher order operators:

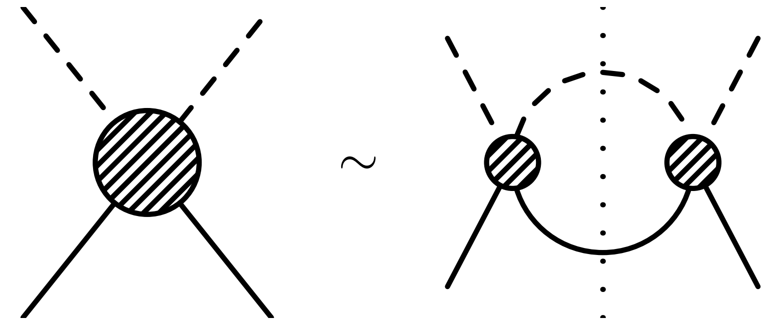
$$\begin{aligned}&\frac{\boxed{A}}{\Lambda^4}\partial_\mu\varrho\partial^\mu\varrho\partial_\nu\phi\partial^\nu\phi+\frac{\boxed{B}}{\Lambda^4}\partial_\mu\varrho\partial_\nu\varrho\partial^\mu\phi\partial^\nu\phi\\+&\frac{\boxed{C}}{\Lambda^4}\partial_\mu\varrho\partial^\mu\varrho\partial_\nu\varrho\partial^\nu\varrho+\frac{\boxed{D}}{\Lambda^4}\partial_\mu\phi\partial^\mu\phi\partial_\nu\phi\partial^\nu\phi,\end{aligned}$$

Possible channels



Slide by Mario Herrero-Valea

$a) \quad \varrho\phi \to \varrho\phi$



Computing the amplitude we get
$$\Sigma_{\mathrm{IR}} = \frac{1}{2}\frac{\partial^2 \mathcal{A}(s)}{\partial s^2} = \frac{2A+3B}{2\Lambda^4}\,.$$

$$\boxed{\frac{2A+3B}{\Lambda^4} \gtrsim \frac{(\xi_h - \xi_\chi)^2}{2\pi^2 M_P^4 (1+6\xi_\chi)^2} \log\left(\frac{E_{\mathrm{UV}}}{E_{\mathrm{IR}}}\right)}$$

We can insert tree-level unitarity cut-off $\Lambda_{\mathrm{tree}} = M_P/\xi_h$

$$2A+3B \gtrsim \frac{1}{2\pi^2 \xi_h^2} \log\left(\frac{E_{\mathrm{UV}}}{E_{\mathrm{IR}}}\right)$$

$b)\quad \varrho\varrho \to \varrho\varrho$

$$\boxed{C \gtrsim \frac{1}{96\pi^2\xi_h^2}\log\left(\frac{E_{\mathrm{UV}}}{E_{\mathrm{IR}}}\right)}$$

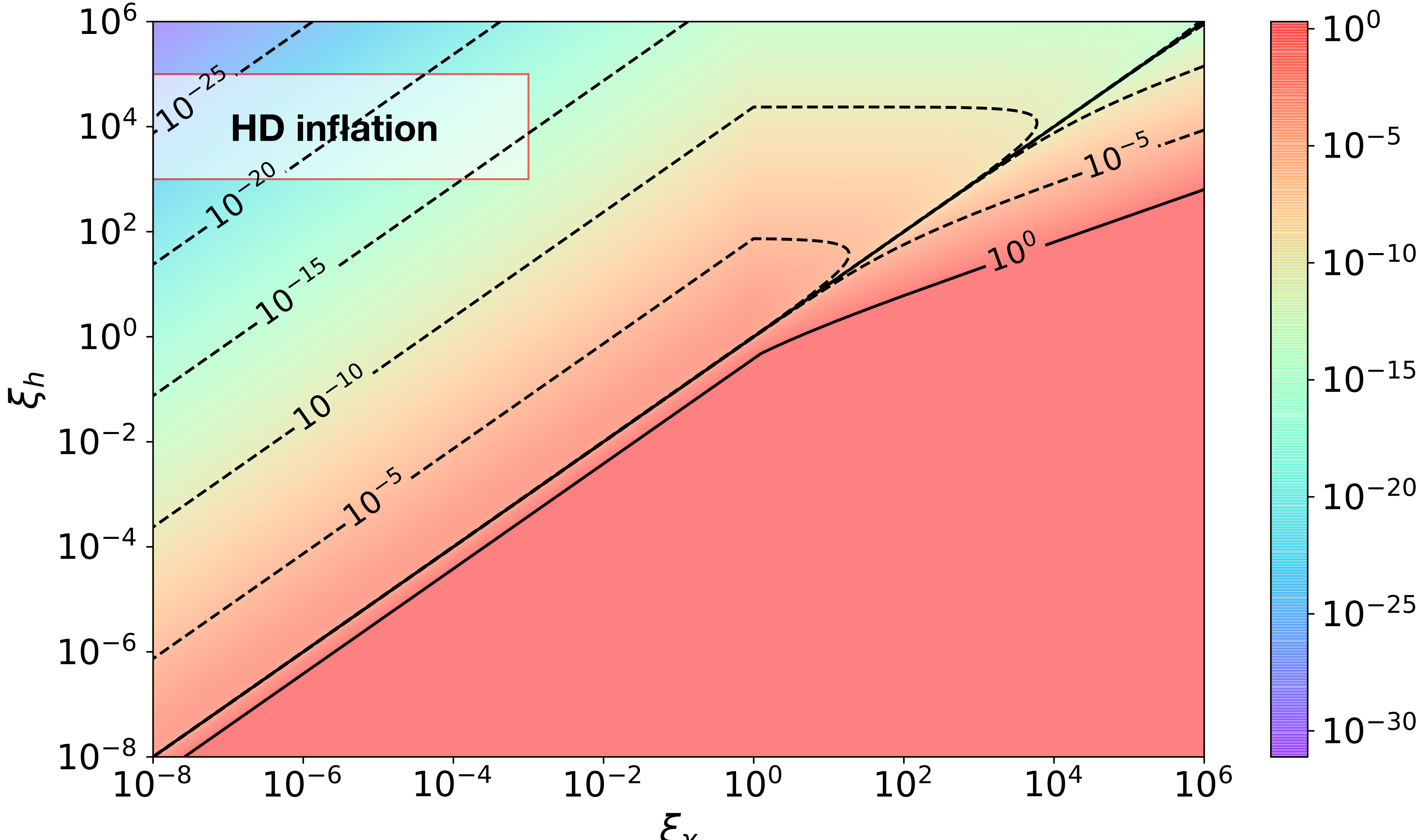
$c)\quad \phi\phi \to \phi\phi$ **is dominated by** $\lambda\phi^4$

$$\frac{24D}{\Lambda^4} + \mathcal{O}(\lambda^2) \gtrsim \frac{81\lambda^2}{4\pi^2 E_{IR}^4}$$

loop corrections

$$\boxed{\frac{24D}{\Lambda^4} > 0}$$

2A + 3B as a function of non-minimal couplings



Conclusions

- We found a non-trivial consistency check successfully satisfied by the Higgs-Dilaton model.
- Once the hierarchy between the non-minimal couplings is assumed, a large value of ξ_h is actually favoured by our results.
- Graviton exchange is not included (yet).

Backup slides

Change of variables

$$S = \int d^4x \sqrt{|g|} \left(-\frac{1}{2}(\xi_h h^2 + \xi_\chi \chi^2) R + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi + \frac{1}{2}\partial_\mu h \partial^\mu h - V(\varphi, \chi) \right).$$

$$\tilde{g}_{\mu\nu} = f(h,\chi) g_{\mu\nu}, \qquad f(h,\chi) = \frac{\xi_h h^2 + \xi_\chi \chi^2}{M_P^2}$$

$$S = \int d^4x \sqrt{|\tilde{g}|} \left(-\frac{M_P^2}{2}\tilde{R} + \frac{1}{2}K(h,\chi) - U(h,\chi) \right)$$

$$K(h,\chi) = \kappa_{AB} \tilde{g}^{\mu\nu} \partial_\mu S^A \partial_\nu S^B \qquad U(h,\chi) = \frac{V(h,\chi)}{(f(h,\chi))^2}$$

$$\kappa_{AB} = \frac{1}{f(h,\chi)} \left(\delta_{AB} + \frac{3M_P^2}{2} \frac{\partial_A (f(h,\chi))^{\frac{1}{2}} \partial_B (f(h,\chi))^{\frac{1}{2}}}{f(h,\chi)} \right)$$

Change of variables

We go to polar variables:

$$\rho = \frac{M_P}{2} \log\left(\frac{(1+6\xi_h)h^2 + (1+6\xi_\chi)\chi^2}{M_P^2}\right), \qquad \tan\theta = \sqrt{\frac{1+6\xi_h}{1+6\xi_\chi}}\frac{h}{\chi}$$

$$K = \left(\frac{1+6\xi_h}{\xi_h}\right)\frac{\partial_\mu\rho\partial^\mu\rho}{\sin^2\theta + \frac{(1+6\xi_h)\xi_\chi}{(1+6\xi_\chi)\xi_h}\cos^2\theta} + \frac{(1+6\xi_h)}{(1+6\xi_\chi)}\frac{M_P^2}{\xi_h}\frac{\left(\tan^2\theta + \frac{\xi_\chi}{\xi_h}\right)\partial_\mu\theta\partial^\mu\theta}{\cos^2\theta\left(\tan^2\theta + \frac{(1+6\xi_h)\xi_\chi}{(1+6\xi_\chi)\xi_h}\right)^2}$$

$$U(\theta) = \frac{\lambda M_P^4}{4\xi_h^2}\left(\frac{\sin^2\theta}{\sin^2\theta + \frac{(1+6\xi_h)\xi_\chi}{(1+6\xi_\chi)\xi_h}\cos^2\theta}\right)^2$$

Change of variables

$$\theta = \arcsin\left(\sqrt{\frac{(1+6\xi_h)\xi_\chi\phi^2}{M_P^2(1+6\xi_\chi)+(\xi_\chi-\xi_h)\phi^2}}\right), \qquad \rho = \varrho\sqrt{\frac{\xi_\chi}{1+6\xi_h}}$$

$$U(\phi) = \frac{\lambda}{4}\phi^4$$

$$K = \frac{1}{2}\left(1+\frac{\xi_\chi-\xi_h}{M_P^2(1+6\xi_\chi)}\phi^2\right)\partial_\mu\varrho\partial^\mu\varrho + \frac{1}{2}\left(1+\frac{2\xi_h+6\xi_h^2-\xi_\chi}{M_P^2(1+6\xi_\chi)}\phi^2+\mathcal{O}(\phi^4)\right)\partial_\mu\phi\partial^\mu\phi$$