Scattering Amplitudes and the Positive Grassmannian

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Niels Bohr Institute

based on work in collaboration with

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Organization and Outline

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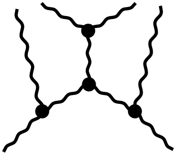

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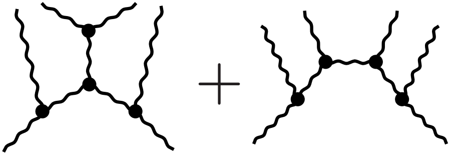

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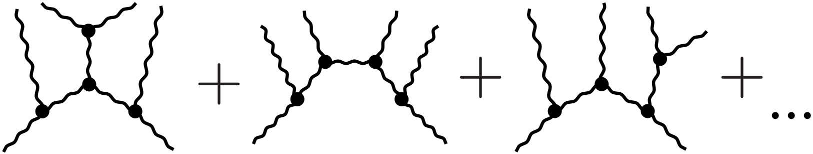

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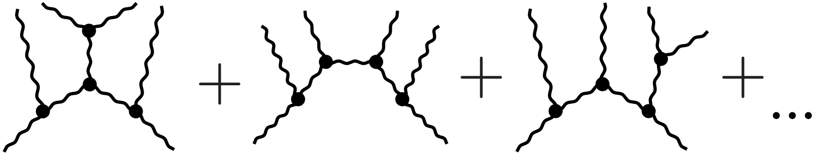

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gluons. The cross section for the scattering of two gluons with momenta p_1, p_2 into four gluons with momenta p_3, p_4, p_5, p_6 is obtained from eq. (5) by setting $I=2$ and replacing the momenta p_3, p_4, p_5, p_6 by $-p_3, -p_4, -p_5, -p_6$.

As the result of the computation of two hundred and forty Feynman diagrams, we obtain

$$A_{\binom{0}{2}}(p_1, p_2, p_3, p_4, p_5, p_6) = (\mathscr{D}^\dagger, \mathscr{D}^\dagger_\rho, \mathscr{D}^\dagger_\sigma, \mathscr{D}^\dagger_\tau)_{\binom{0}{2}} \cdot \begin{pmatrix} K & K_\rho & K_\sigma & K_\tau \\ K_\rho & K & K_\tau & K_\sigma \\ K_\sigma & K_\tau & K & K_\rho \\ K_\tau & K_\sigma & K_\rho & K \end{pmatrix} \cdot \begin{pmatrix} \mathscr{D} \\ \mathscr{D}_\rho \\ \mathscr{D}_\sigma \\ \mathscr{D}_\tau \end{pmatrix}_{\binom{0}{2}}, \tag{6}$$

where $\mathscr{D}$, $\mathscr{D}_\rho$, $\mathscr{D}_\sigma$ and $\mathscr{D}_\tau$ are 11-component complex vector functions of the momenta p_1, p_2, p_3, p_4, p_5 and p_6, and K, K_ρ, K_σ and K_τ are constant 11×11 symmetric matrices. The vectors $\mathscr{D}_\rho$, $\mathscr{D}_\sigma$ and $\mathscr{D}_\tau$ are obtained from the vector $\mathscr{D}$ by the permutations $(p_2 \leftrightarrow p_3)$, $(p_5 \leftrightarrow p_6)$ and $(p_2 \leftrightarrow p_3, p_5 \leftrightarrow p_6)$, respectively, of the momentum variables in $\mathscr{D}$. The individual components of the vector $\mathscr{D}$ represent the sums of all contributions proportional to the appropriately chosen eleven basis color factors. The matrices K, which are the suitable sums over the color indices of products of the color bases, contain two independent structures, proportional to $N^4(N^2-1)$ and $N^2(N^2-1)$, respectively (N is the number of colors, $N=3$ for QCD):

$$K = \tfrac{1}{8}g^8 N^4(N^2-1)K^{(4)} + \tfrac{1}{2}g^8 N^2(N^2-1)K^{(2)}. \tag{7}$$

Here g denotes the gauge coupling constant. The matrices $K^{(4)}$ and $K^{(2)}$ are given in table 1. The vector $\mathscr{D}$ is related to the thirty-three diagrams $D^{\mathrm{G}}(I=1-33)$ for two-gluon to four-scalar scattering, eleven diagrams $D^{\mathrm{F}}(I=1-11)$ for two-fermion to four-scalar scattering and sixteen diagrams $D^{\mathrm{S}}(I=1-16)$ for two-scalar to four-scalar scattering, in the following way:

$$\begin{aligned} \mathscr{D}_0 &= \frac{2s_{14}}{\sqrt{|s_{15}s_{45}s_{16}s_{46}|}\,s_{23}s_{56}}\{t_{123}^2 C^{\mathrm{G}}\cdot D_0^{\mathrm{G}} - 4s_{14}t_{123}E(p_5+p_6, p_6)C^{\mathrm{F}}\cdot D_0^{\mathrm{F}} \\ &\quad - 2s_{14}G(p_5+p_6, p_5+p_6)C^{\mathrm{S}}\cdot D_0^{\mathrm{S}}\}, \\ \mathscr{D}_2 &= \frac{s_{56}}{s_{23}}C^{\mathrm{G}}\cdot D_2^{\mathrm{G}}, \end{aligned} \tag{8}$$

where the constant matrices $C^{\mathrm{G}}(11\times 33)$, $C^{\mathrm{F}}(11\times 11)$ and $C^{\mathrm{S}}(11\times 16)$ are given in table 2. The Lorentz invariants s_{ij} and t_{ijk} are defined as $s_{ij}=(p_i+p_j)^2$, $t_{ijk}=(p_i+p_j+p_k)^2$ and the complex functions E and G are given by

$$\begin{aligned} E(p_i, p_j) &= \tfrac{1}{2}\{(p_1p_4)(p_ip_j) - (p_1p_i)(p_jp_4) - (p_1p_j)(p_ip_4) + i\varepsilon_{\mu\nu\rho\lambda}p_1^\mu p_i^\nu p_j^\rho p_4^\lambda\}/(p_1p_4), \\ G(p_i, p_j) &= E(p_i, p_5)E(p_j, p_6), \end{aligned} \tag{9}$$

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TABLE 1
Matrices $K(I, J)[I = 1\text{-}11, J = 1\text{-}11]$.

Matrix $K^{(4)}$											Matrix $K^{(2)}$										
8	4	−2	2	−1	2	0	1	0	0	−1	0	0	0	0	0	0	0	0	3	3	−3
4	8	−1	1	−1	0	2	1	0	1	−1	0	0	0	0	0	0	0	0	3	3	0
−2	−1	8	4	4	1	1	2	2	1	2	0	0	0	0	0	0	0	0	0	0	0
2	1	4	8	2	−1	−1	4	1	1	1	0	0	0	0	0	0	0	0	0	0	0
−1	−1	4	2	8	1	2	4	−2	−1	4	0	0	0	0	0	0	0	0	0	0	0
2	0	1	−1	1	8	4	−1	0	1	0	0	0	0	0	0	0	0	0	3	3	0
0	2	1	−1	2	4	8	−2	0	0	0	0	0	0	0	0	0	0	0	3	3	0
1	1	2	4	4	−1	−2	8	−1	−1	2	0	0	0	0	0	0	0	0	0	0	0
0	0	2	1	−2	0	0	−1	8	4	−2	3	3	0	0	0	3	3	0	0	0	0
0	1	1	1	−1	1	0	−1	4	8	−1	3	3	0	0	0	3	3	0	0	0	0
−1	−1	2	1	4	0	0	2	−2	−1	8	−3	0	0	0	0	0	0	0	0	0	0

Matrix $K_\rho^{(4)}$											Matrix $K_\rho^{(2)}$										
0	0	0	0	1	1	0	1	1	0	−1	3	3	0	3	0	0	0	3	0	0	0
0	0	0	0	2	0	1	1	2	1	−2	3	3	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	1	1	0	0	3	0	0	3	0	0	3	3	0
0	0	0	0	1	0	0	2	0	1	0	3	0	0	0	0	3	0	0	0	0	0
1	2	0	1	0	1	2	2	0	0	2	0	0	0	0	0	0	0	0	3	3	0
1	0	1	0	1	4	2	0	0	0	−1	0	0	3	3	0	0	0	0	3	3	0
0	1	1	0	2	2	4	0	0	0	−2	0	0	0	0	0	0	0	0	0	3	0
1	1	1	2	2	0	0	4	0	0	0	3	0	0	0	0	0	0	0	3	0	0
1	2	0	0	0	0	0	0	0	2	−1	0	0	3	0	3	3	0	3	0	0	0
0	1	1	1	0	0	0	0	2	4	0	0	0	3	0	3	3	3	0	0	0	0
−1	−2	1	0	2	−1	−2	0	−1	0	4	0	0	0	0	0	0	0	0	0	0	0

Matrix $K_\sigma^{(4)}$											Matrix $K_\sigma^{(2)}$										
4	2	0	2	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	3	0	−3
2	4	0	1	0	0	1	1	0	1	0	0	0	0	0	3	0	0	3	0	0	0
0	0	4	2	2	1	1	1	2	1	0	0	0	0	0	0	0	3	0	0	0	0
2	1	2	0	1	2	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	2	1	0	0	0	0	4	2	2	0	3	0	0	0	0	3	0	0	0	0
1	0	1	2	0	0	0	0	1	2	0	0	0	0	0	0	3	3	0	0	0	−3
0	1	1	1	0	0	0	0	2	4	0	0	0	3	0	3	3	3	0	0	0	0
1	1	1	0	0	0	0	0	2	0	1	0	3	0	0	0	0	0	3	0	0	0
0	0	2	1	4	1	2	2	0	0	−4	3	0	0	0	0	0	0	0	0	0	0
0	1	1	0	2	2	4	0	0	0	−2	0	0	0	0	0	0	0	0	0	3	0
0	0	0	0	2	0	0	1	−4	−2	4	−3	0	0	0	0	−3	0	0	0	0	0

Matrix $K_\tau^{(4)}$											Matrix $K_\tau^{(2)}$										
0	1	−1	−1	1	1	0	1	2	0	0	3	3	0	0	0	3	3	0	0	0	0
1	0	−2	−1	2	0	1	1	4	2	0	3	3	0	0	0	3	3	0	0	0	0
−1	−2	0	0	0	1	1	1	−1	1	0	0	0	3	3	3	0	0	3	0	0	0
−1	−1	0	1	0	2	1	0	1	−1	0	0	0	3	3	3	0	0	3	0	0	3
1	2	0	0	1	−1	−1	0	−2	2	1	0	0	3	3	3	0	0	3	0	0	0
1	0	1	2	−1	0	1	−2	2	4	−1	3	3	0	0	0	3	3	0	0	0	−3
0	1	1	1	−1	1	0	−1	4	8	−1	3	3	0	0	0	3	3	0	0	0	0
1	1	1	0	0	−2	−1	0	2	−2	0	0	0	3	3	3	0	0	3	0	0	0
2	4	−1	1	−2	2	4	2	1	0	−2	0	0	0	0	0	0	0	0	3	3	0
0	2	1	−1	2	4	8	−2	0	0	0	0	0	0	0	0	0	0	0	3	3	0
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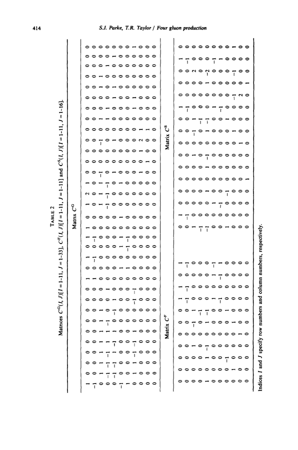

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where ε is the totally antisymmetric tensor, $\varepsilon_{xyzt}=1$. For the future use, we define one more function,

$$F(p_i, p_j)=\{(p_1p_4)(p_ip_j)+(p_1p_i)(p_jp_4)-(p_1p_j)(p_ip_4)\}/(p_1p_4)\,. \tag{10}$$

Note that when evaluating A_0 and A_2 at crossed configurations of the momenta, care must be taken with the implicit dependence of the functions E, F and G on the momenta p_1, p_4, p_5, p_6.

The diagrams D_2^G are listed below:

$$D_2^G(1)=\frac{\delta_2}{s_{14}s_{25}s_{36}}\{[(p_2-p_5)(p_3-p_6)][(p_1-p_4)(p_3+p_6)]-[(p_2-p_5)(p_3+p_6)]$$
$$\times[(p_1-p_4)(p_3-p_6)]+[(p_2+p_5)(p_3-p_6)][(p_1-p_4)(p_2-p_5)]\}\,,$$

$$D_2^G(2)=\frac{1}{s_{25}s_{36}}\{2E(p_2-p_5,p_3-p_6)-2E(p_3-p_6,p_2-p_5)+\delta_2[(p_2-p_5)(p_3-p_6)]\}\,,$$

$$D_2^G(3)=\frac{4}{s_{25}s_{36}t_{125}}\{[(p_1+p_2-p_5)(p_4+p_3-p_6)]E(p_2,p_3)$$
$$-[(p_1+p_2-p_5)(p_4-p_3+p_6)]E(p_2,p_6)$$
$$-[(p_1-p_2+p_5)(p_4+p_3-p_6)]E(p_5,p_3)$$
$$+[(p_1-p_2+p_5)(p_4-p_3+p_6)]E(p_5,p_6)$$
$$\dot{-}[p_1(p_2-p_5)]E(p_3-p_6,p_3+p_6)-[p_4(p_3-p_6)]E(p_2+p_5,p_2-p_5)$$
$$+\delta_2[p_1(p_2-p_5)][p_4(p_3-p_6)]\}\,,$$

$$D_2^G(4)=\frac{-2}{s_{36}t_{125}}\{E(p_3-p_6,p_3+p_6)-\delta_2[p_4(p_3-p_6)]\}\,,$$

$$D_2^G(5)=\frac{-2}{s_{25}t_{125}}\{E(p_2+p_5,p_2-p_5)-\delta_2[p_1(p_2-p_5)]\}\,,$$

$$D_2^G(6)=\frac{\delta_2}{t_{125}}\,,$$

$$D_2^G(7)=\frac{4}{s_{12}s_{36}t_{125}}\{[(p_1+p_2-p_5)(p_4+p_3-p_6)]E(p_2,p_3)$$
$$-[(p_1+p_2-p_5)(p_4-p_3+p_6)]E(p_2,p_6)-[p_4(p_3-p_6)]E(p_2,p_2-p_5)\}\,,$$

$$D_2^G(8)=\frac{4}{s_{34}s_{25}t_{125}}\{[(p_1+p_2-p_5)(p_4+p_3-p_6)]E(p_2,p_3)$$
$$-[(p_1-p_2+p_5)(p_4+p_3-p_6)]E(p_5,p_3)-[p_1(p_2-p_5)]E(p_3-p_6,p_3)\}\,,$$

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$$D_2^G(9) = \frac{4}{s_{15}s_{36}t_{125}}\{[(p_1-p_2+p_5)(p_4+p_3-p_6)]E(p_5,p_3)$$
$$-[(p_1-p_2+p_5)(p_4-p_3+p_6)]E(p_5,p_6)+[p_4(p_3-p_6)]E(p_5,p_2-p_5)\},$$

$$D_2^G(10) = \frac{4}{s_{25}s_{46}t_{125}}\{[(p_1+p_2-p_5)(p_4-p_3+p_6)]E(p_2,p_6)$$
$$-[(p_1-p_2+p_5)(p_4-p_3+p_6)]E(p_5,p_6)+[p_1(p_2-p_5)]E(p_3-p_6,p_6)\},$$

$$D_2^G(11) = \frac{\delta_2}{s_{36}t_{124}}[s_{35}-s_{56}+s_{36}],$$

$$D_2^G(12) = \frac{-\delta_2}{s_{36}t_{145}}[s_{23}-s_{26}-s_{36}],$$

$$D_2^G(13) = \frac{\delta_2}{s_{14}s_{36}t_{124}}[s_{12}-s_{24}][s_{35}-s_{56}+s_{36}],$$

$$D_2^G(14) = \frac{\delta_2}{s_{14}s_{36}t_{145}}[s_{15}-s_{45}][s_{23}-s_{26}-s_{36}],$$

$$D_2^G(15) = \frac{\delta_2}{s_{14}s_{36}}(p_1-p_4)(p_3-p_6),$$

$$D_2^G(16) = \frac{-4}{s_{12}s_{36}t_{124}}[s_{35}-s_{56}+s_{36}]E(p_2,p_2),$$

$$D_2^G(17) = \frac{4}{s_{36}s_{45}t_{145}}[s_{23}-s_{26}-s_{36}]E(p_5,p_5),$$

$$D_2^G(18) = \frac{-4}{s_{12}s_{36}s_{45}}[2(p_1+p_2)(p_3-p_6)-s_{36}]E(p_2,p_5),$$

$$D_2^G(19) = \frac{-2}{s_{12}s_{36}}E(p_2,p_3-p_6),$$

$$D_2^G(20) = \frac{2}{s_{36}s_{45}}E(p_3-p_6,p_5),$$

$$D_2^G(21) = \frac{-4}{s_{25}s_{34}t_{134}}[s_{26}-s_{56}+s_{25}]E(p_3,p_3),$$

$$D_2^G(22) = \frac{4}{s_{16}s_{25}t_{146}}[s_{23}-s_{35}-s_{25}]E(p_6,p_6),$$

$$D_2^G(23) = \frac{4}{s_{16}s_{25}s_{34}}[2(p_1+p_6)(p_2-p_5)+s_{25}]E(p_6,p_3),$$

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$$D_2^{\mathrm{G}}(24)=\frac{-2}{s_{25}s_{34}}E(p_2-p_5,p_3)\,,$$

$$D_2^{\mathrm{G}}(25)=\frac{2}{s_{16}s_{25}}E(p_6,p_2-p_5)\,,$$

$$D_2^{\mathrm{G}}(26)=\frac{-2}{s_{12}t_{125}}E(p_2,p_2-p_5)\,,$$

$$D_2^{\mathrm{G}}(27)=\frac{2}{s_{46}t_{125}}E(p_3-p_6,p_6)\,,$$

$$D_2^{\mathrm{G}}(28)=\frac{2}{s_{15}t_{125}}E(p_5,p_2-p_5)\,,$$

$$D_2^{\mathrm{G}}(29)=\frac{-2}{s_{34}t_{125}}E(p_3-p_6,p_3)\,,$$

$$D_2^{\mathrm{G}}(30)=\frac{4}{s_{12}s_{34}t_{125}}[(p_1+p_2-p_5)(p_4+p_3-p_6)-t_{125}]E(p_2,p_3)\,,$$

$$D_2^{\mathrm{G}}(31)=\frac{4}{s_{12}s_{46}t_{125}}[(p_1+p_2-p_5)(p_4-p_3+p_6)+t_{125}]E(p_2,p_6)\,,$$

$$D_2^{\mathrm{G}}(32)=\frac{4}{s_{15}s_{34}t_{125}}[(p_1-p_2+p_5)(p_4+p_3-p_6)+t_{125}]E(p_5,p_3)\,,$$

$$D_2^{\mathrm{G}}(33)=\frac{4}{s_{15}s_{46}t_{125}}[(p_1-p_2+p_5)(p_4-p_3+p_6)-t_{125}]E(p_5,p_6)\,,\qquad(11)$$

where $\delta_2=1$.

The diagrams D_0^{G} are obtained from D_2^{G} by replacing δ_2 by $\delta_0=0$ and the functions $E(p_i,p_j)$ by $G(p_i,p_j)$.

The diagrams D_0^{F} are listed below:

$$D_0^{\mathrm{F}}(1)=\frac{4}{s_{15}s_{34}t_{125}}\{F(p_5,p_6)E(p_3,p_5)-F(p_5,p_3)E(p_6,p_5)$$
$$+[F(p_6,p_3)+s_{34}]E(p_5,p_5)\}\,,$$

$$D_0^{\mathrm{F}}(2)=\frac{-4}{s_{16}s_{25}s_{34}}\{[F(p_6,p_2)+\tfrac{1}{2}s_{16}]E(p_3,p_5)$$
$$+[F(p_2,p_3)+\tfrac{1}{2}s_{34}]E(p_6,p_5)-F(p_6,p_3)E(p_2,p_5)\}\,,$$

$$D_0^{\mathrm{F}}(3)=\frac{4}{s_{15}s_{36}t_{125}}\{F(p_5,p_6)E(p_3,p_5)-F(p_5,p_3)E(p_6,p_5)$$
$$-[F(p_3,p_6)-\tfrac{1}{2}s_{36}-\tfrac{1}{2}s_{34}+\tfrac{1}{2}s_{46}]E(p_5,p_5)\}\,,$$

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$$D_0^F(4) = \frac{4}{s_{25}s_{34}t_{125}}\{F(p_2,p_3)E(p_5,p_5) - F(p_5,p_3)E(p_2,p_5) + [F(p_5,p_2) - \tfrac{1}{2}s_{25} - \tfrac{1}{2}s_{12} + \tfrac{1}{2}s_{15}]E(p_3,p_5)\},$$

$$D_0^F(5) = \frac{2}{s_{16}s_{25}t_{146}}[s_{35} - s_{23} + s_{25}]E(p_6,p_5),$$

$$D_0^F(6) = \frac{2}{s_{25}s_{34}t_{134}}[s_{56} - s_{26} - s_{25}]E(p_3,p_5),$$

$$D_0^F(7) = \frac{4}{s_{25}s_{36}t_{125}}\{[F(p_5,p_2) - \tfrac{1}{2}s_{25} - \tfrac{1}{2}s_{12} + \tfrac{1}{2}s_{15}]E(p_3,p_5) + [F(p_2,p_3) + \tfrac{1}{4}t_{125}]E(p_5,p_5) - [F(p_5,p_3) + \tfrac{1}{4}t_{125}]E(p_2,p_5)\},$$

$$D_0^F(8) = \frac{1}{s_{14}s_{36}}E(p_3 - p_6, p_5),$$

$$D_0^F(9) = \frac{2}{s_{14}s_{36}t_{124}}[s_{35} - s_{56} + s_{36}]E(p_2,p_5),$$

$$D_0^F(10) = \frac{2}{s_{14}s_{36}t_{145}}[s_{23} - s_{26} - s_{36}]E(p_5,p_5),$$

$$D_0^F(11) = \frac{1}{2s_{14}s_{25}s_{36}}\{[s_{23} + s_{35} - s_{26} - s_{56}]E(p_2 - p_5, p_5) - [s_{23} + s_{26} - s_{35} - s_{56}]E(p_3 - p_6, p_5) - [s_{23} + s_{56} - s_{35} - s_{26}]E(p_2 + p_5, p_5)\}. \tag{12}$$

The diagrams D_0^S are listed below:

$$D_0^S(1) = \frac{1}{s_{25}s_{36}t_{125}}[s_{34} - s_{46} + s_{36}][s_{12} - s_{15} - s_{25}],$$

$$D_0^S(2) = \frac{1}{s_{14}s_{36}t_{124}}[s_{12} - s_{24} - s_{14}][s_{35} - s_{56} + s_{36}],$$

$$D_0^S(3) = \frac{1}{s_{14}s_{36}t_{145}}[s_{15} - s_{45} + s_{14}][s_{23} - s_{26} - s_{36}],$$

$$D_0^S(4) = \frac{1}{s_{15}s_{36}t_{125}}[s_{15} + s_{25} - s_{12}][s_{34} - s_{46} + s_{36}],$$

$$D_0^S(5) = \frac{1}{s_{15}s_{34}t_{156}}[s_{56} - s_{15} - s_{16}][s_{23} - s_{24} - s_{34}],$$

$$D_0^S(6) = \frac{1}{s_{15}s_{34}t_{125}}[s_{46} - s_{34} - s_{36}][s_{12} - s_{25} - s_{15}],$$

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$$D_0^S(7) = \frac{1}{s_{25}s_{34}t_{125}}[s_{36} - s_{46} + s_{34}][s_{12} - s_{15} - s_{25}]\,,$$

$$D_0^S(8) = \frac{1}{s_{16}s_{25}t_{146}}[s_{25} + s_{35} - s_{23}][s_{14} - s_{46} + s_{16}]\,,$$

$$D_0^S(9) = \frac{1}{s_{25}s_{34}t_{134}}[s_{14} + s_{34} - s_{13}][s_{26} - s_{56} + s_{25}]\,,$$

$$D_0^S(10) = \frac{1}{s_{25}s_{36}}(p_2 - p_5)(p_3 - p_6)\,,$$

$$D_0^S(11) = \frac{1}{s_{14}s_{36}}(p_1 - p_4)(p_3 - p_6)\,,$$

$$D_0^S(12) = \frac{1}{s_{16}s_{25}}(p_6 - p_1)(p_2 - p_5)\,,$$

$$D_0^S(13) = \frac{1}{s_{15}s_{34}}(p_5 - p_1)(p_3 - p_4)\,,$$

$$D_0^S(14) = \frac{1}{s_{25}s_{34}}(p_2 - p_5)(p_3 - p_4)\,,$$

$$\begin{aligned} D_0^S(15) = \frac{1}{s_{14}s_{25}s_{36}}\{&[(p_2 + p_5)(p_3 - p_6)][(p_1 - p_4)(p_2 - p_5)] \\ &+ [(p_2 - p_5)(p_3 - p_6)][(p_1 - p_4)(p_3 + p_6)] \\ &+ [(p_1 + p_4)(p_2 - p_5)][(p_1 - p_4)(p_3 - p_6)]\}\,, \end{aligned}$$

$$\begin{aligned} D_0^S(16) = \frac{2}{s_{16}s_{34}s_{25}}\{&[(p_2 - p_5)(p_3 + p_4)][(p_1 - p_6)(p_3 - p_4)] \\ &+ [(p_1 + p_6)(p_3 - p_4)][(p_1 - p_6)(p_2 - p_5)] \\ &+ [(p_1 - p_6)(p_2 + p_5)][(p_3 - p_4)(p_2 - p_5)]\}\,. \end{aligned} \qquad (13)$$

The preceding list completes the result. Let us recapitulate now the numerical procedure of calculating the full cross section. First the diagrams D are calculated by using eqs. (11)–(13). The result is substituted to eq. (8) to obtain the vectors $\mathscr{D}_0$ and $\mathscr{D}_2$. After generating the vectors $\mathscr{D}_{0_\rho}$, $\mathscr{D}_{0_\sigma}$, $\mathscr{D}_{0_\tau}$, $\mathscr{D}_{2_\rho}$, $\mathscr{D}_{2_\sigma}$ and $\mathscr{D}_{2_\tau}$ by the appropriate permutations of momenta, eq. (6) is used to obtain the functions A_0 and A_2. Finally, the total cross section is calculated by using eq. (5). The FORTRAN 5 program based on such a scheme generates ten Monte Carlo points in less than a second on the heterotic CDC CYBER 175/875.

Given the complexity of the final result, it is very important to have some reliable testing procedures available for numerical calculations. Usually in QCD, the multi-gluon amplitudes are tested by checking the gauge invariance. Due to the specifics

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of our calculation, the most powerful test does not rely on the gauge symmetry, but on the appropriate permutation symmetries. The function $A_0(p_1, p_2, p_3, p_4, p_5, p_6)$ must be symmetric under arbitrary permutations of the momenta (p_1, p_2, p_3) and separately, (p_4, p_5, p_6), whereas the function $A_2(p_1, p_2, p_3, p_4, p_5, p_6)$ must be symmetric under the permutations of (p_1, p_2, p_3, p_4) and separately, (p_5, p_6). This test is extremely powerful, because the required permutation symmetries are hidden in our supersymmetry relations, eqs. (1) and (3), and in the structure of amplitudes involving different species of particles. Another, very important test relies on the absence of the double poles of the form $(s_{ij})^{-2}$ in the cross section, as required by general arguments based on the helicity conservation. Further, in the leading $(s_{ij})^{-1}$ pole approximation, the answer should reduce to the two goes to three cross section [3, 4], convoluted with the appropriate Altarelli-Parisi probabilities [5]. Our result has succesfully passed both these numerical checks.

Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

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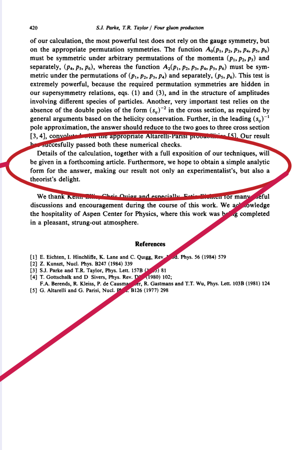

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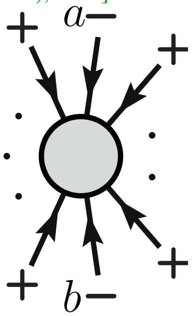

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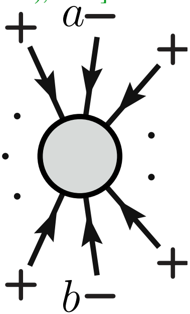

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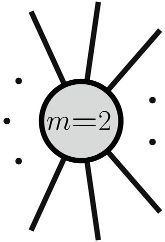
$$\mathcal{A}_n^{(2)} = \prod_{I=1}^{4}\left\{\bigoplus_{a<b}\langle a\,b\rangle\widetilde{\eta}_a^I\widetilde{\eta}_b^I\right\}\frac{1}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle n\,1\rangle}\delta^4\left(\sum p_a^\mu\right)$$



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Useful Lorentz-invariant scalars:

$$\langle ab\rangle \equiv \begin{vmatrix} \lambda_a^1 & \lambda_b^1 \\ \lambda_a^2 & \lambda_b^2\end{vmatrix}, \qquad [ab] \equiv \begin{vmatrix} \widetilde{\lambda}_a^{\dot{1}} & \widetilde{\lambda}_b^{\dot{1}} \\ \widetilde{\lambda}_a^{\dot{2}} & \widetilde{\lambda}_b^{\dot{2}}\end{vmatrix}$$

$$(p_a+p_b)^2 = \langle ab\rangle[ba] \equiv s_{ab}, \qquad \langle a|(b+\ldots+c)|d] \equiv \langle a|\Big(b\rangle[b+\ldots+c\rangle[c\Big)|d].$$

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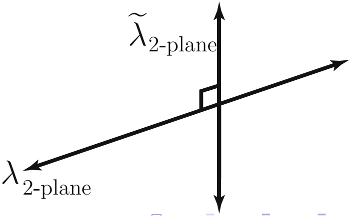

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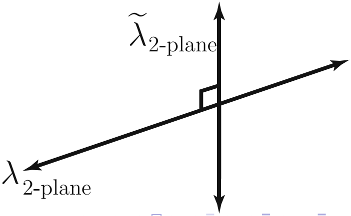

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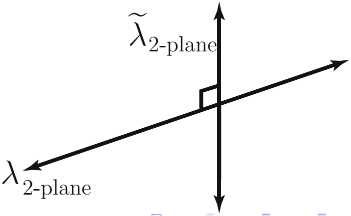

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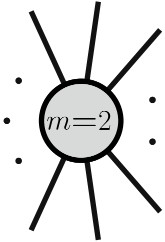

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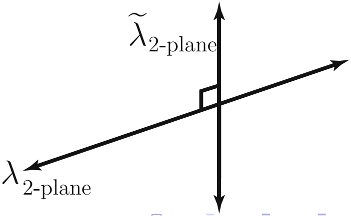

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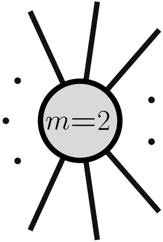

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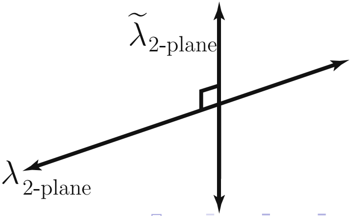

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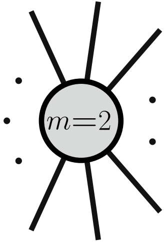

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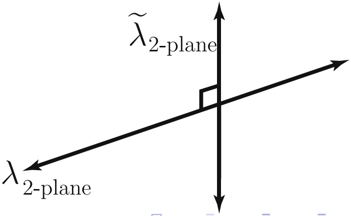

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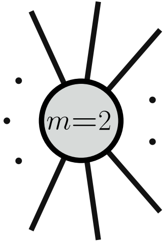

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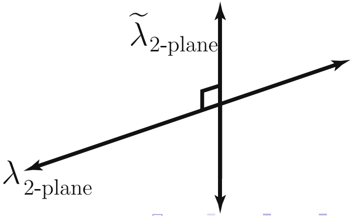

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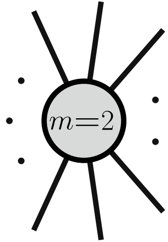
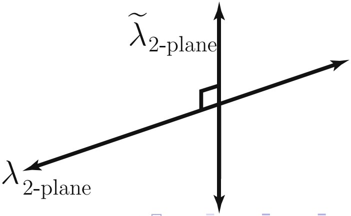


$$\lambda \equiv \begin{pmatrix}\lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \cdots & \lambda_n^1\\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \cdots & \lambda_n^2\end{pmatrix} \in G(2,n)$$



The Grassmannian $G(m,n)$:

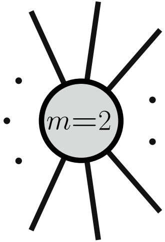
The space of m-planes in n-dimensions

Parke and Taylor's Heroic Computation: Six Months Later

Six months later, they had come upon a "**guess**", not only for their earlier amplitude, but an infinite number of amplitudes! [PRL **56** (1986), 2459]

In modern notation, they suggested that:

$$\mathcal{A}_n^{(2)} = \delta^{2\times 4}(\lambda\cdot\widetilde{\eta})\frac{1}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle n\,1\rangle}\delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})$$



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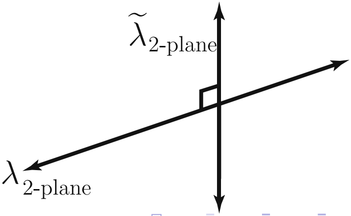

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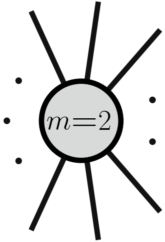

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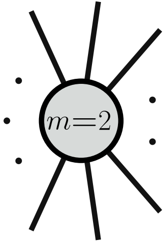

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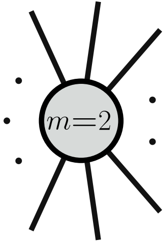

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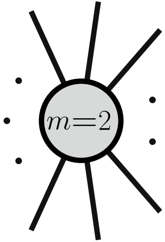

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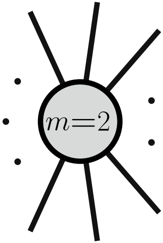

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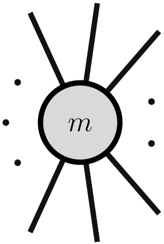

$$\Lambda \equiv \begin{pmatrix} \Lambda_1^1 & \Lambda_2^1 & \Lambda_3^1 & \cdots & \Lambda_n^1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Lambda_1^m & \Lambda_2^m & \Lambda_3^m & \cdots & \Lambda_n^m \end{pmatrix} \in G(m,n)$$

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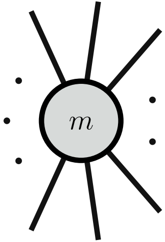

$$C \equiv \begin{pmatrix} c_1^1 & c_2^1 & c_3^1 & \cdots & c_n^1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_1^m & c_2^m & c_3^m & \cdots & c_n^m \end{pmatrix} \in G(m,n)$$

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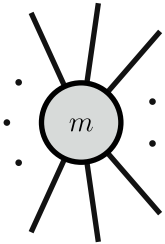

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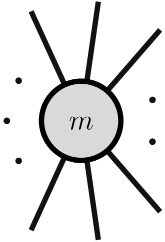

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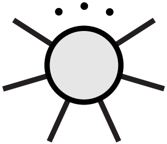

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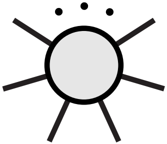

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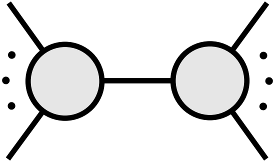

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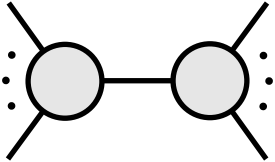

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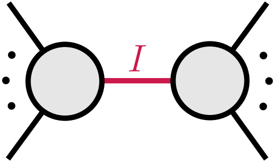

Internal Particles:

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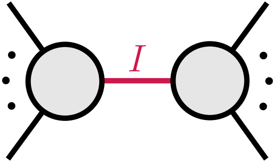

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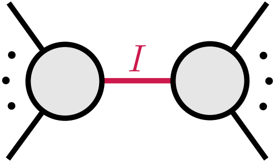

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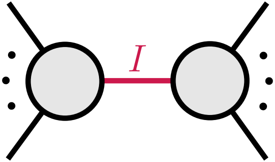

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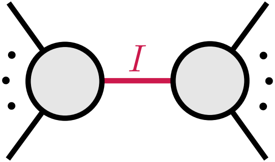

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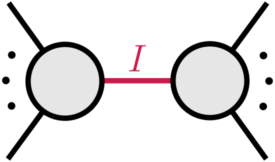

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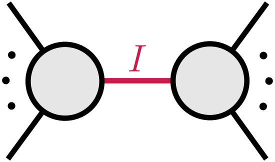

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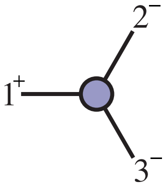

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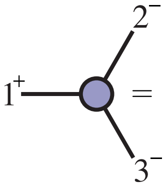

$$= \frac{\langle 23\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})$$

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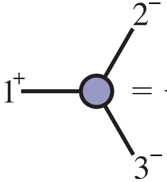

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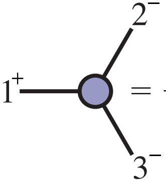

$$= \frac{\langle 23\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\delta^{2\times 2}(\lambda\cdot\tilde{\lambda})$$

λ's are generic

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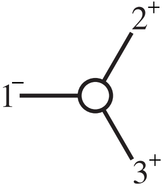

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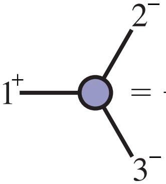

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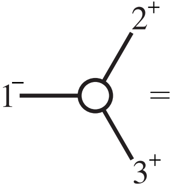

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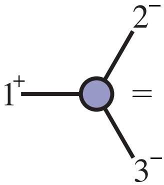

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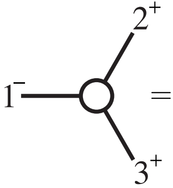

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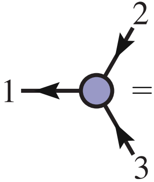

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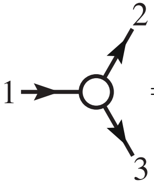

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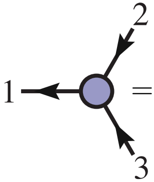

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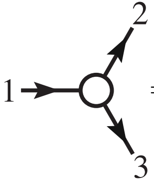

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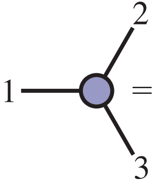

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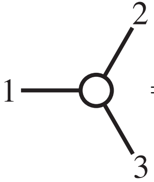

$$= \frac{\delta^{1\times4}(\widetilde{\lambda}^\perp\cdot\widetilde{\eta})}{[12]\,[23]\,[31]}\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})$$

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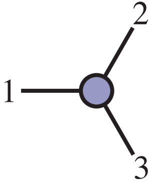

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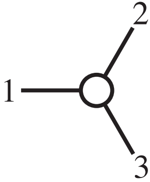

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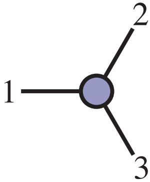

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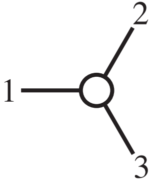

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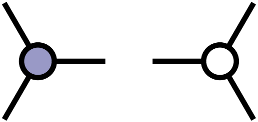

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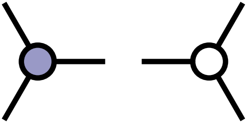

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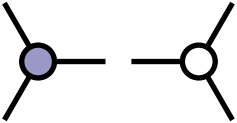

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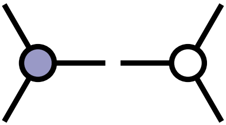

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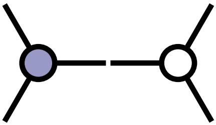

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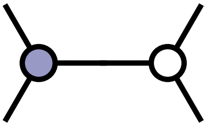

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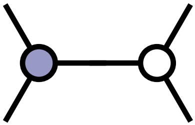

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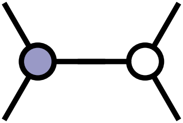

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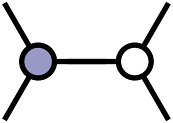

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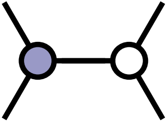

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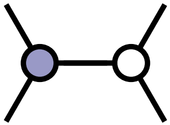

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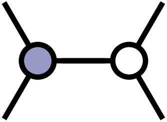

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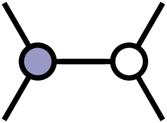

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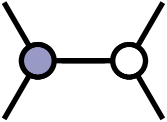

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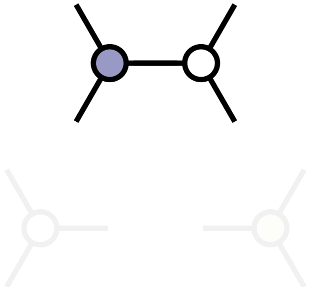

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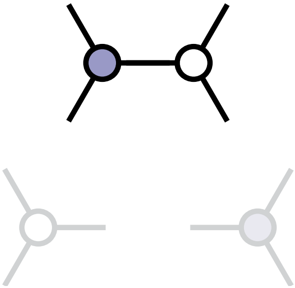

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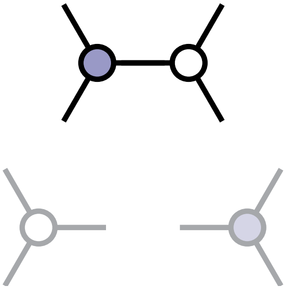

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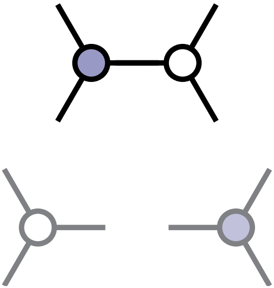

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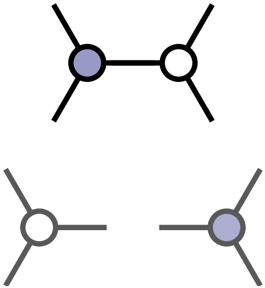

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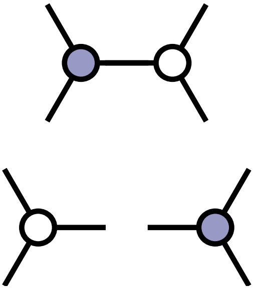

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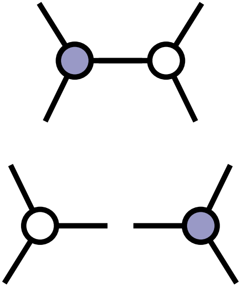

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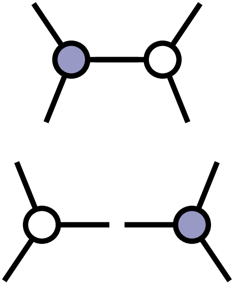

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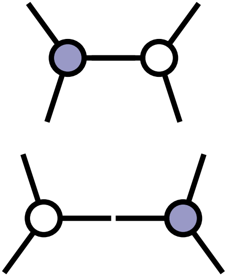

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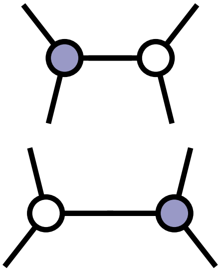

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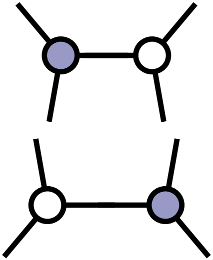

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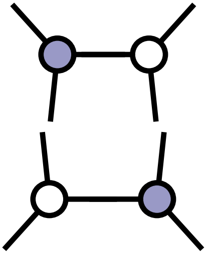

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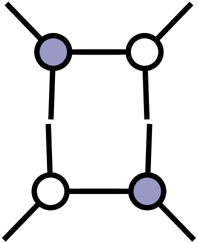

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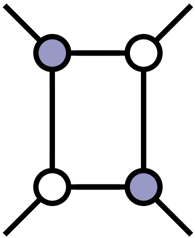

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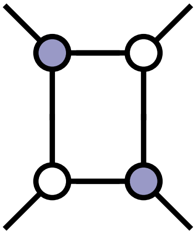

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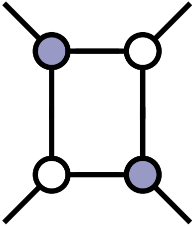

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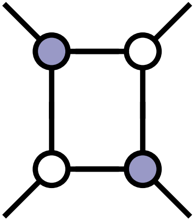

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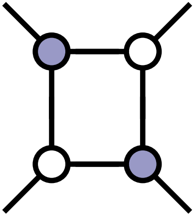

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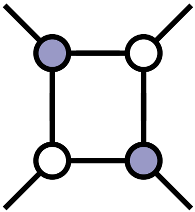

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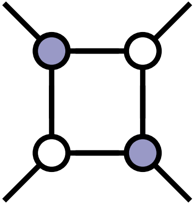

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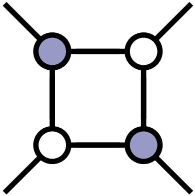

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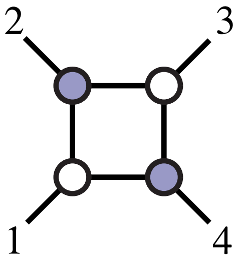

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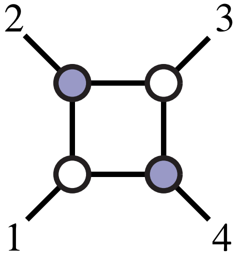

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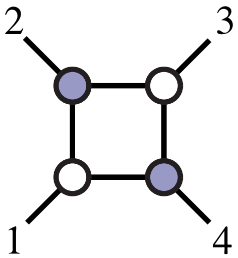

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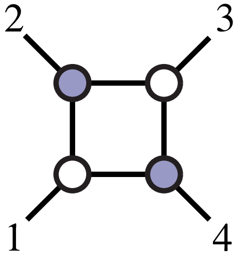

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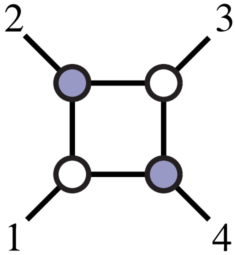

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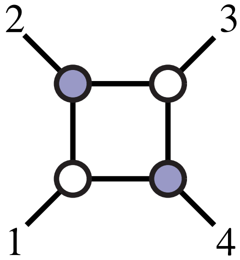

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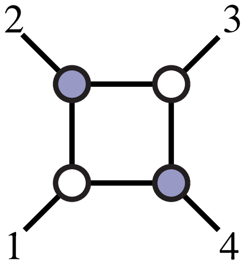

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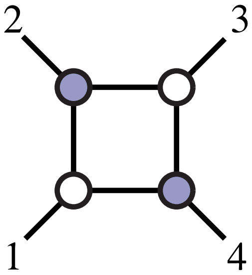

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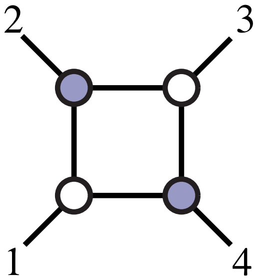

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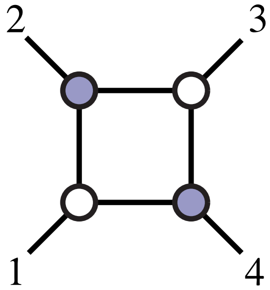

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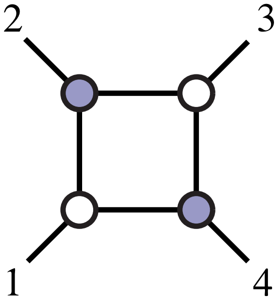

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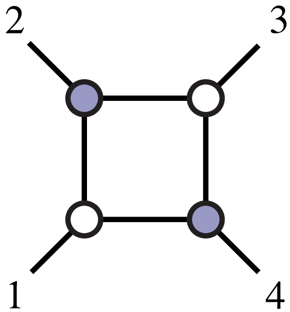

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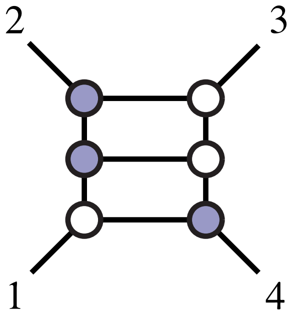

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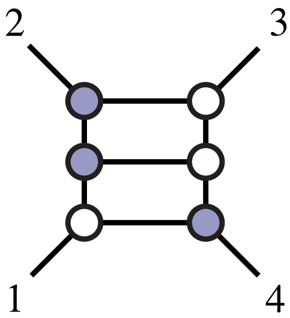

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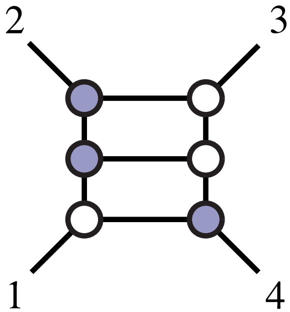

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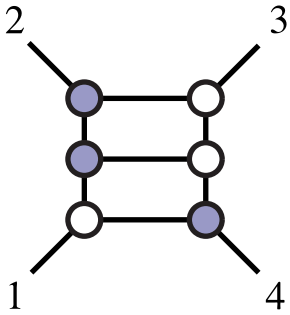

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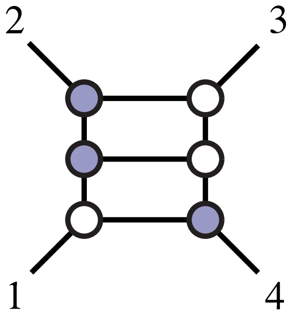

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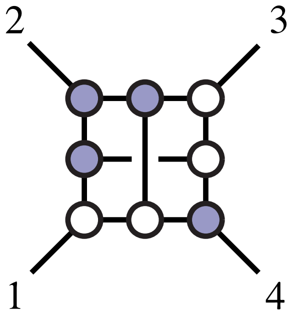

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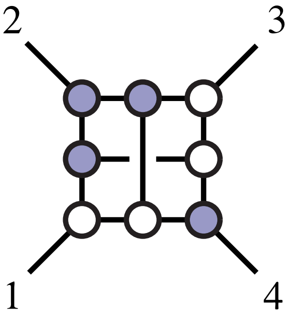

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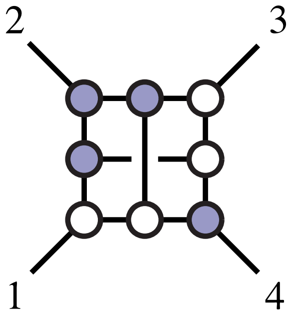

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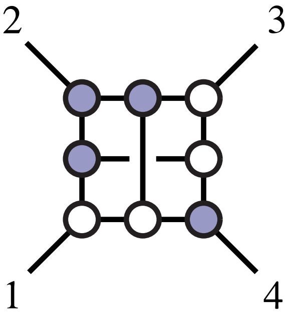

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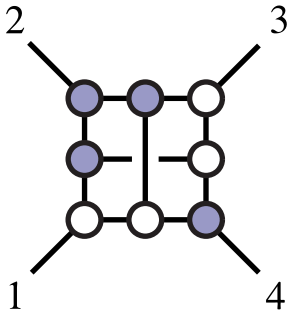

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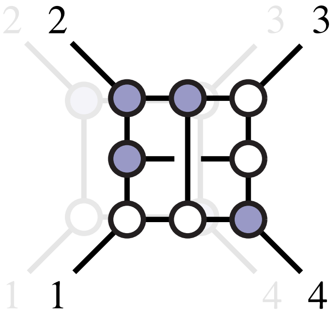

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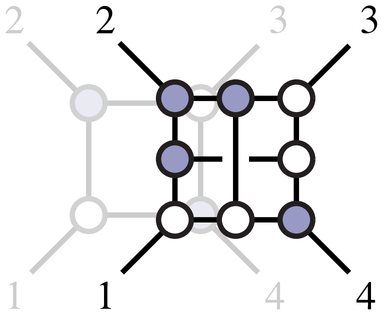

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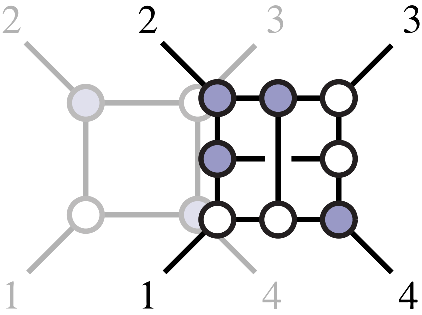

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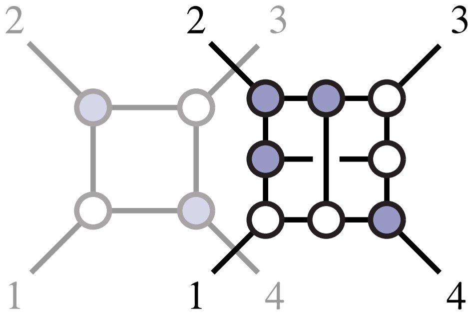

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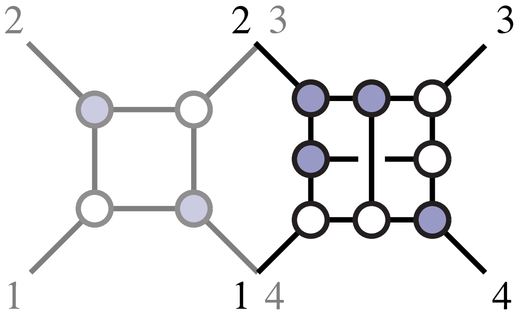

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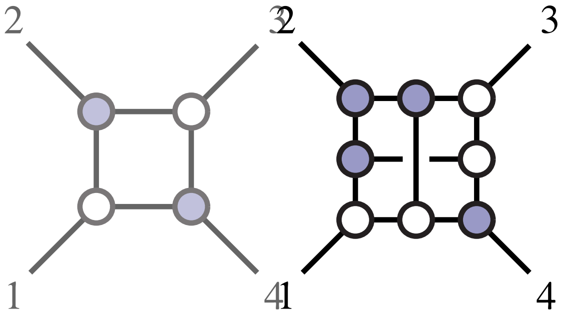

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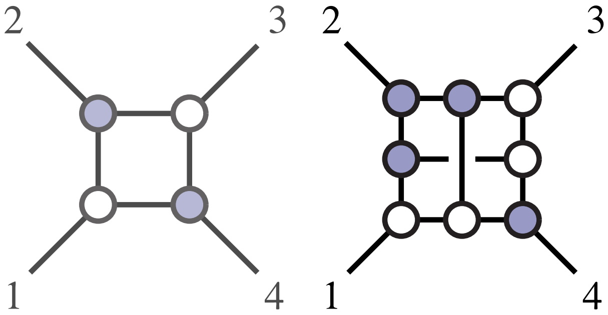

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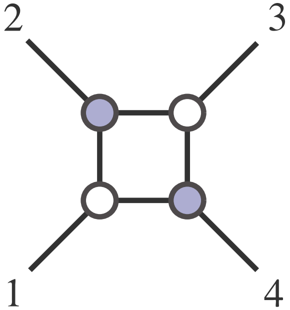

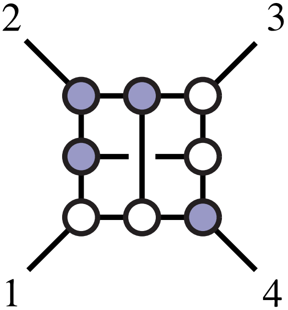

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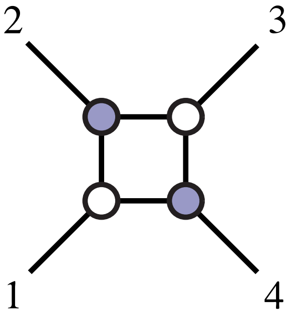

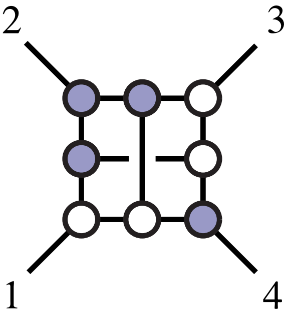

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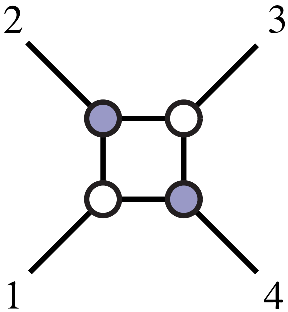

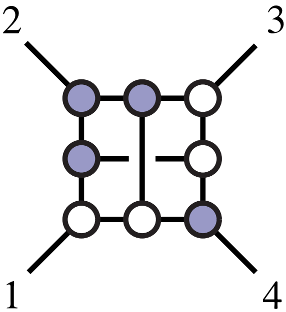

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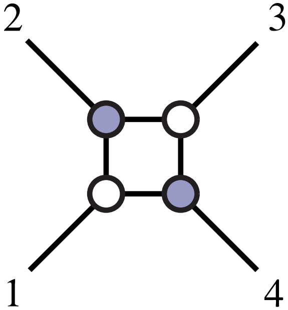

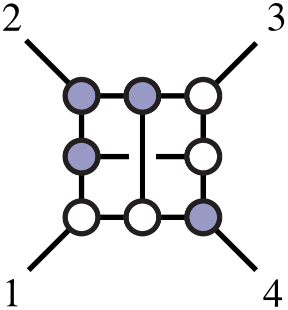

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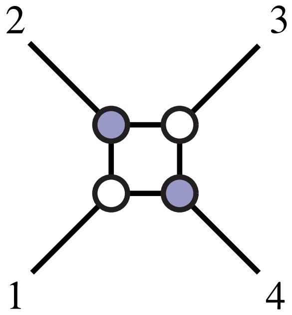

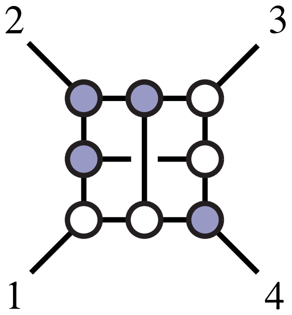

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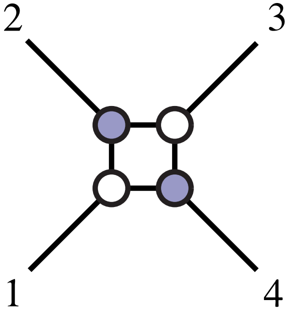

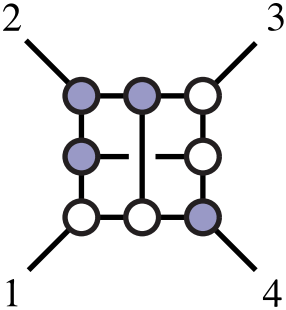

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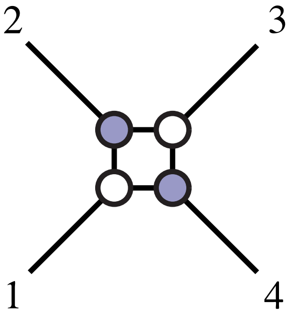

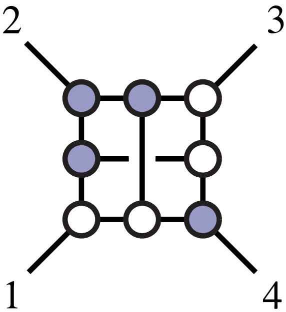

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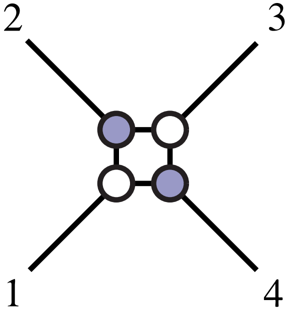

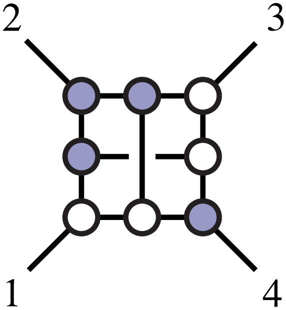

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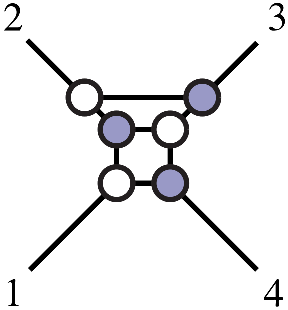

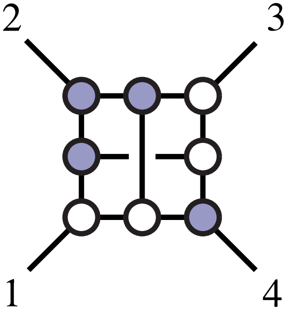

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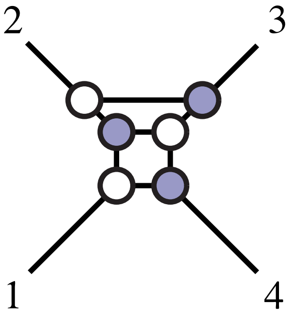

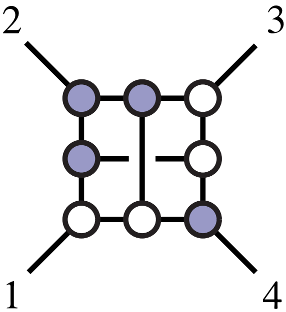

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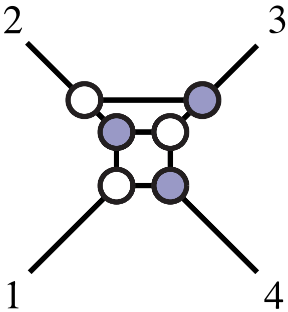

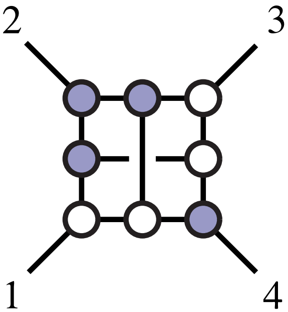

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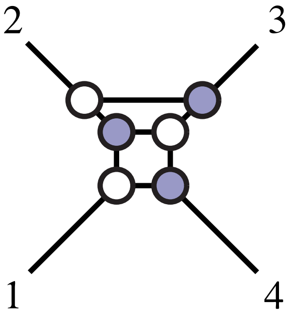

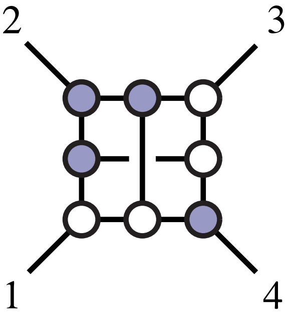

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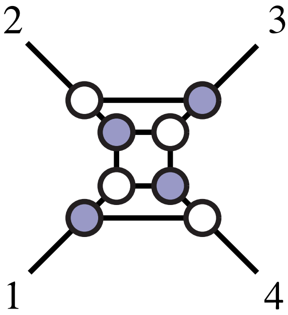

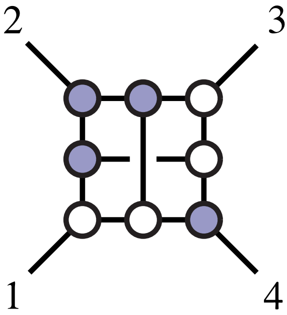

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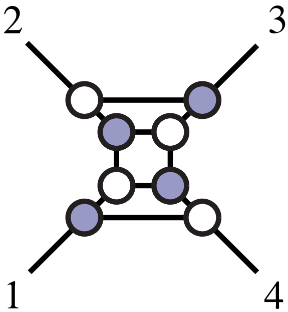

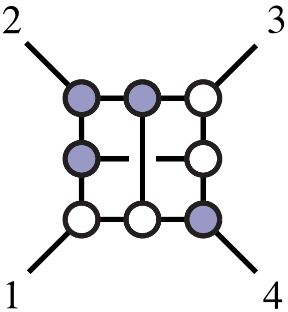

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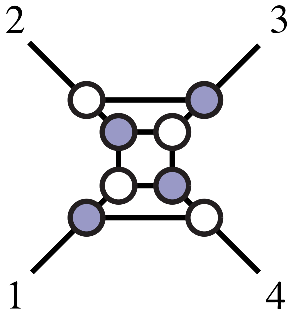

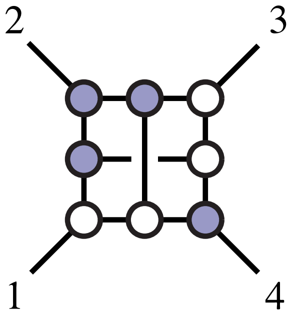

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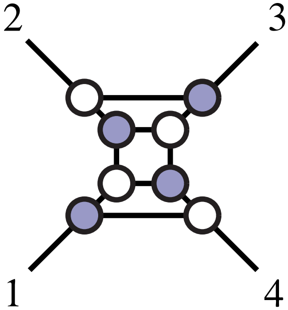

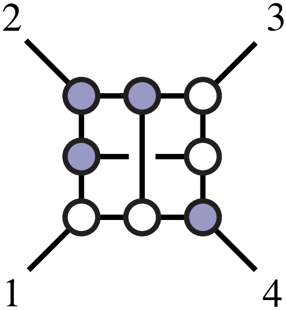

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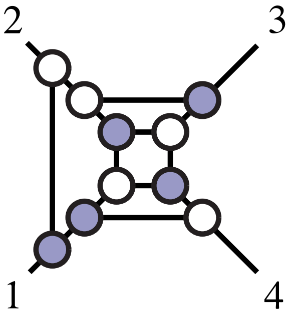

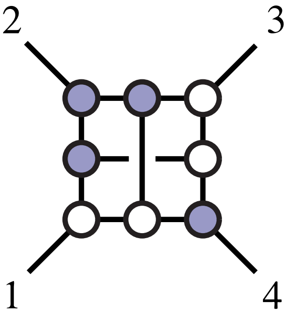

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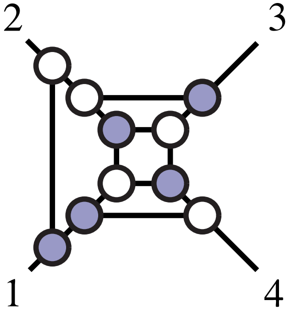

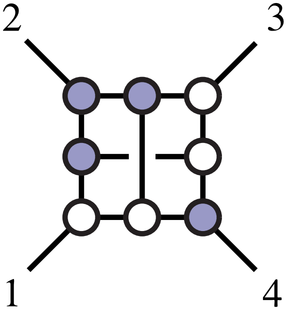

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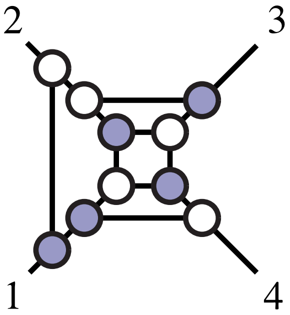

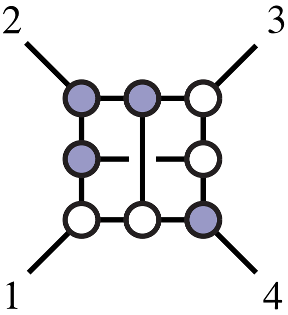

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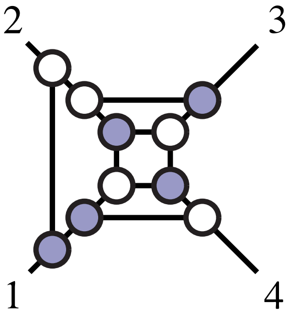

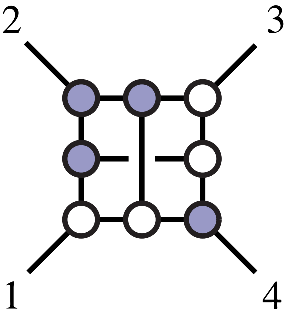

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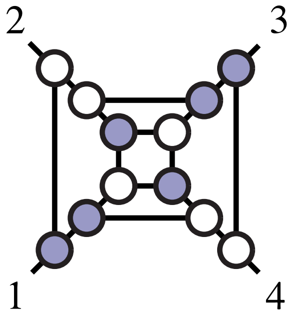

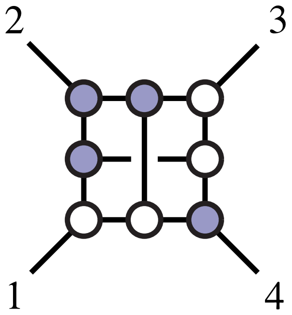

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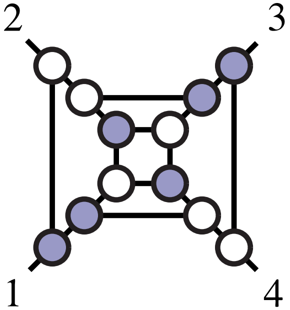

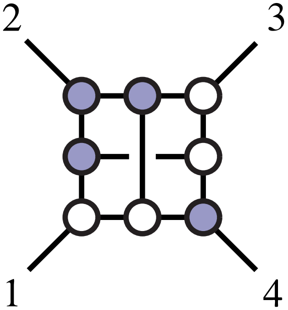

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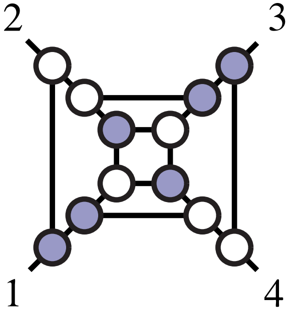

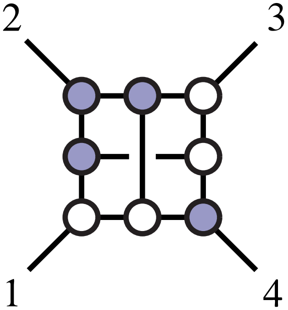

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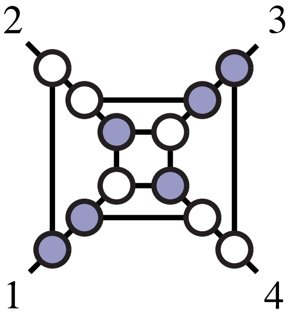

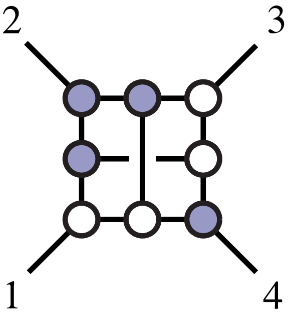

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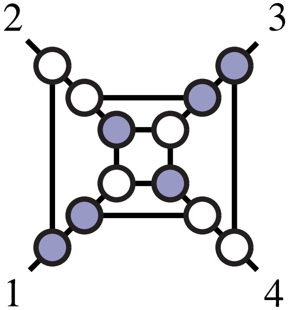

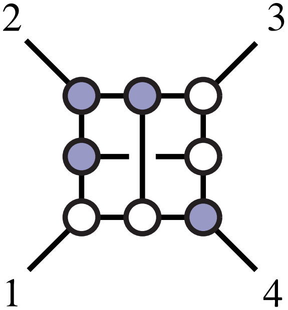

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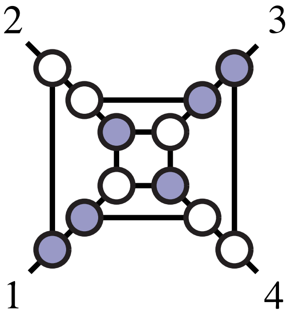

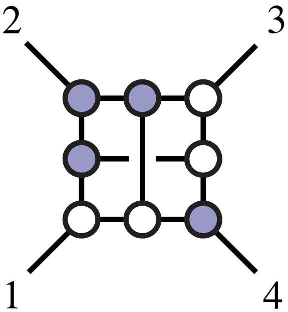

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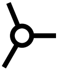

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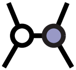

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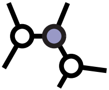

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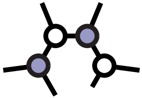

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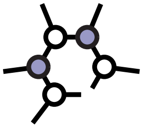

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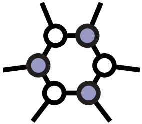

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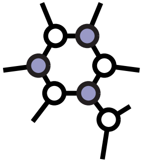

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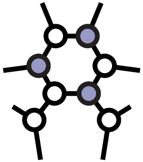

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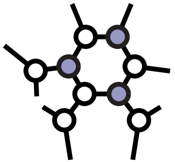

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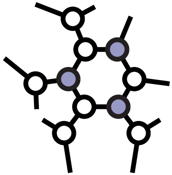

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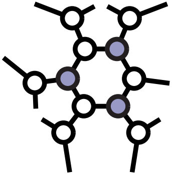

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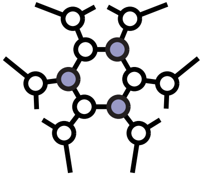

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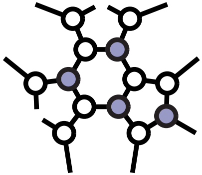

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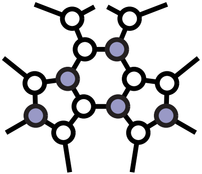

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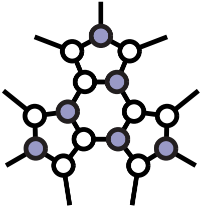

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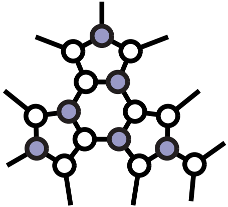

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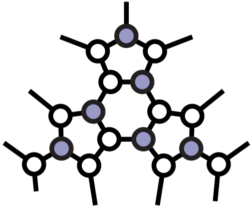

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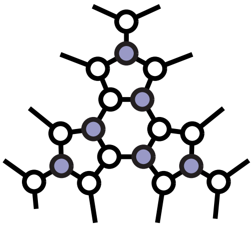

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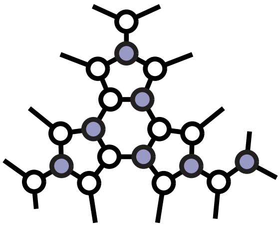

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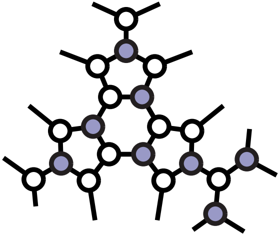

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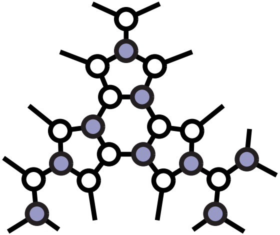

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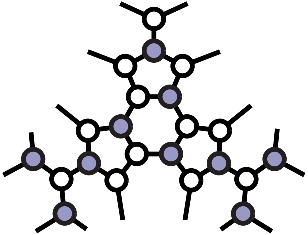

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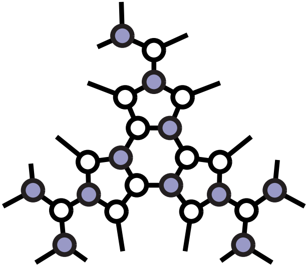

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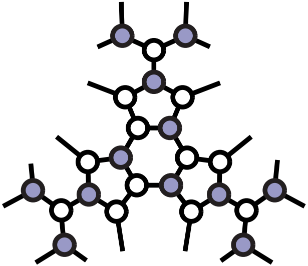

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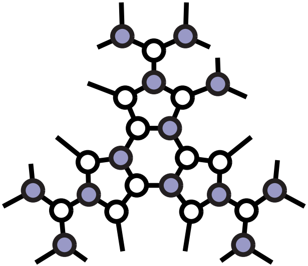

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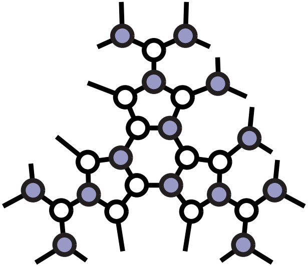

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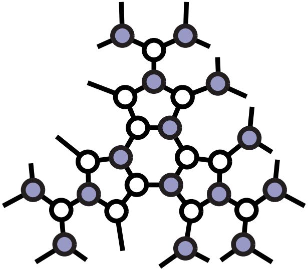

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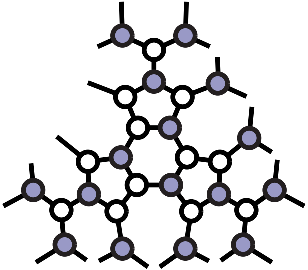

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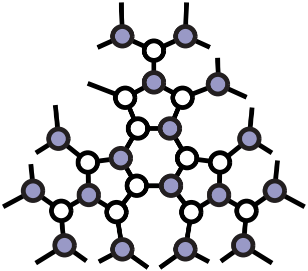

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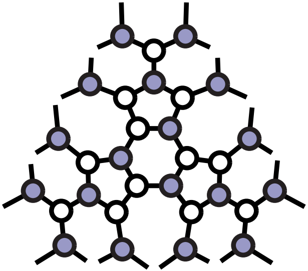

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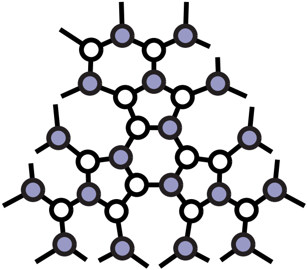

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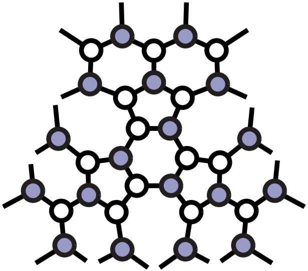

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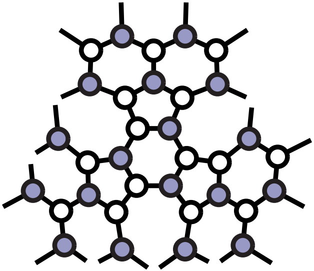

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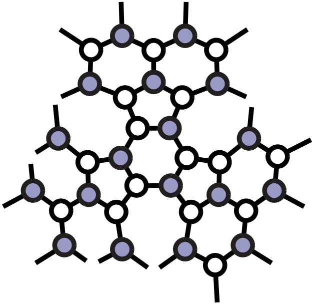

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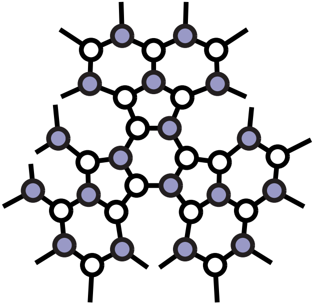

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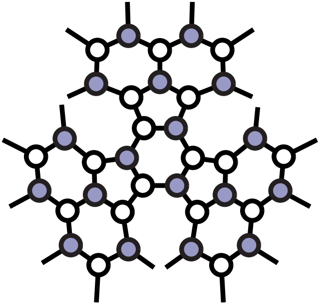

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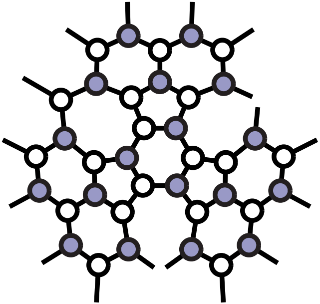

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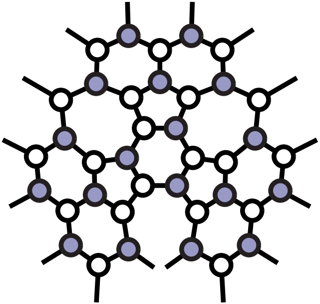

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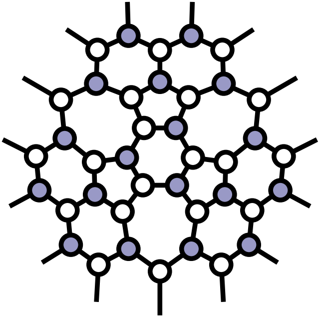

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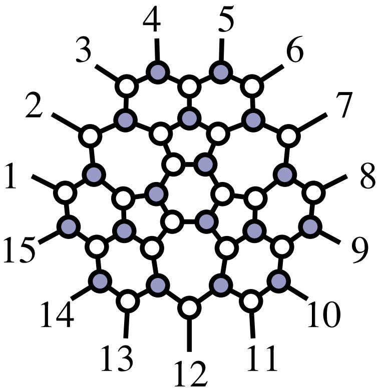

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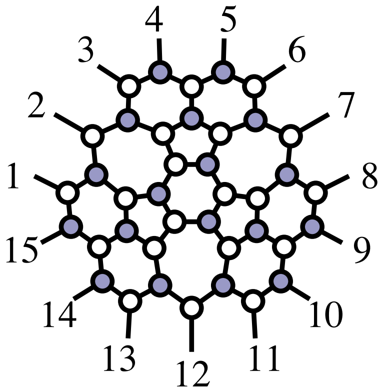

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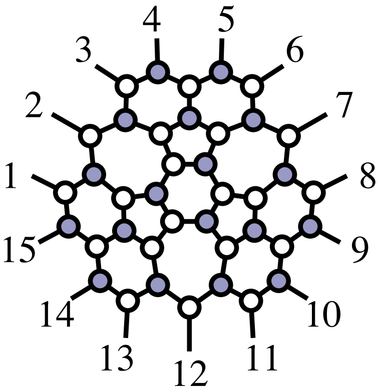

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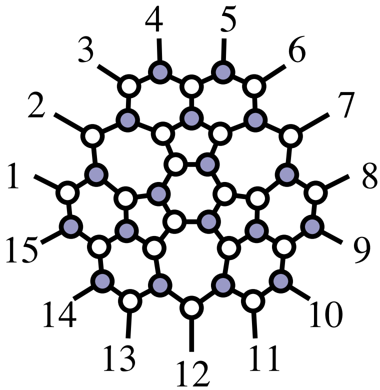

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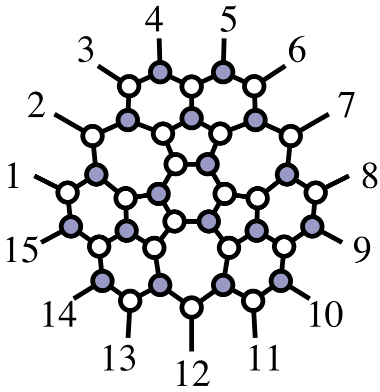

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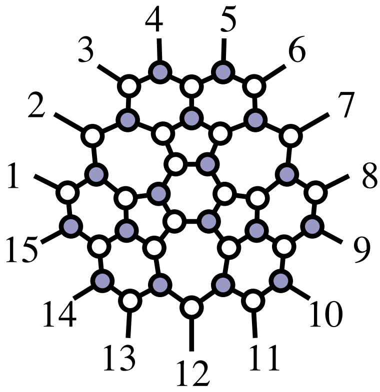

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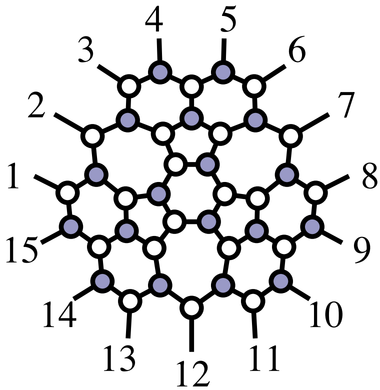

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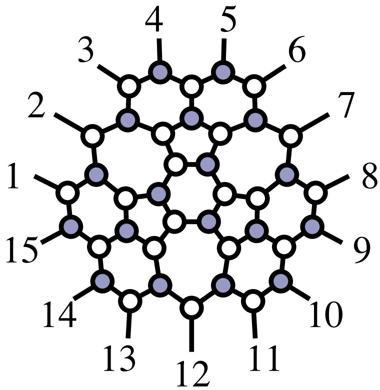

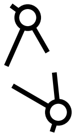

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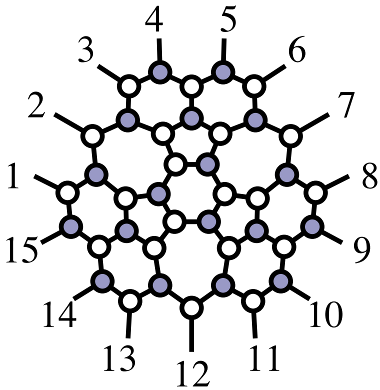

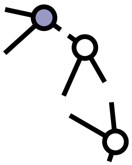

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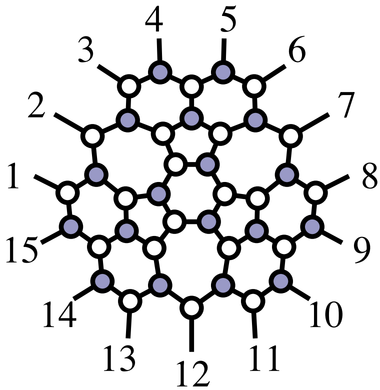

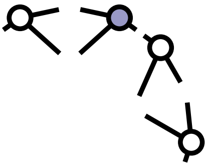

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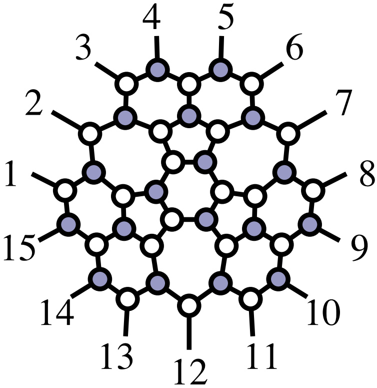

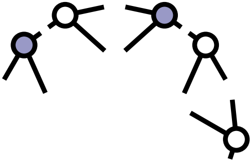

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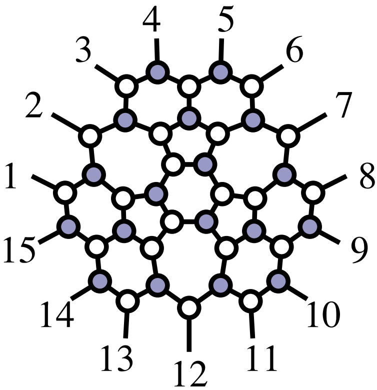

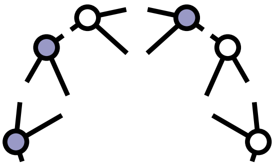

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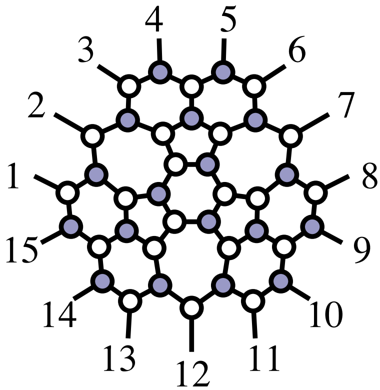

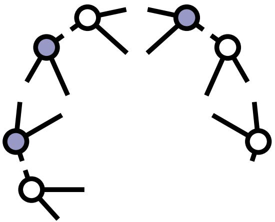

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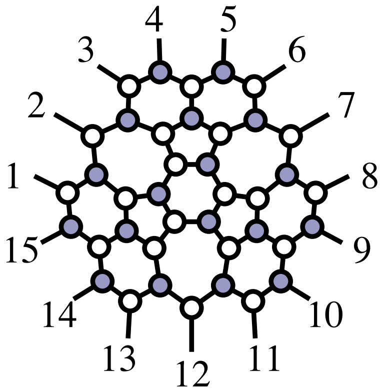

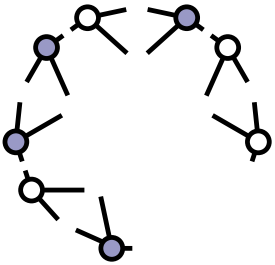

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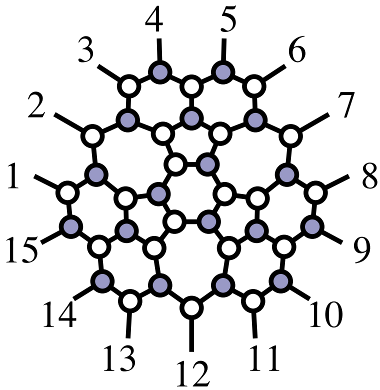

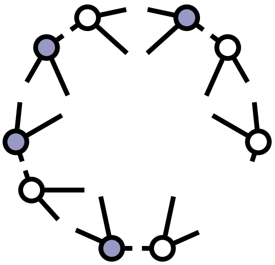

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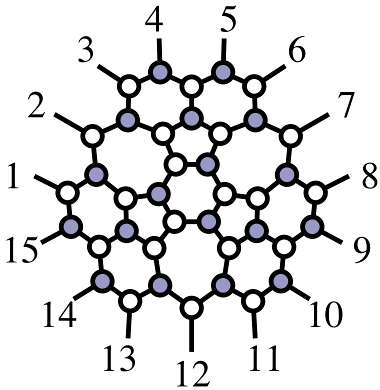

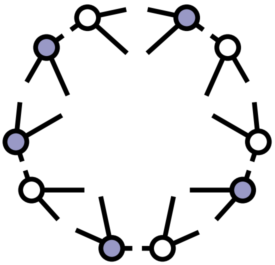

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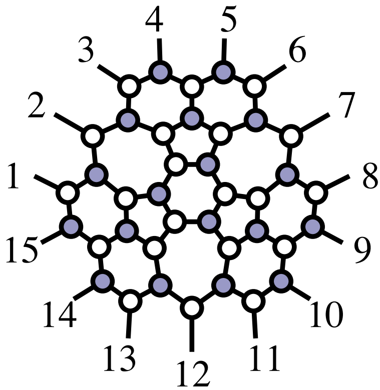

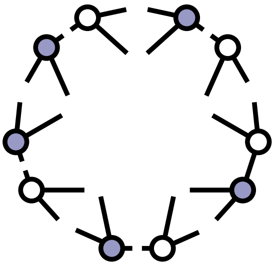

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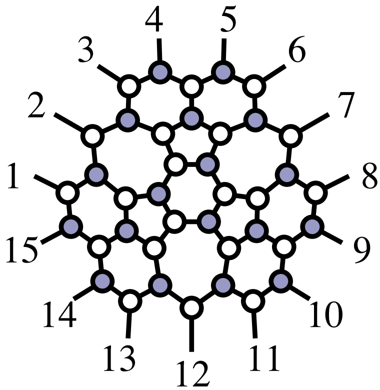

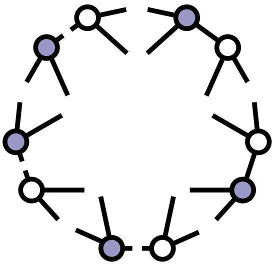

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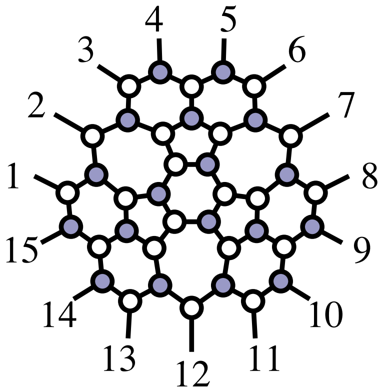

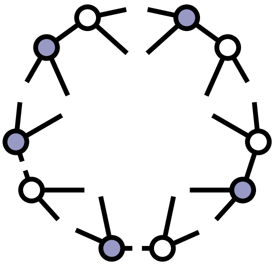

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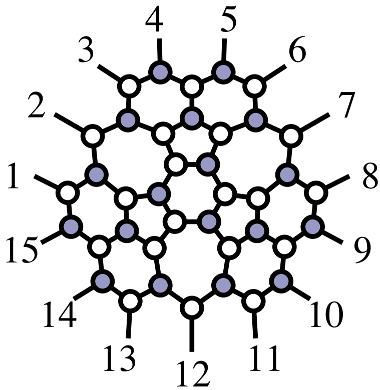

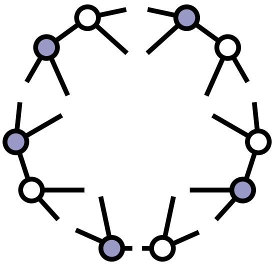

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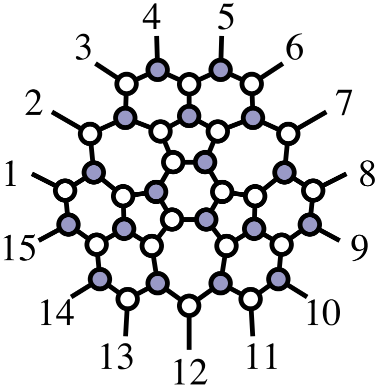

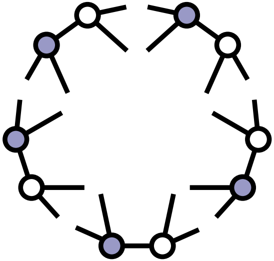

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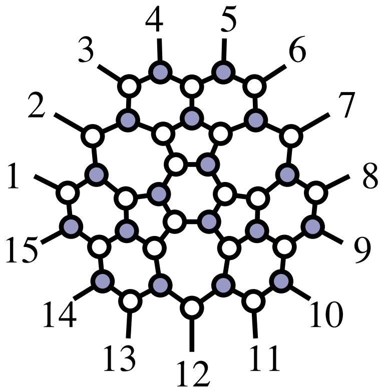

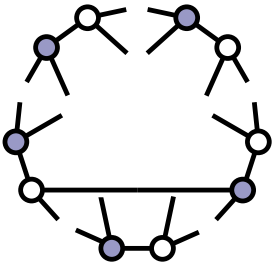

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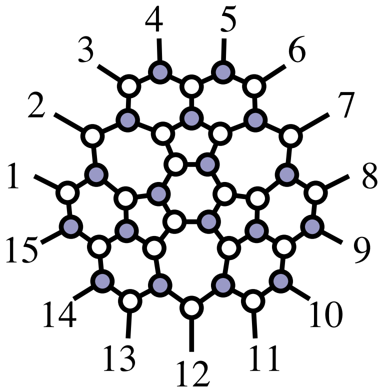

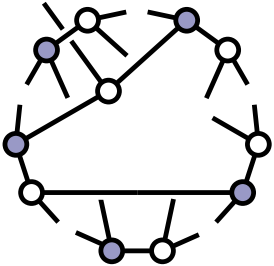

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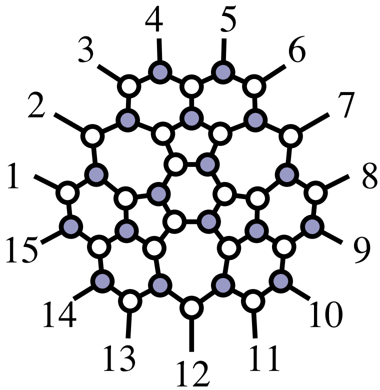

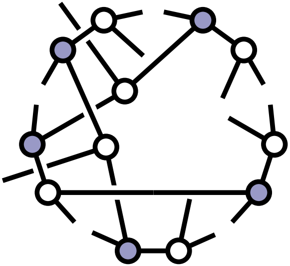

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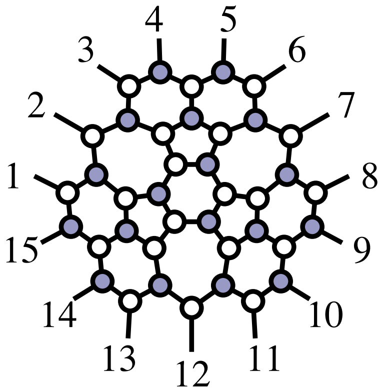

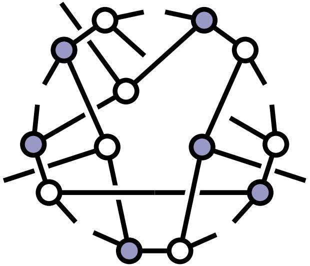

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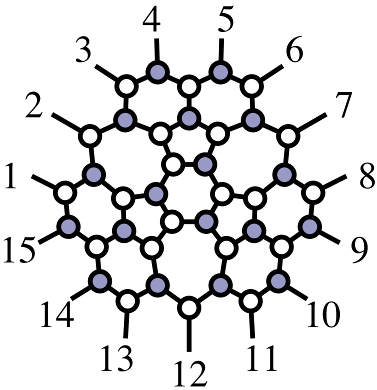

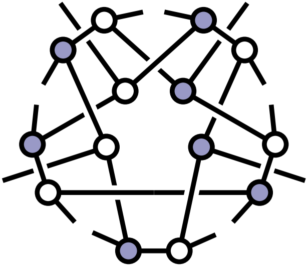

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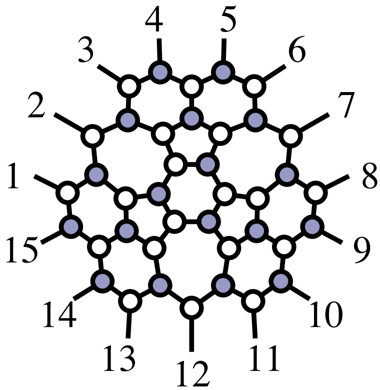

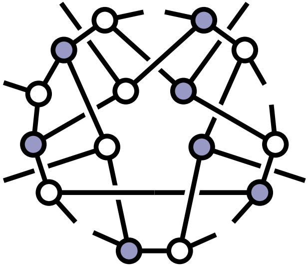

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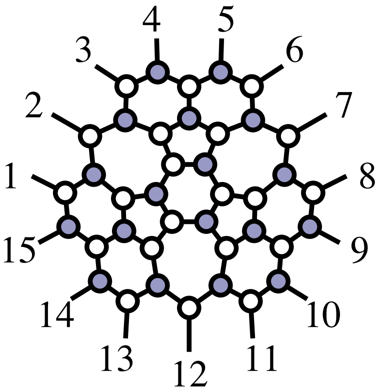

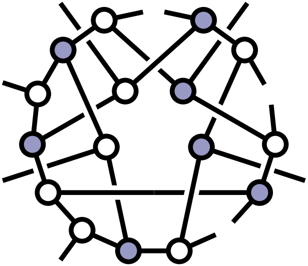

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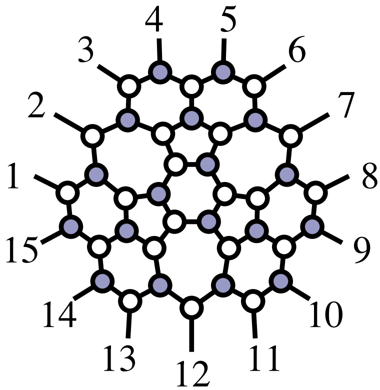

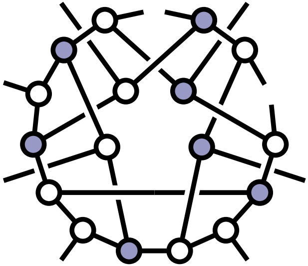

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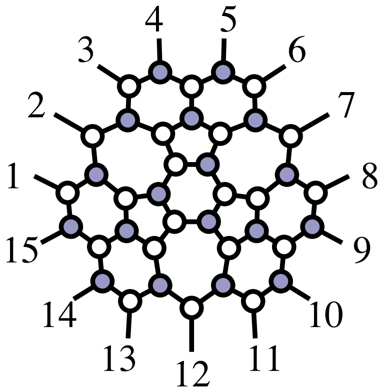

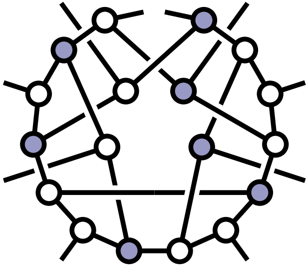

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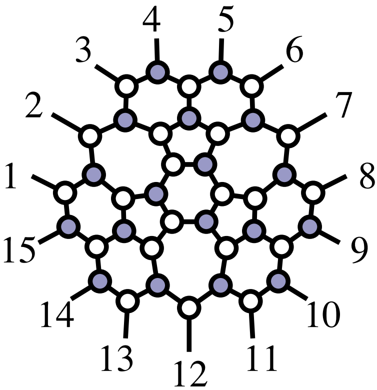

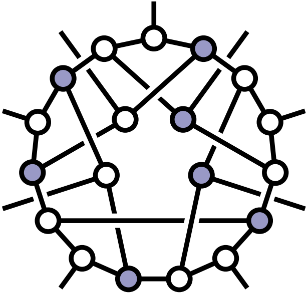

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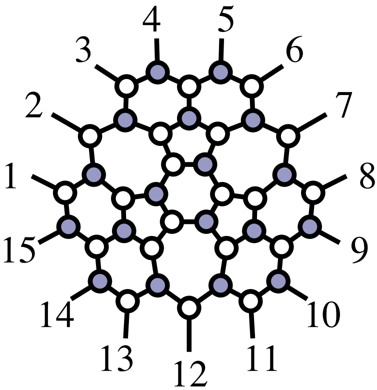

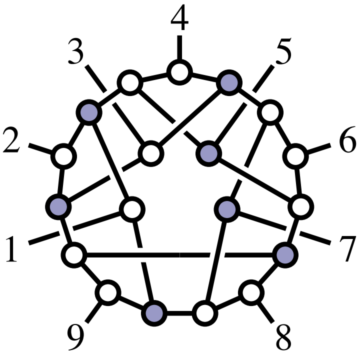

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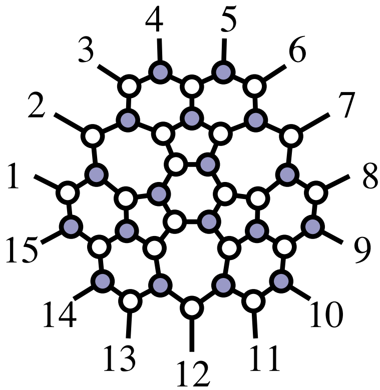

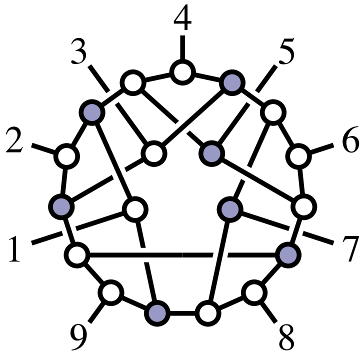

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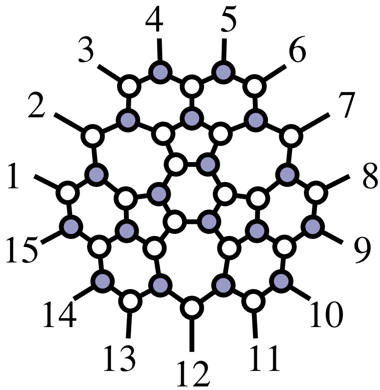

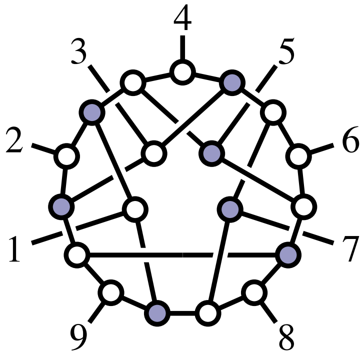

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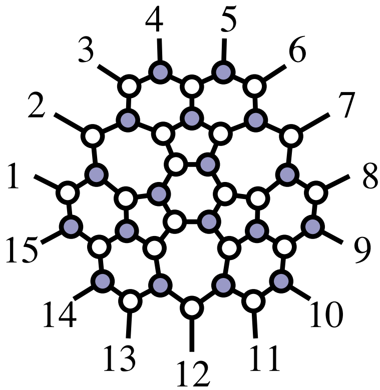

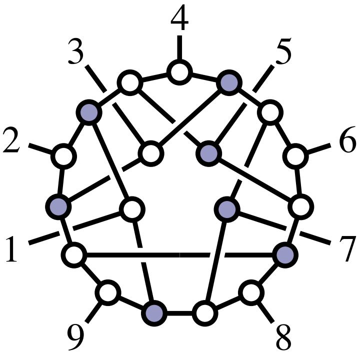

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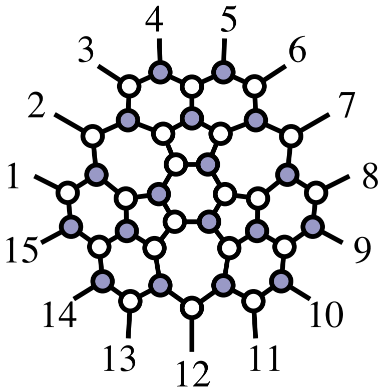

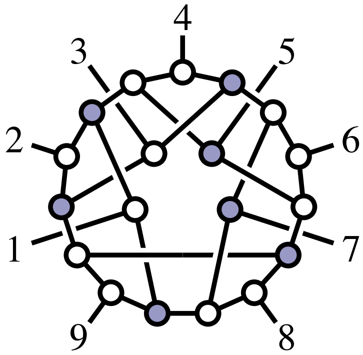

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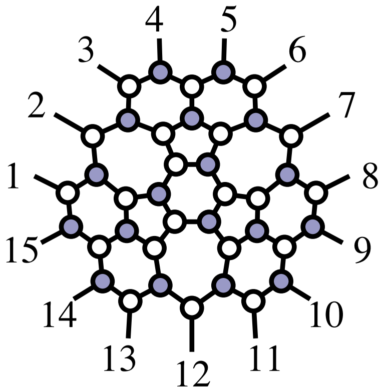

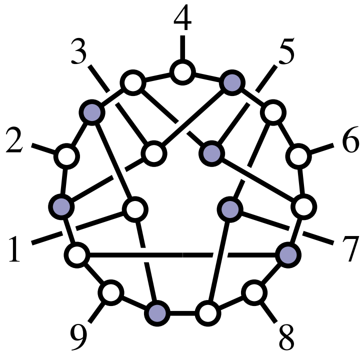

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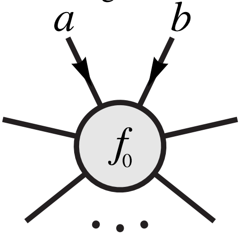

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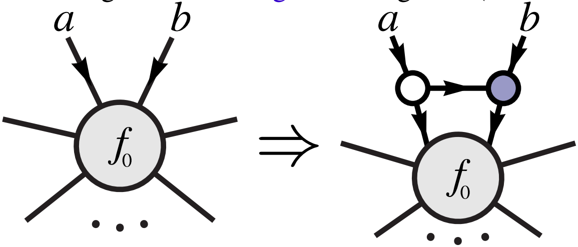

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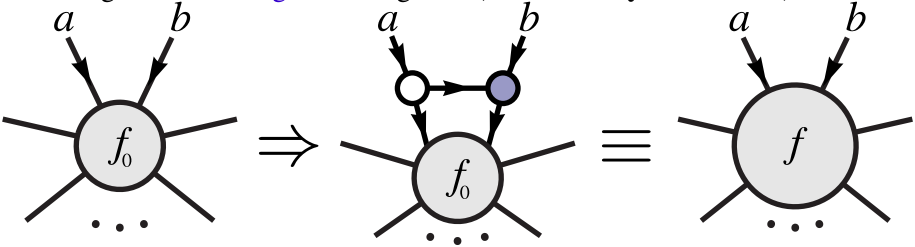

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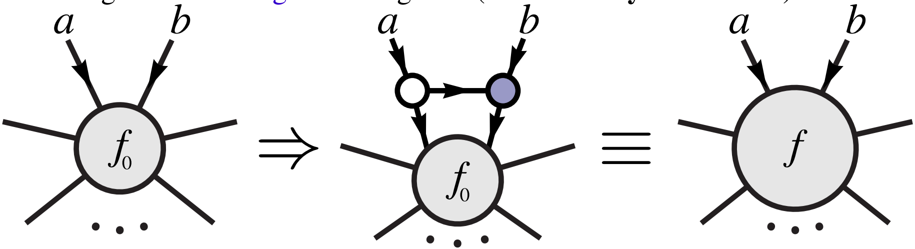

Adding the bridge has the effect of shifting the momenta p_a and p_b flowing into the diagram f_0 according to:

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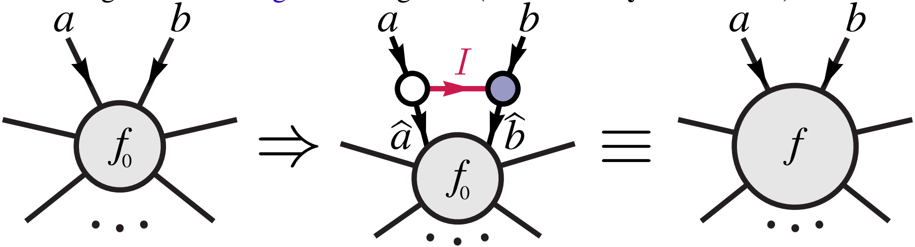

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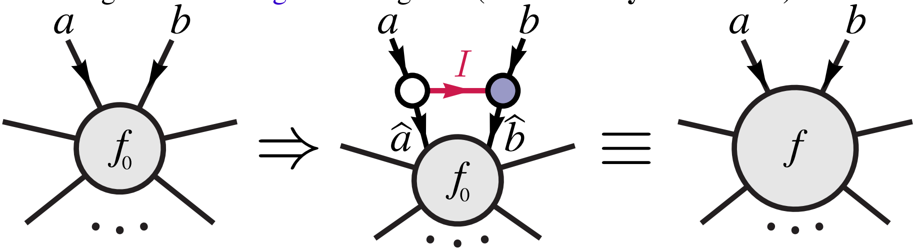

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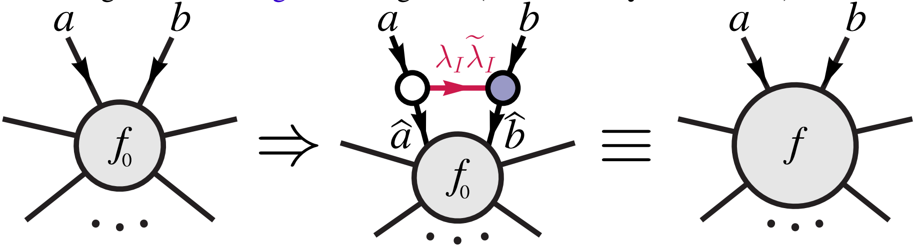

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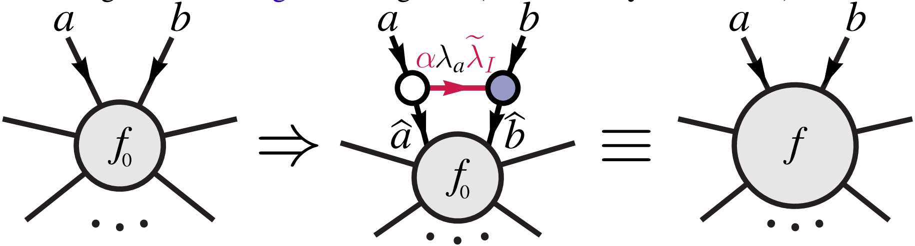

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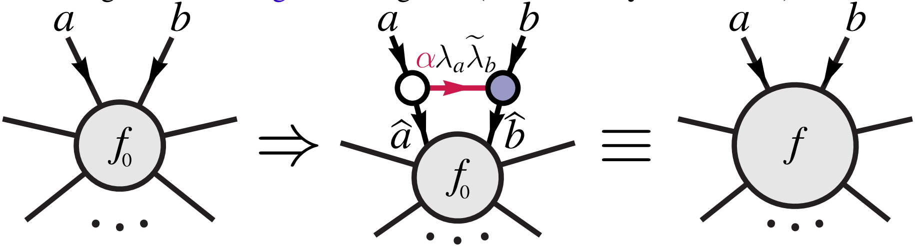

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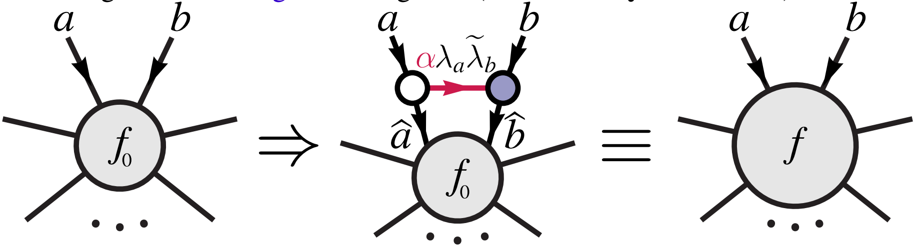

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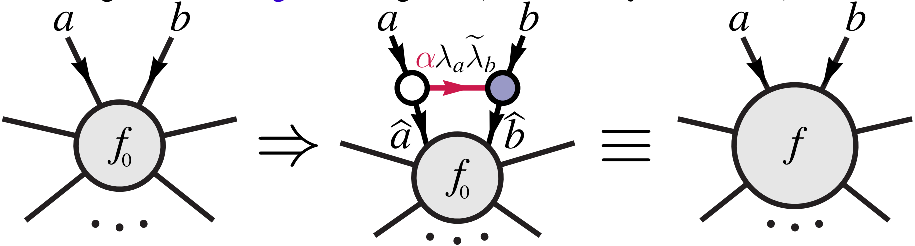

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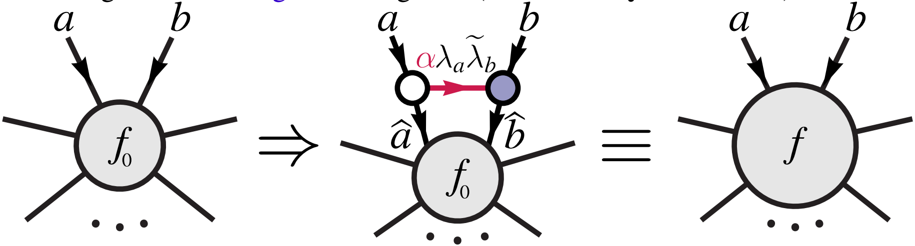

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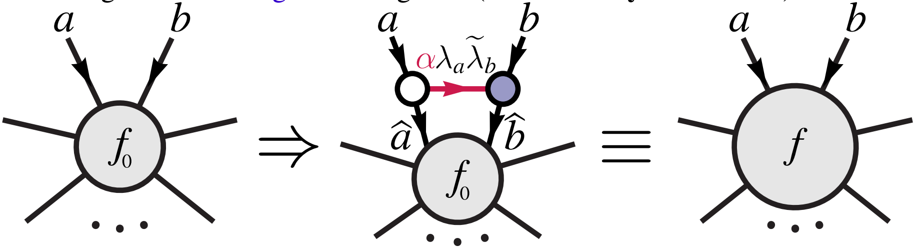

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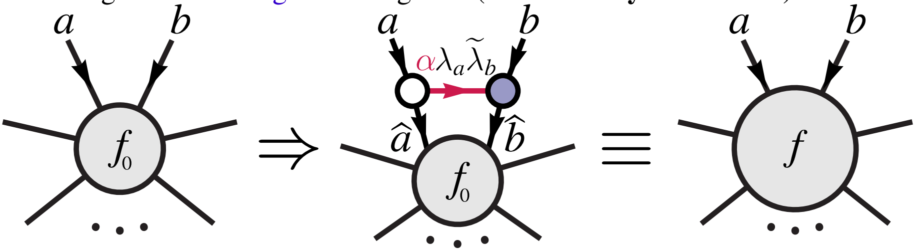

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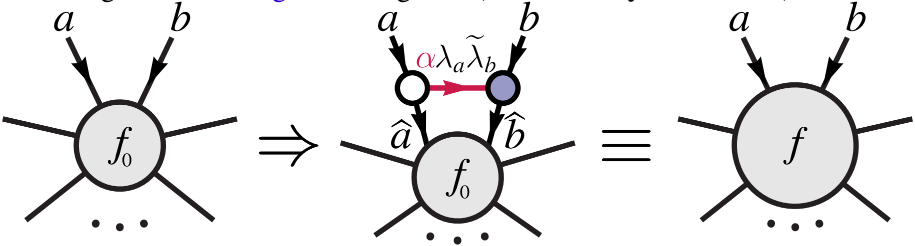

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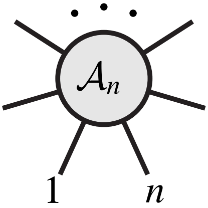

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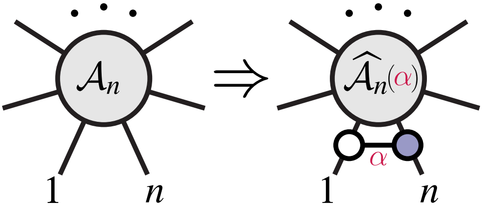

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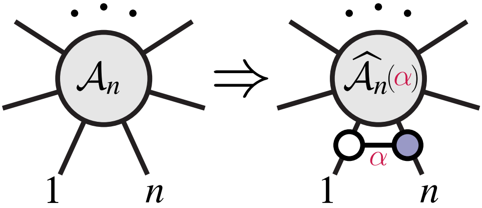

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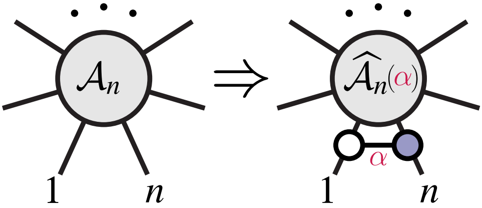

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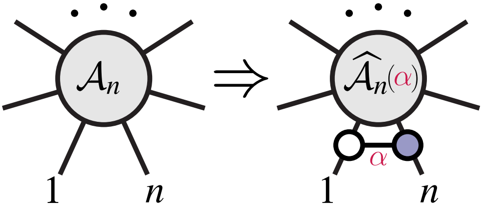

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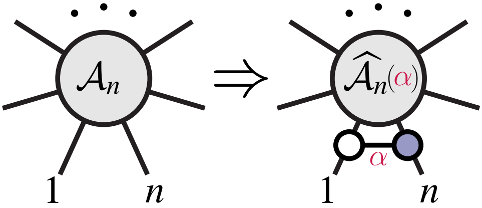

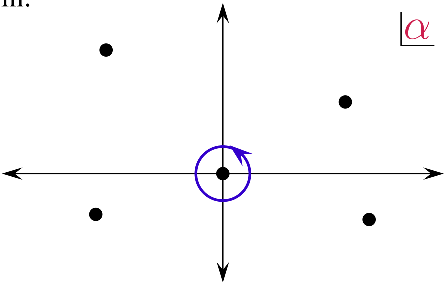

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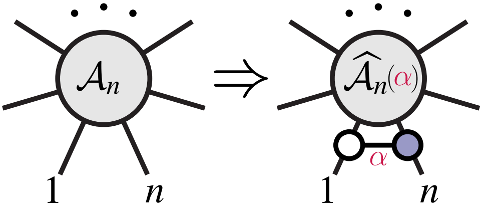

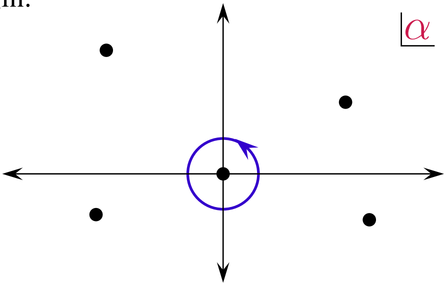

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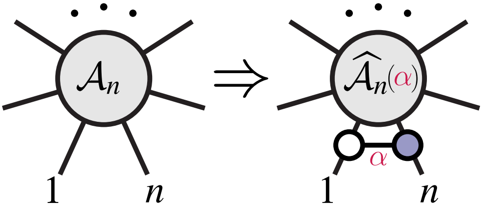

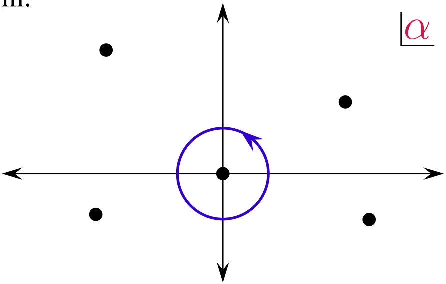

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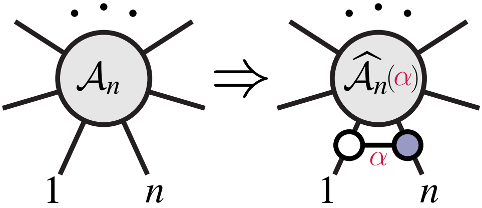

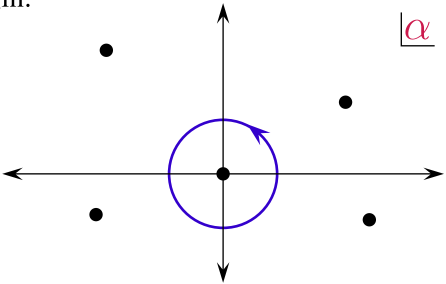

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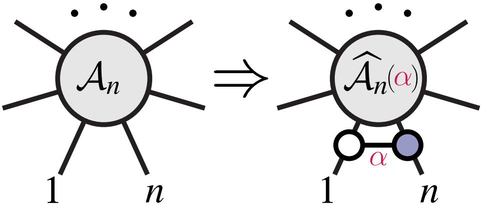

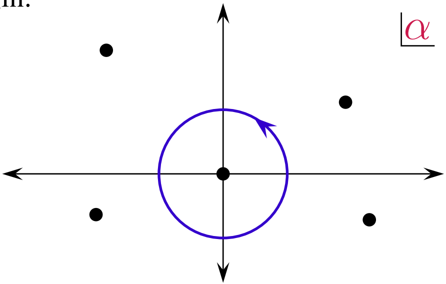

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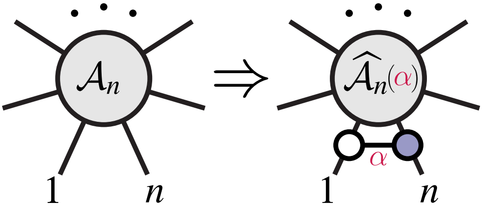

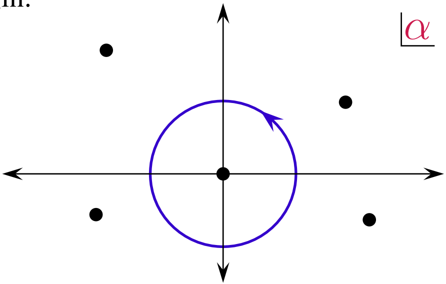

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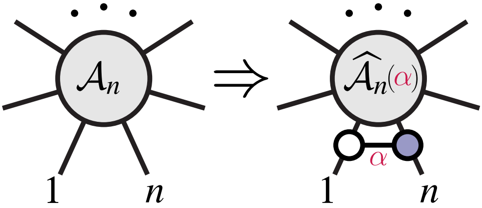

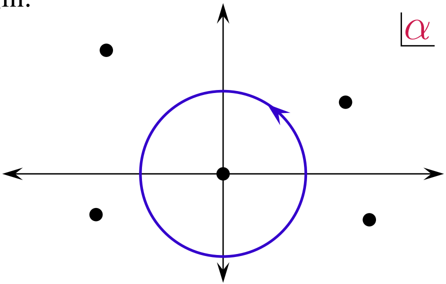

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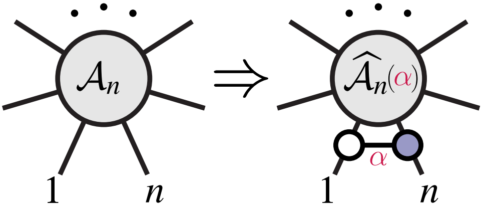

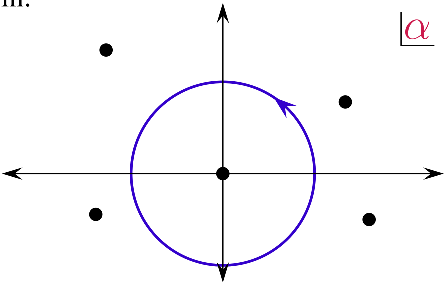

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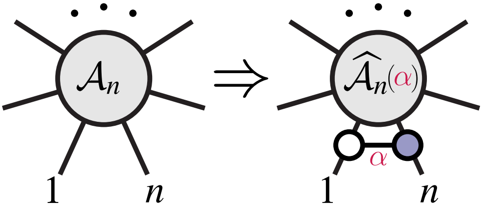

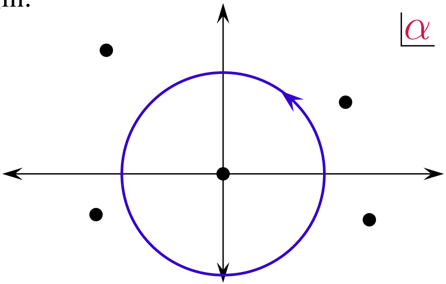

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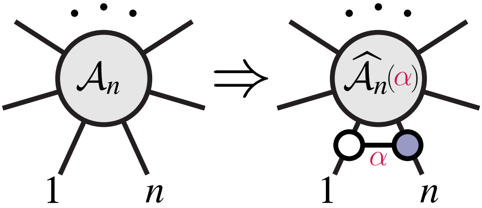

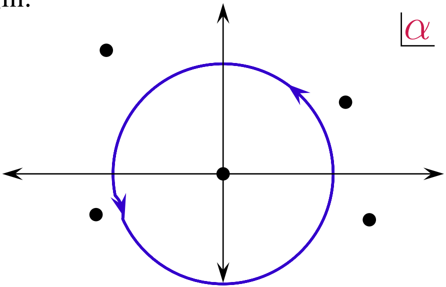

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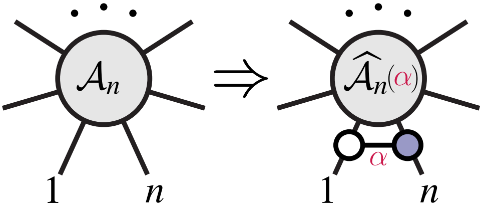

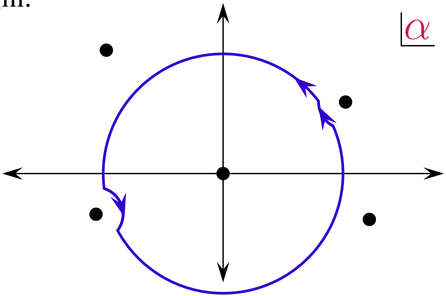

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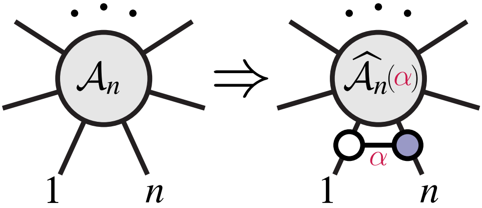

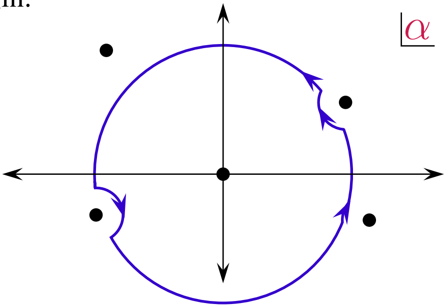

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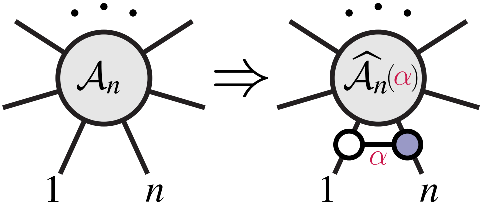

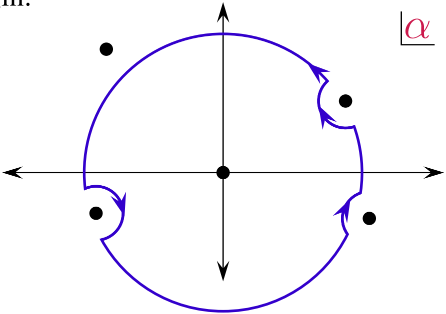

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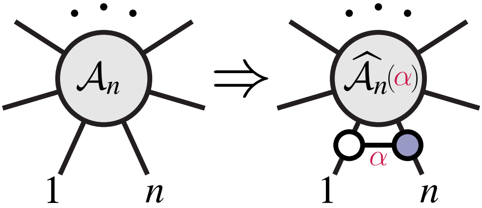

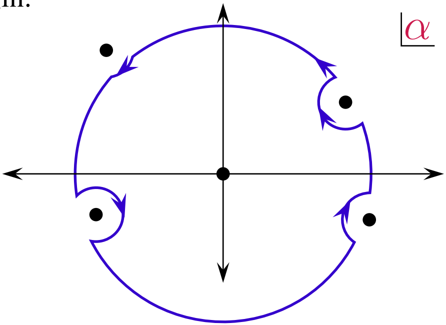

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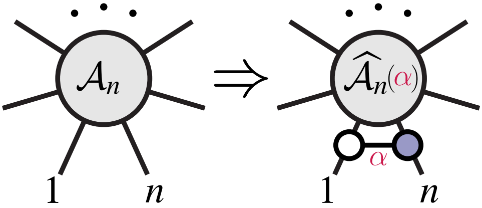

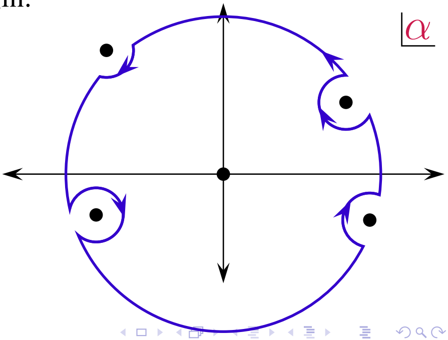

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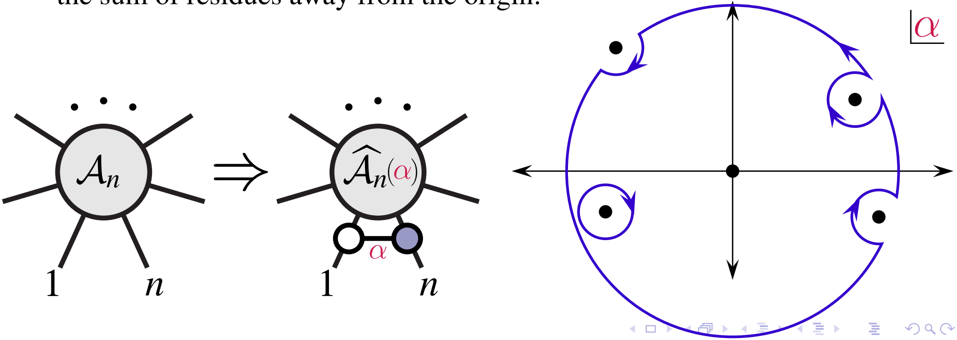

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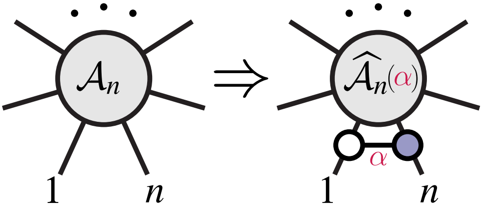

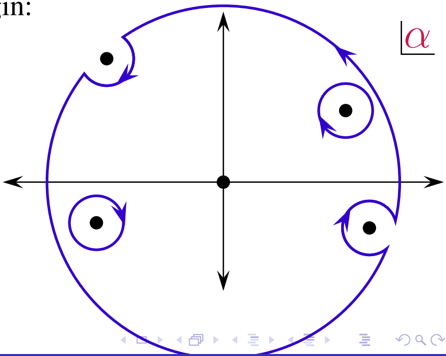

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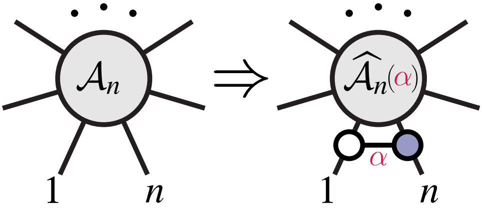

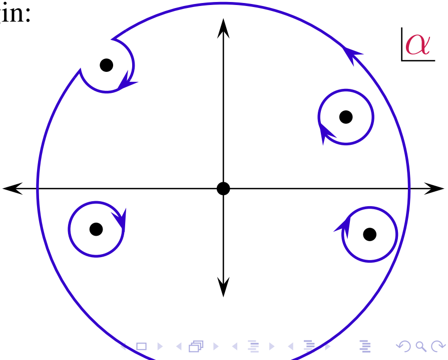

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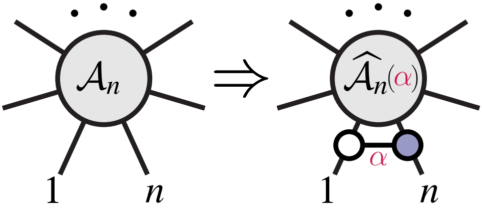

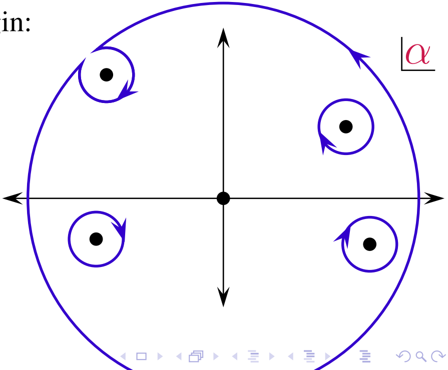

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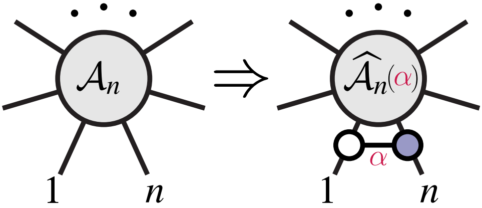

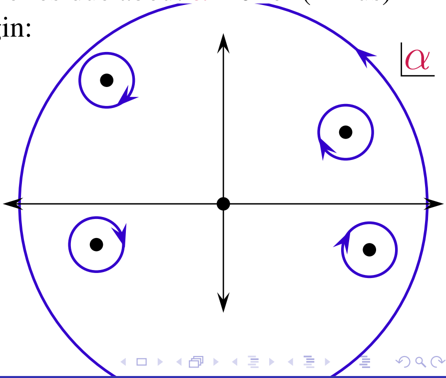

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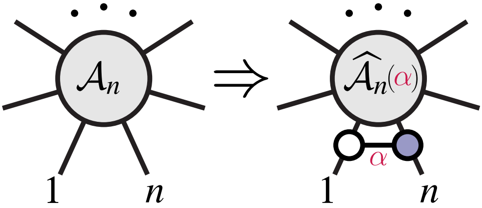

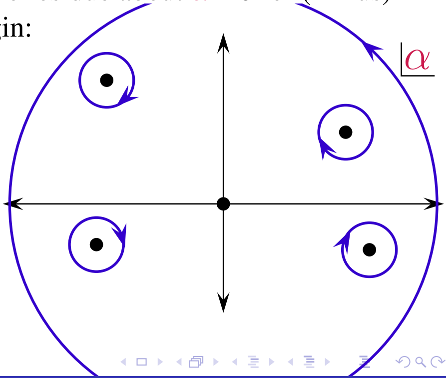

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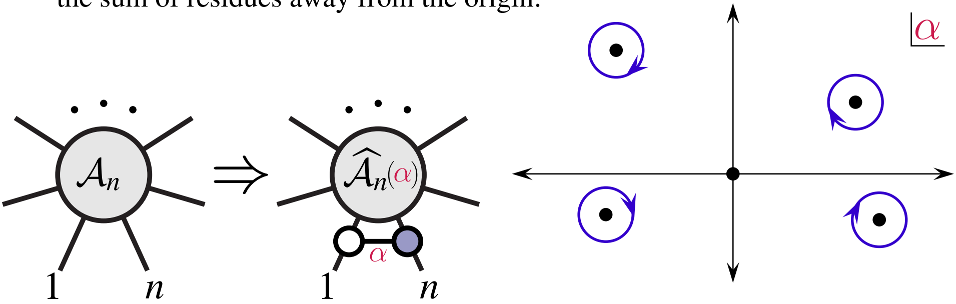

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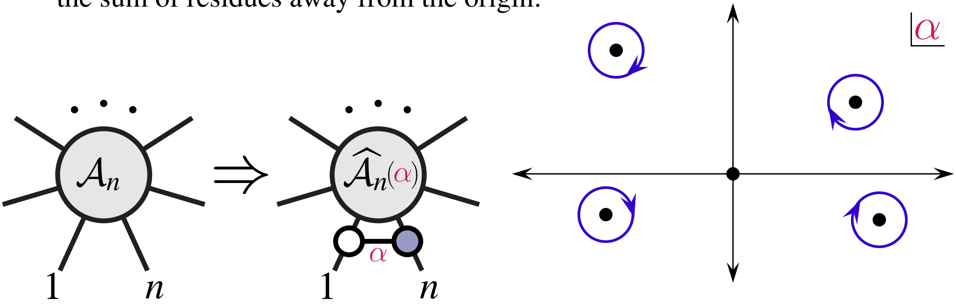

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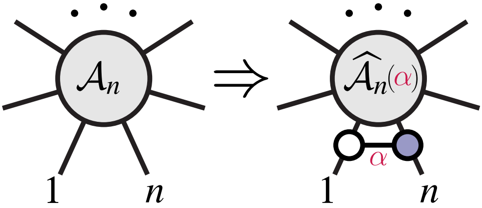

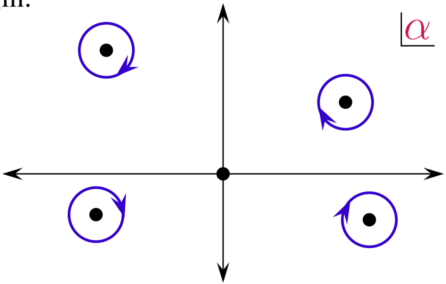

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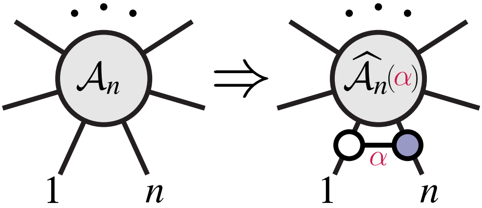

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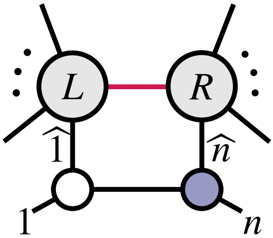

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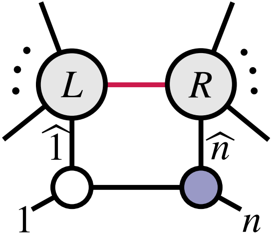

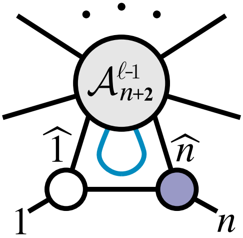

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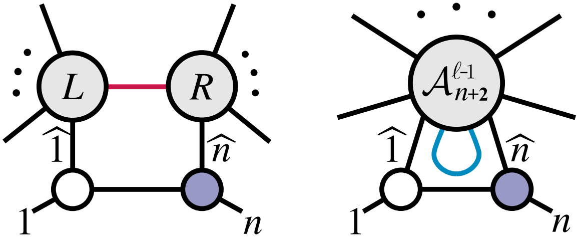

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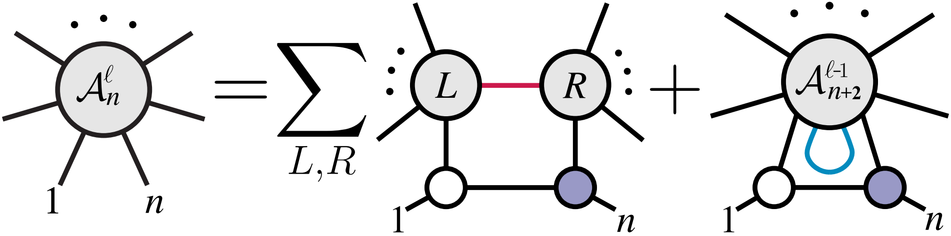

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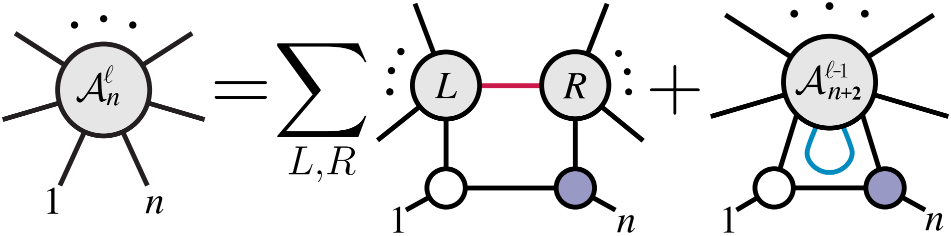

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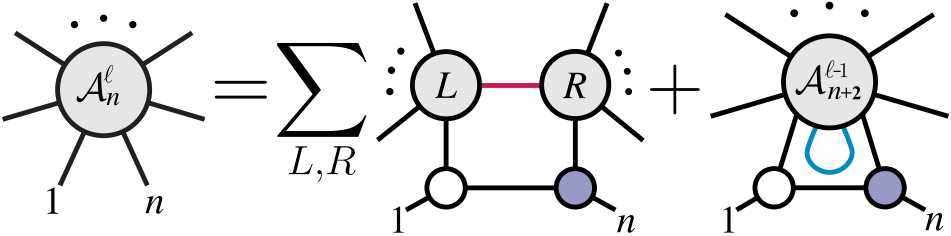

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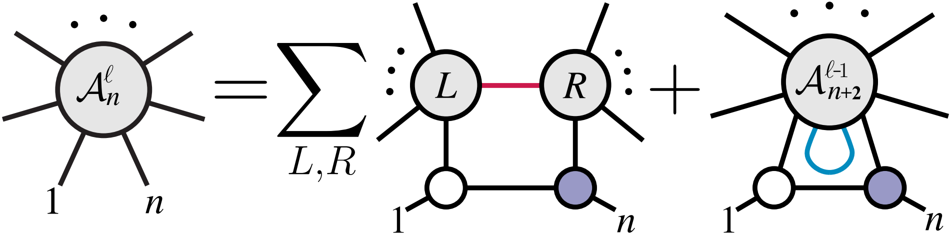

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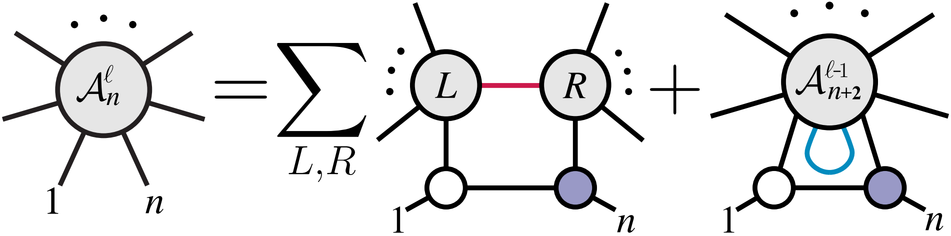

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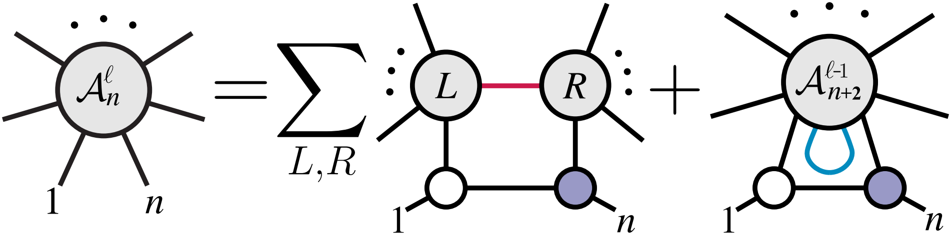

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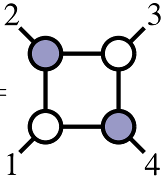
$\mathcal{A}_4^{(2)} =$ 

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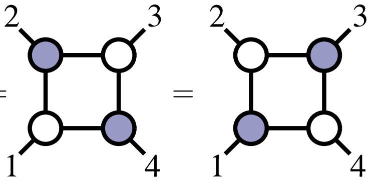

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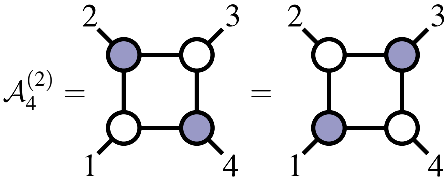

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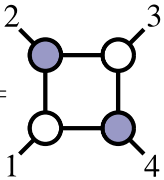
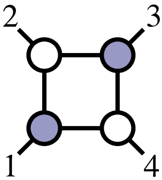
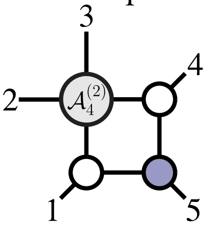
$\mathcal{A}_4^{(2)} =$ 

$=$ 

$\mathcal{A}_5^{(2)} =$ 

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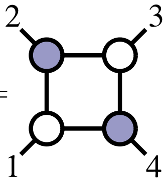
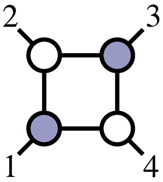
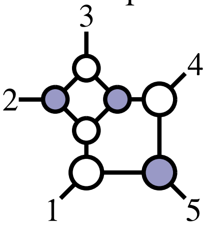
$\mathcal{A}_4^{(2)} =$ 

$=$ 

$\mathcal{A}_5^{(2)} =$ 

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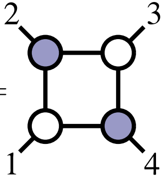
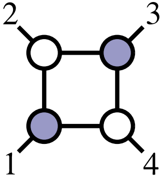
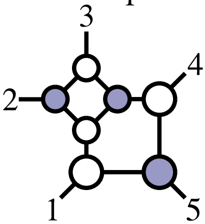
$\mathcal{A}_4^{(2)} =$  $=$ 

$\mathcal{A}_5^{(2)} =$ 

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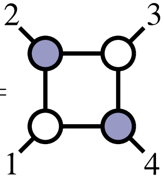
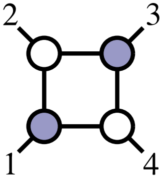
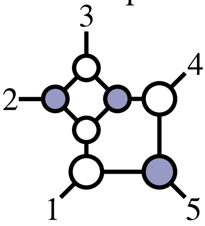
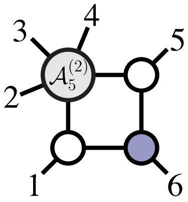
$\mathcal{A}_4^{(2)} =$  $=$ 

$\mathcal{A}_5^{(2)} =$ 

$\mathcal{A}_6^{(2)} =$ 

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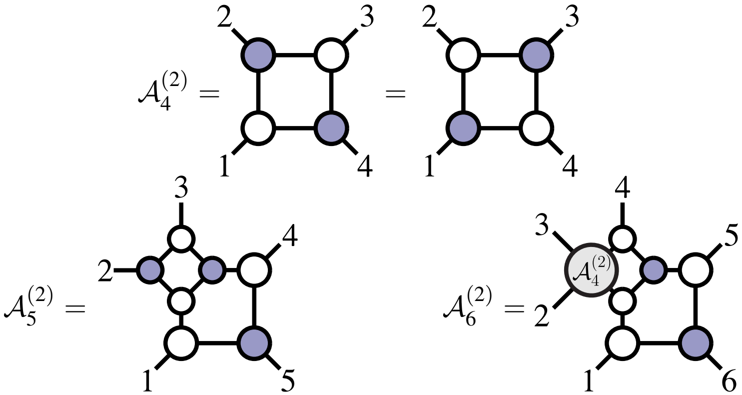

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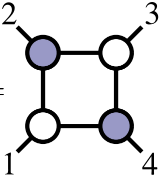
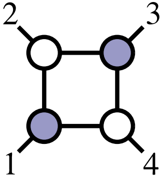
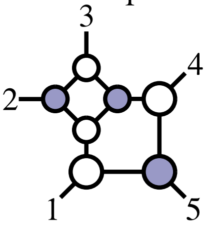
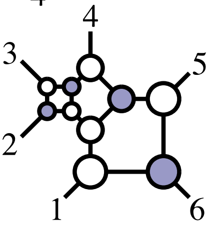
$\mathcal{A}_4^{(2)} =$  $=$ 

$\mathcal{A}_5^{(2)} =$ 

$\mathcal{A}_6^{(2)} =$ 

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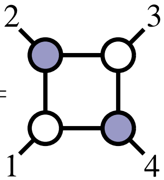
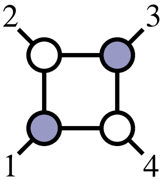
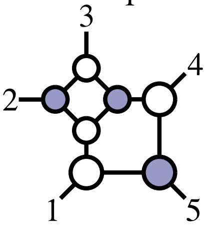
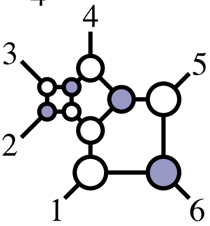
$\mathcal{A}_4^{(2)} =$  $=$ 

$\mathcal{A}_5^{(2)} =$ 

$\mathcal{A}_6^{(2)} =$ 

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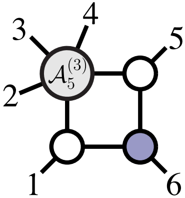
$\mathcal{A}_6^{(3)} =$ 

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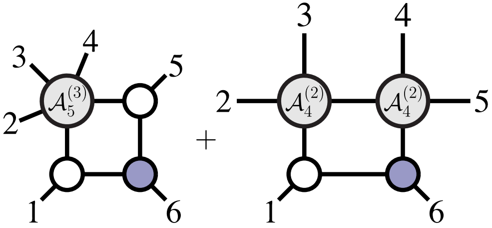
$\mathcal{A}_6^{(3)} =$ 

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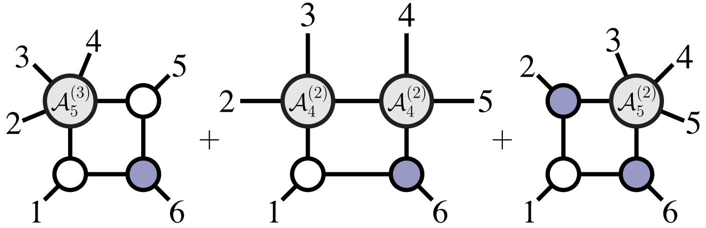

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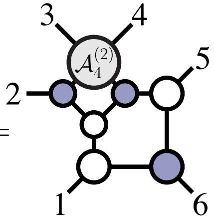
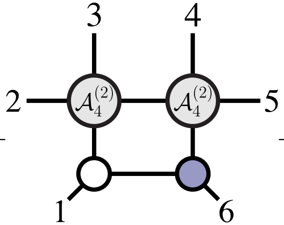
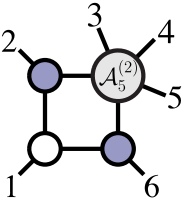
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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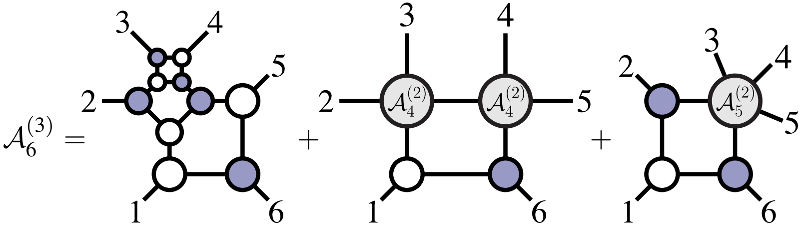

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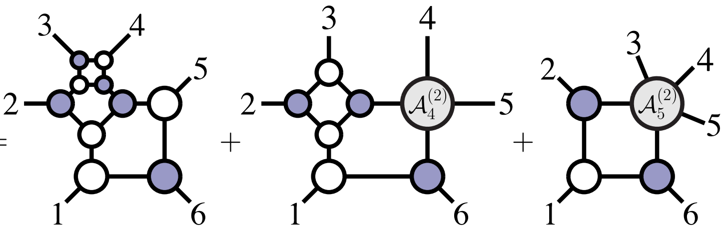

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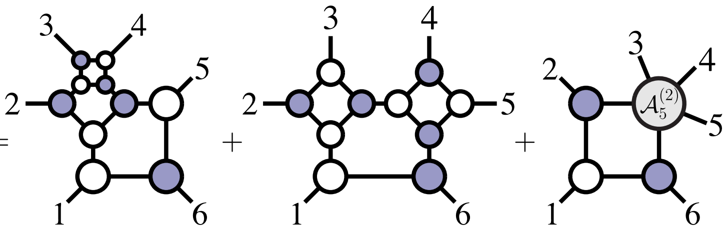
$\mathcal{A}_6^{(3)} =$ 

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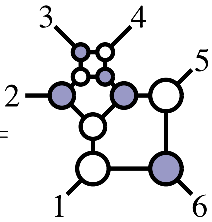
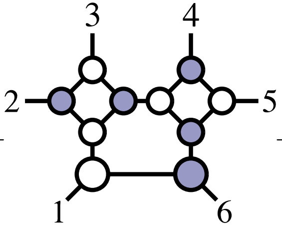
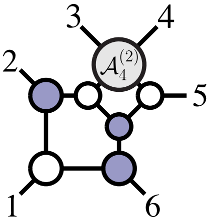
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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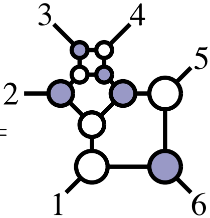
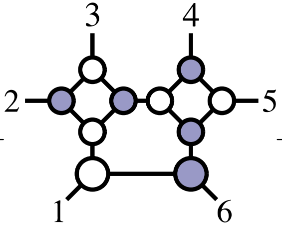
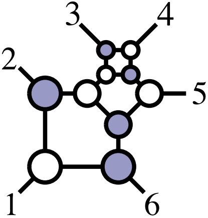
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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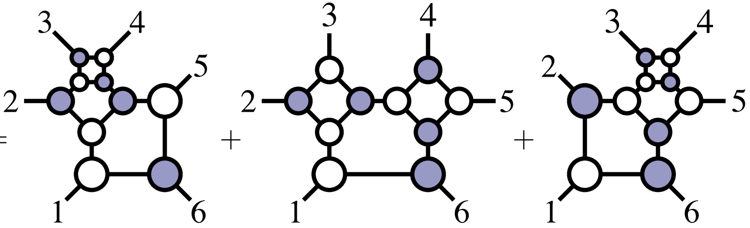

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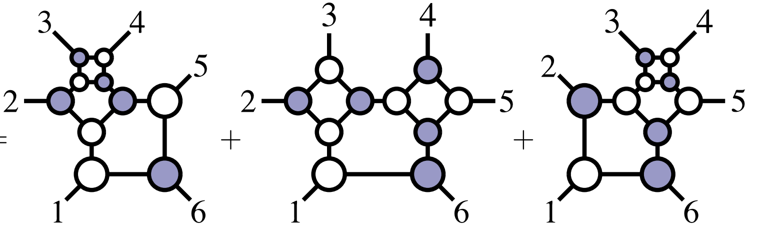

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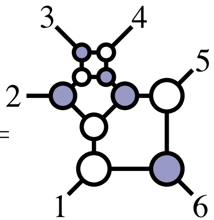
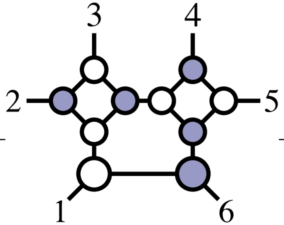
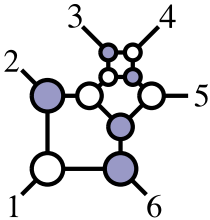
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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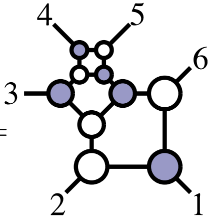
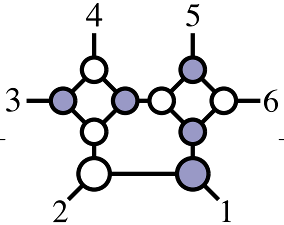
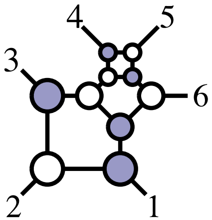
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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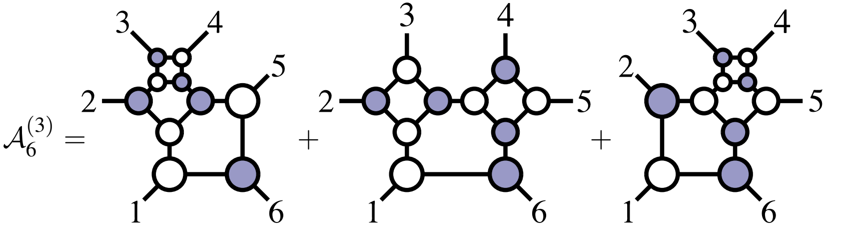

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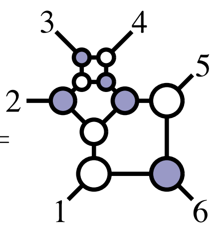
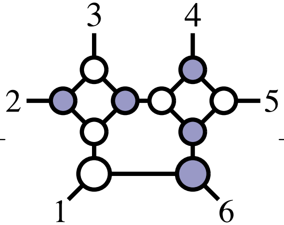
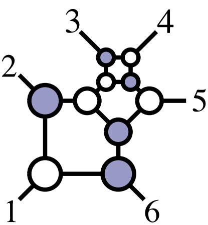
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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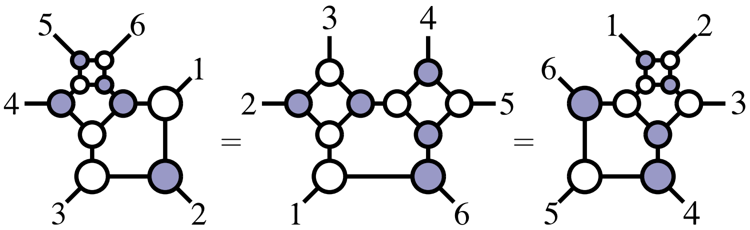

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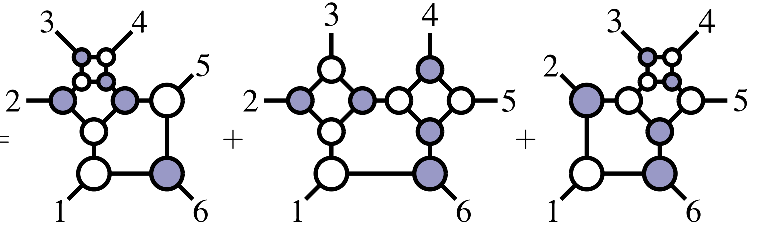

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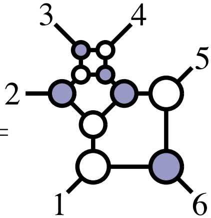
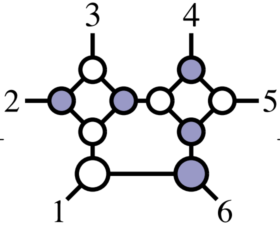
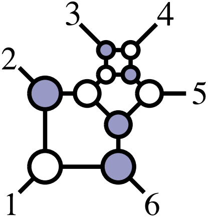
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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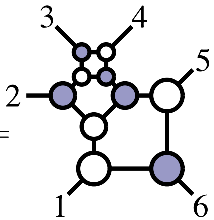
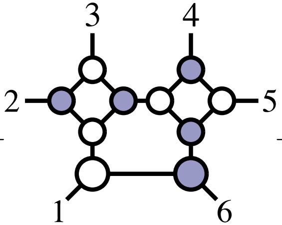
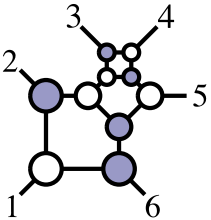
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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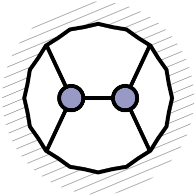

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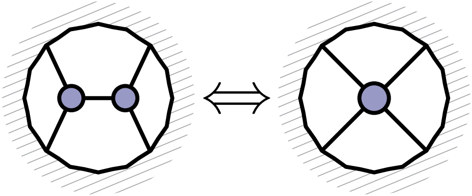

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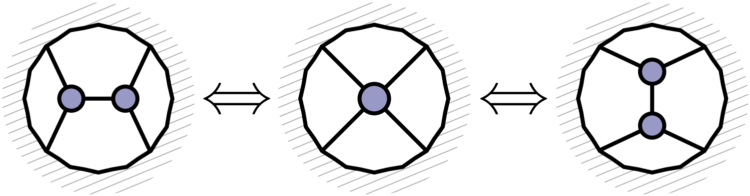

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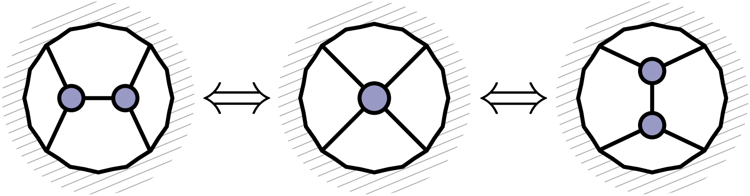

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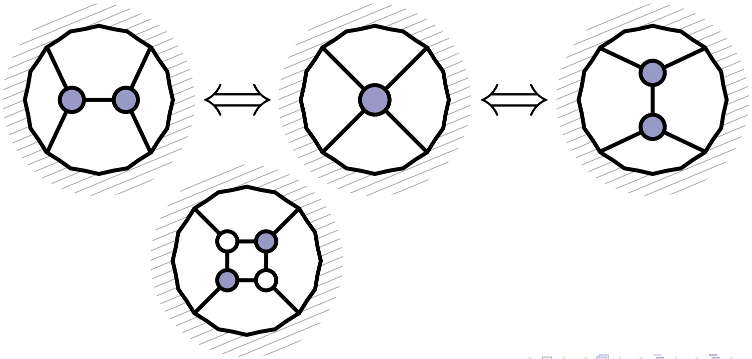

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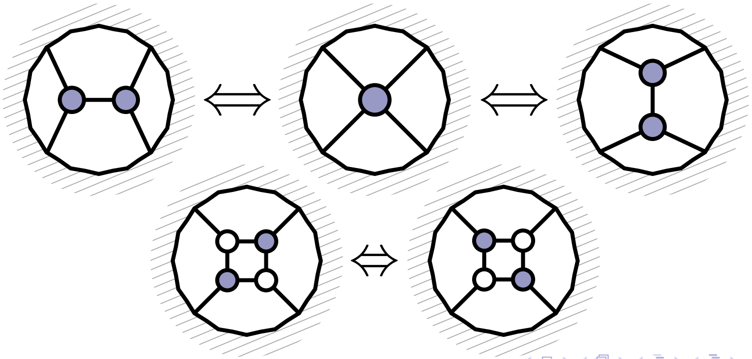

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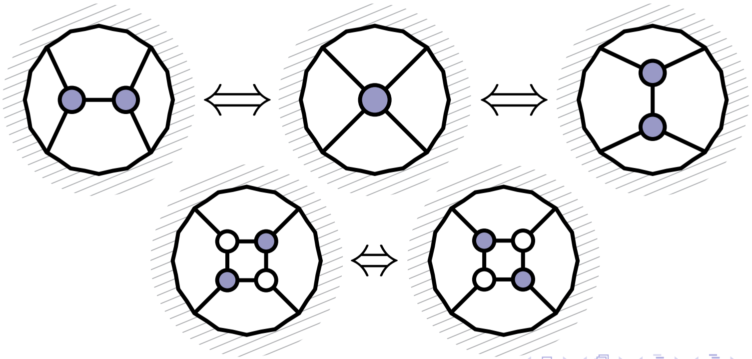

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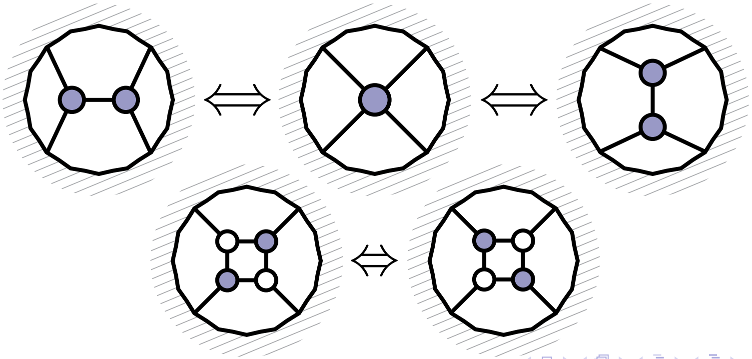

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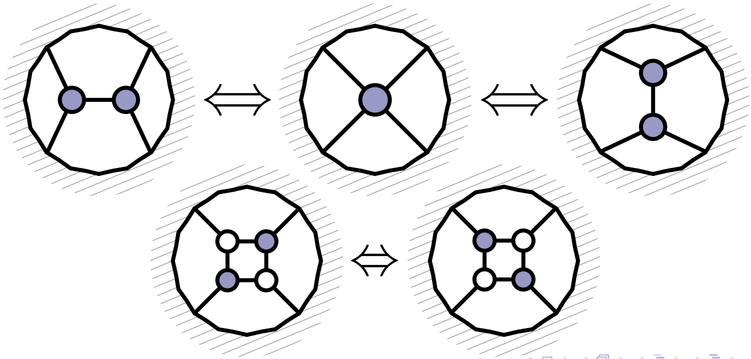

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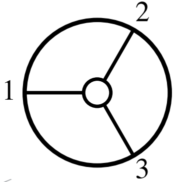

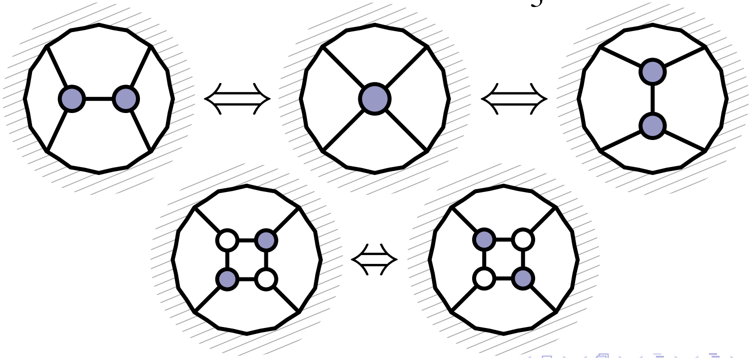

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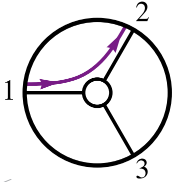

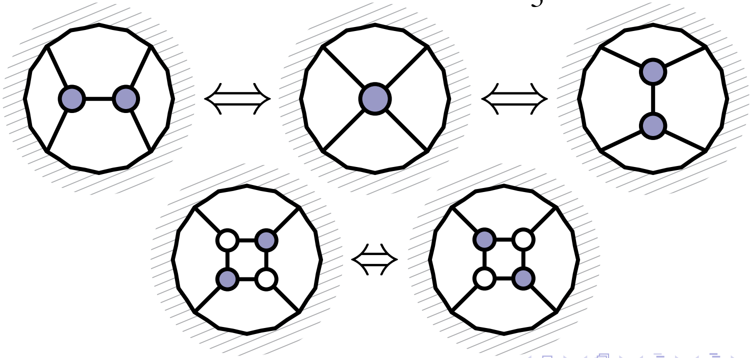

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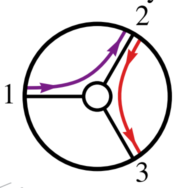

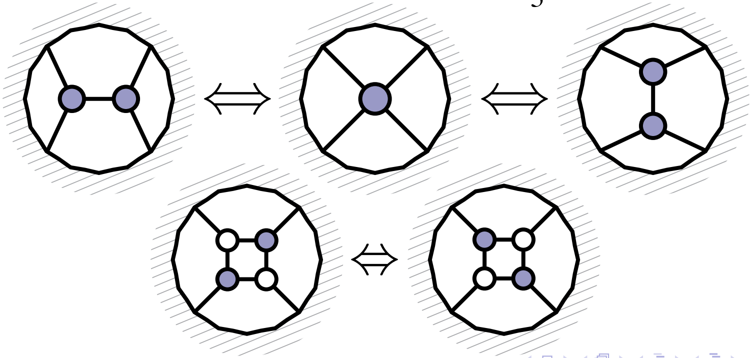

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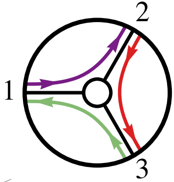

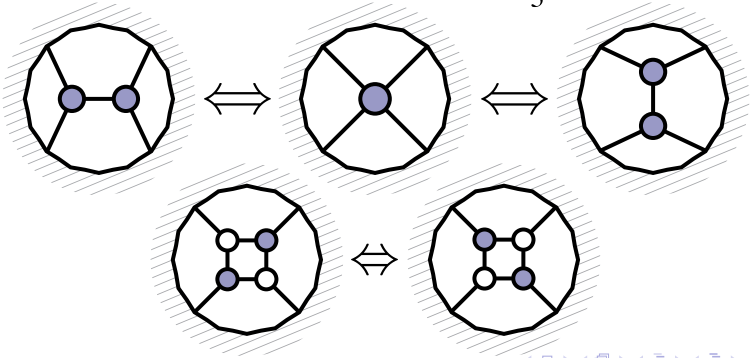

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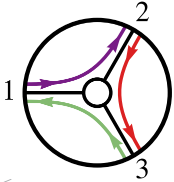

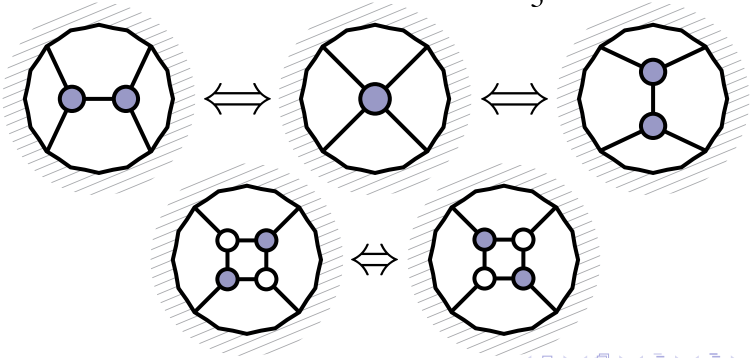

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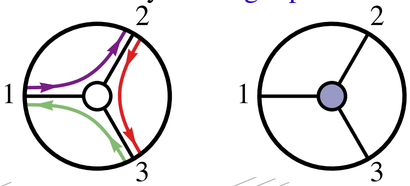

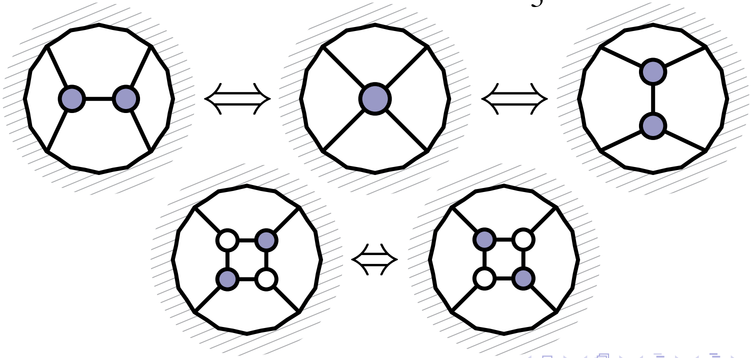

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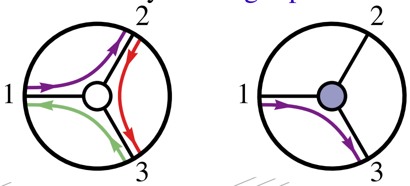

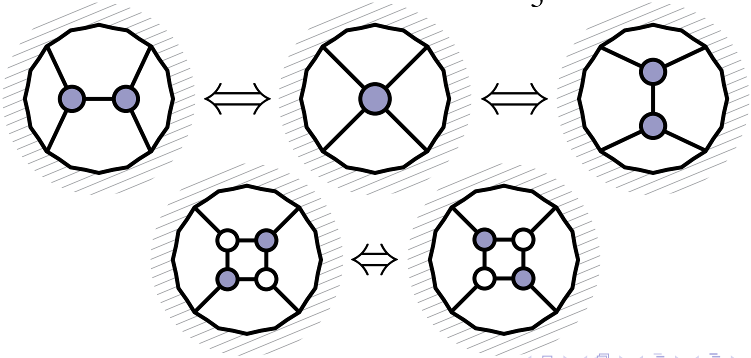

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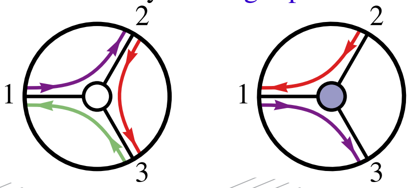

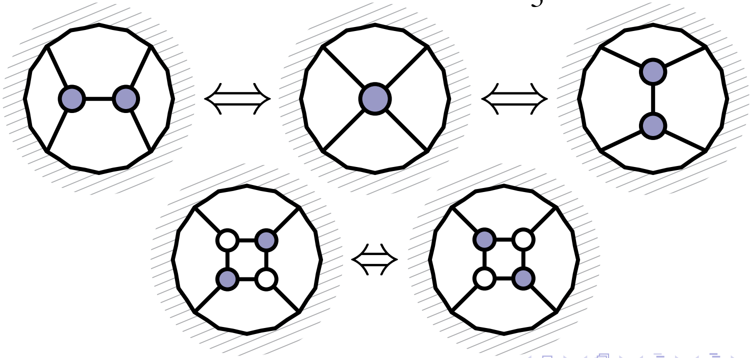

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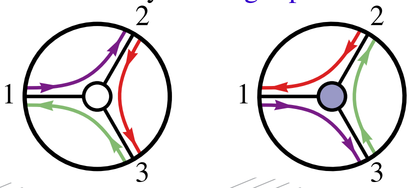

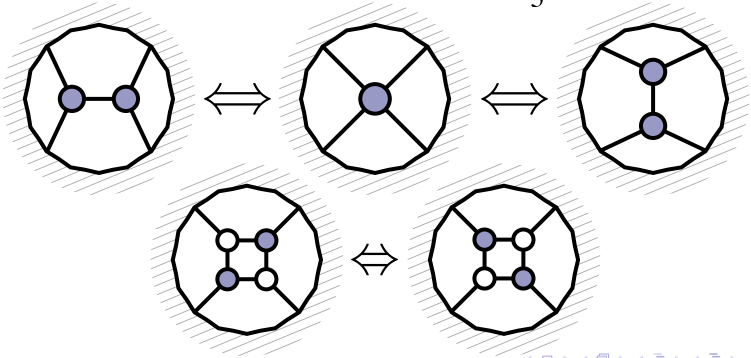

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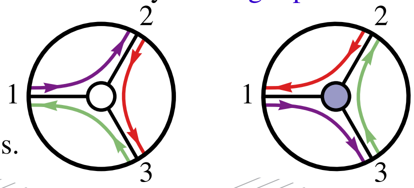

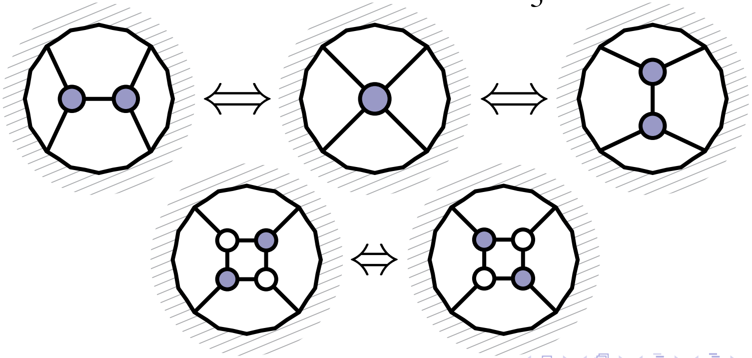

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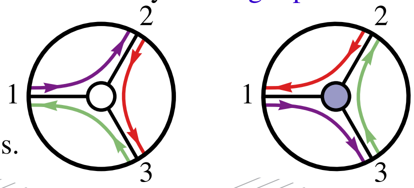

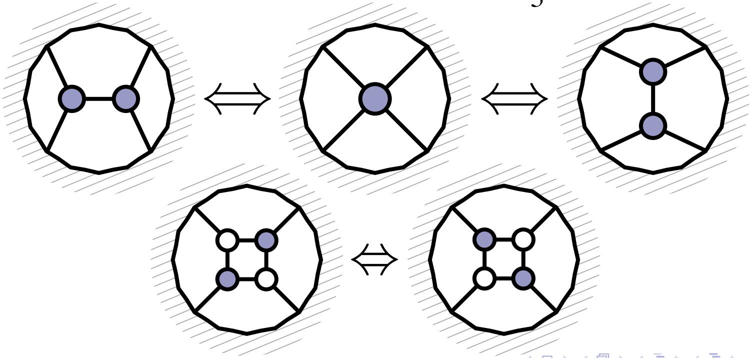

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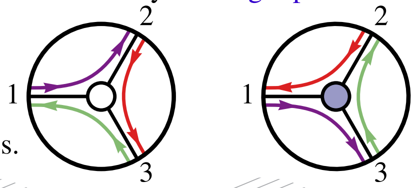

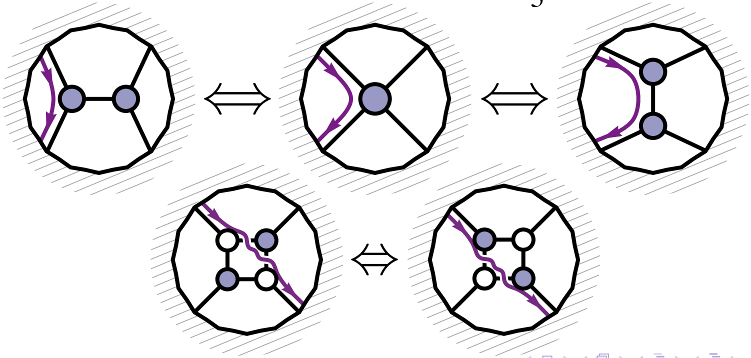

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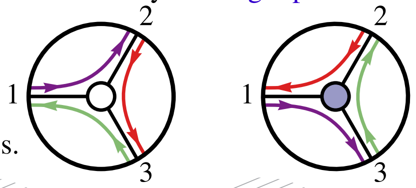

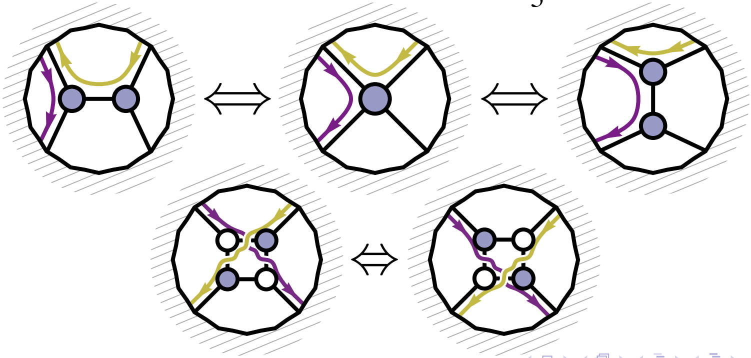

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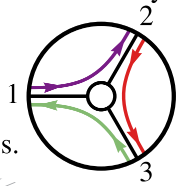

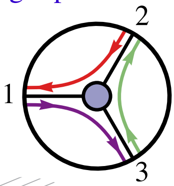

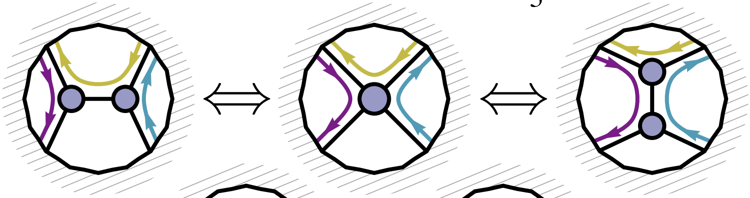

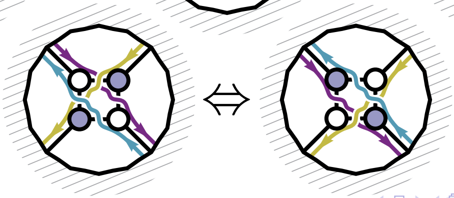

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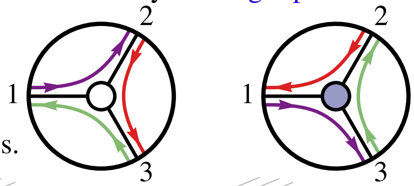

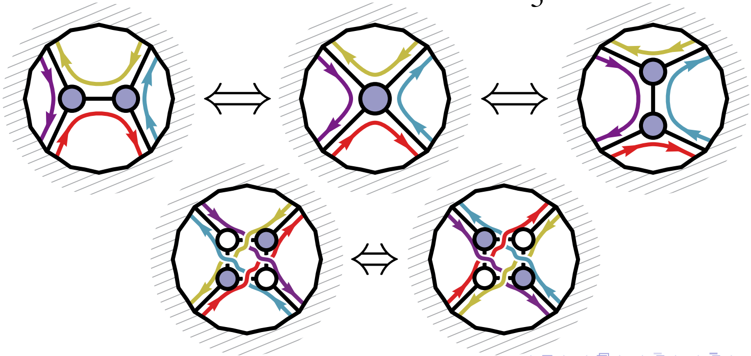

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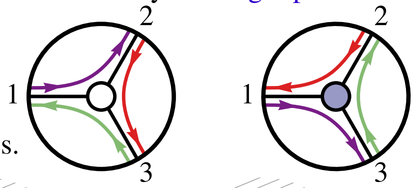

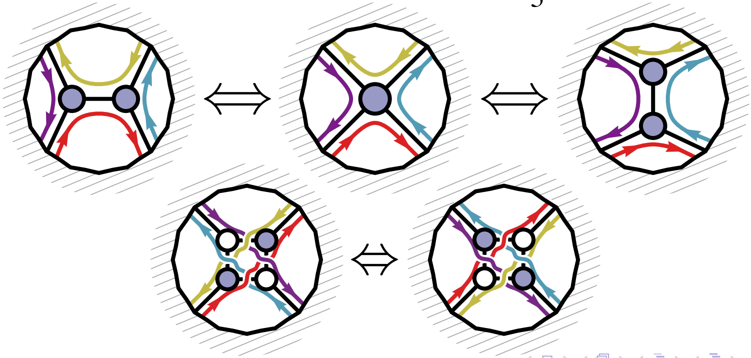

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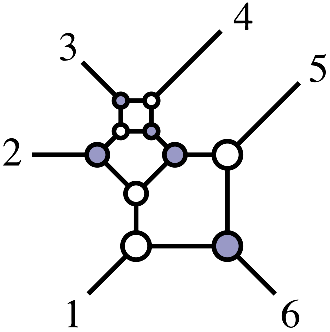

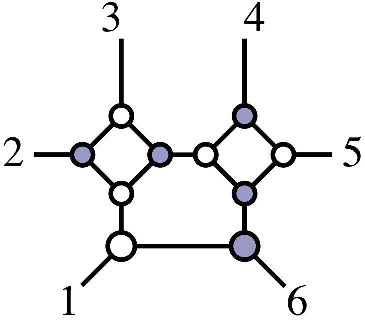

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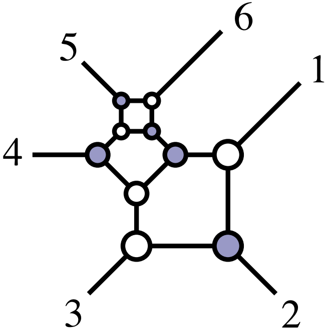

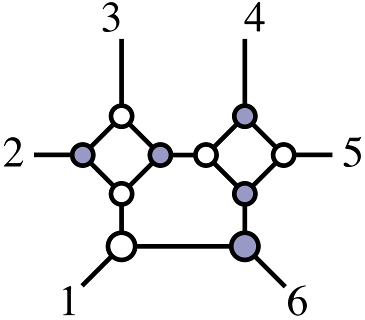

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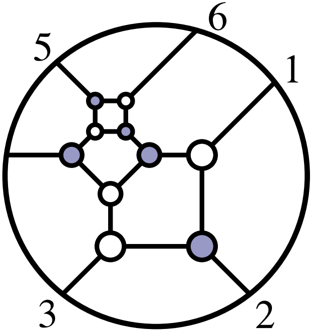

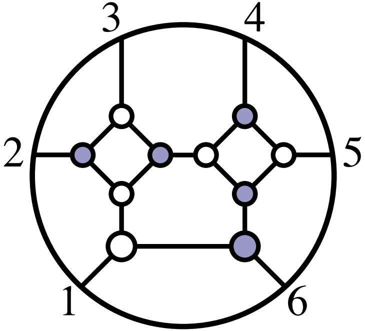

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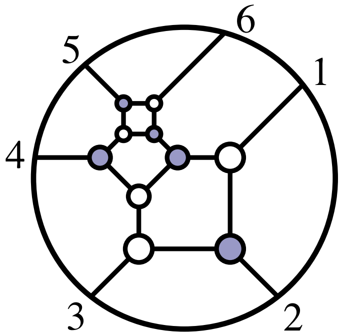

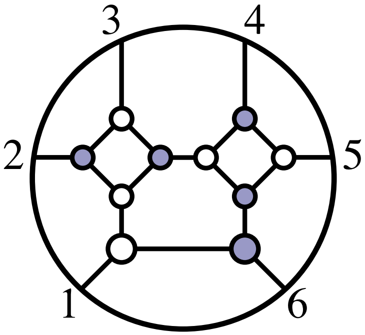

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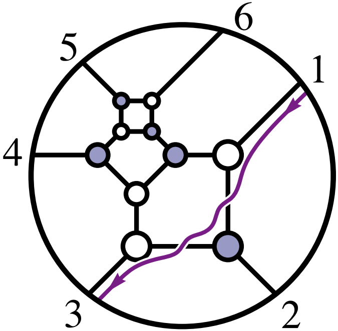

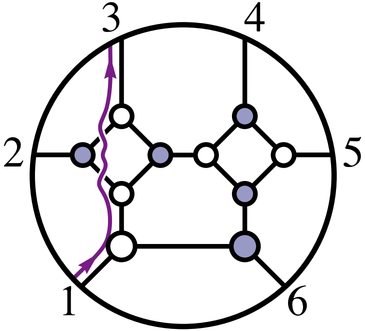

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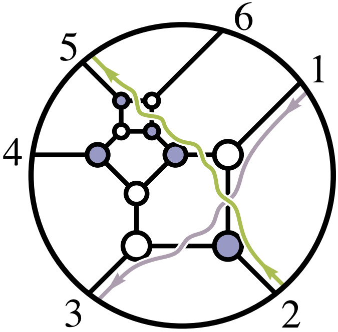

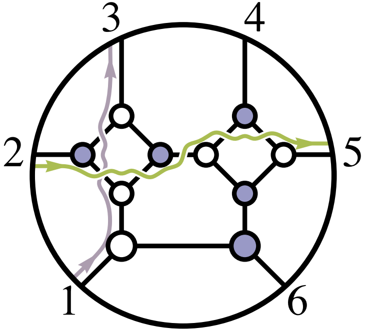

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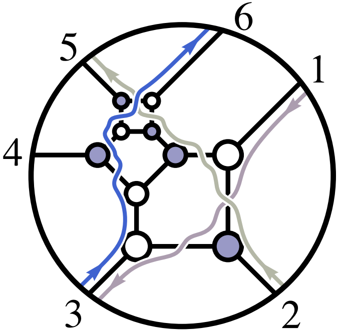

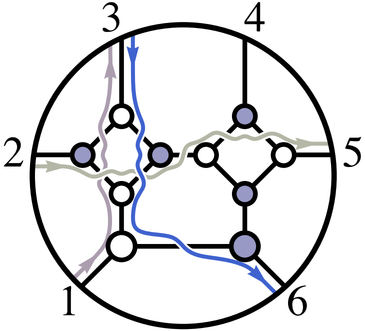

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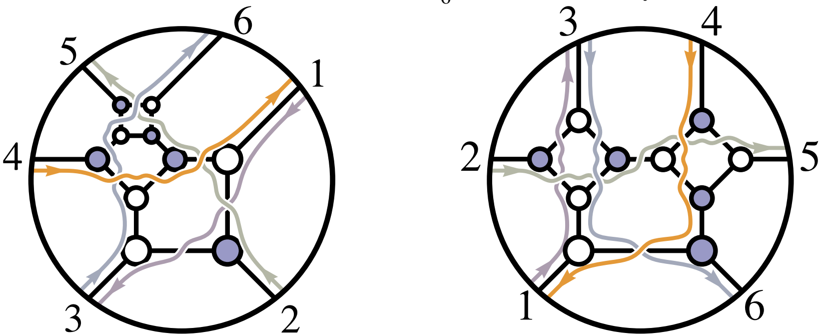

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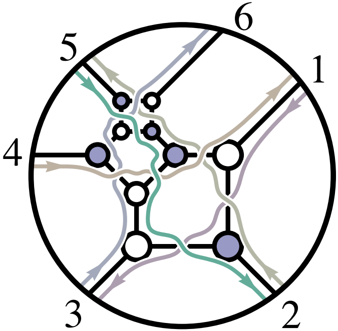

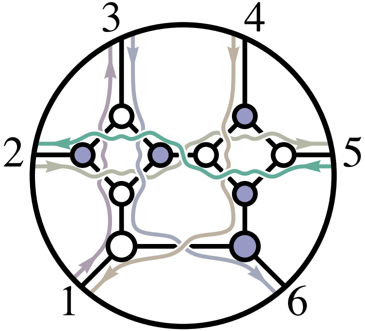

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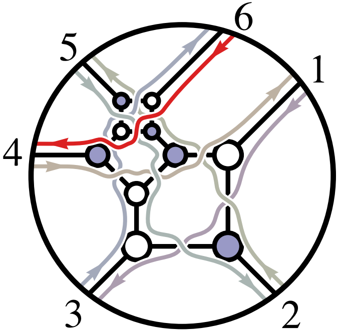

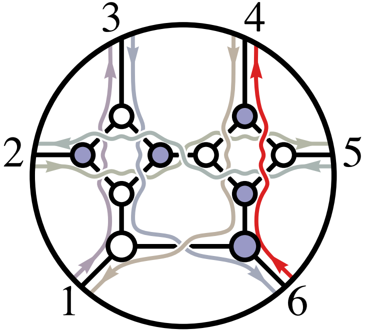

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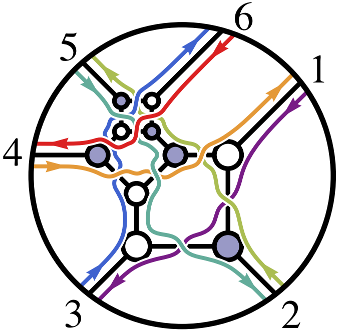

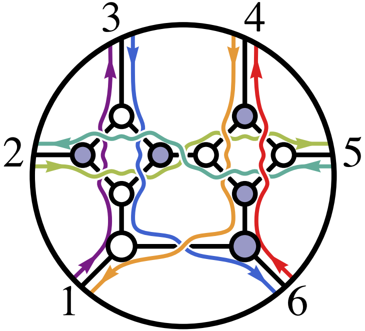

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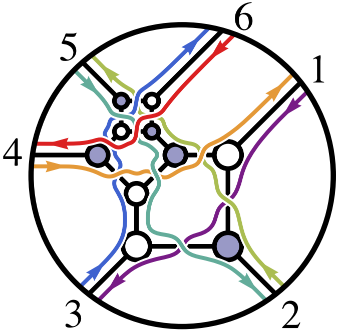

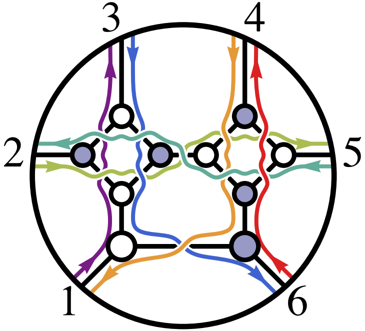

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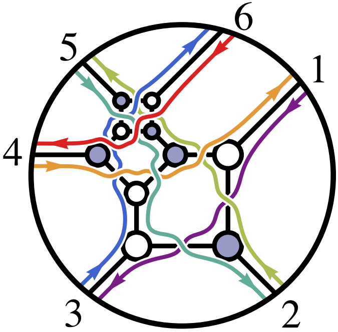

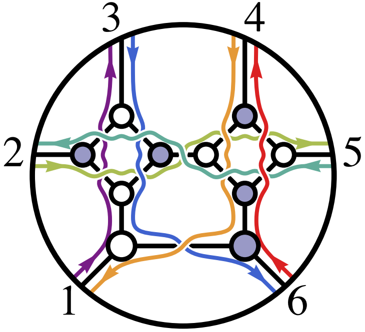

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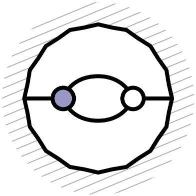

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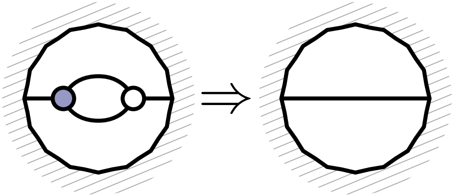

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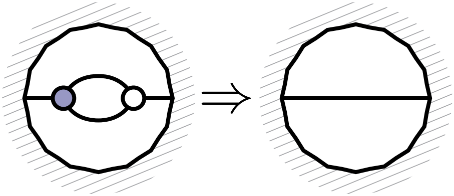

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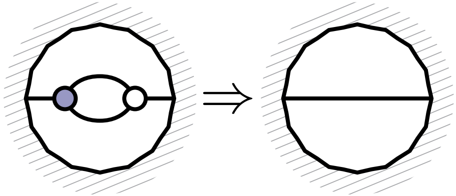

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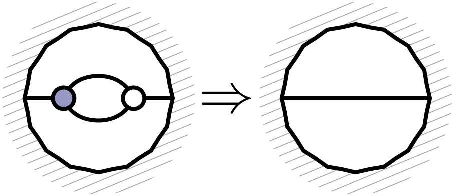

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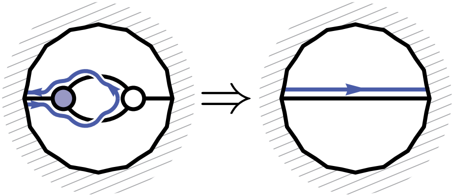

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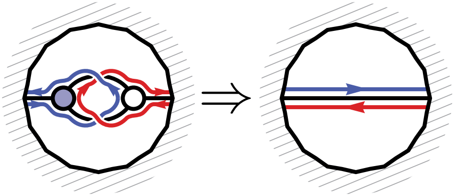

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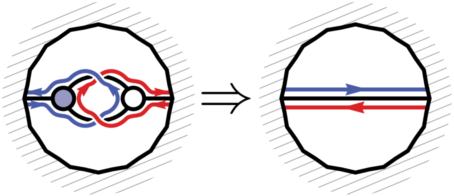
Such factors of $d\alpha/\alpha$ arising from bubble deletion encode loop integrands!



Canonical Coordinates for Computing On-Shell Functions

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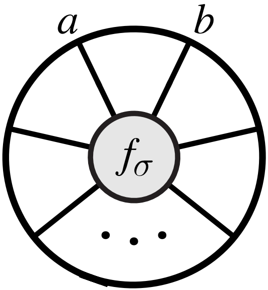

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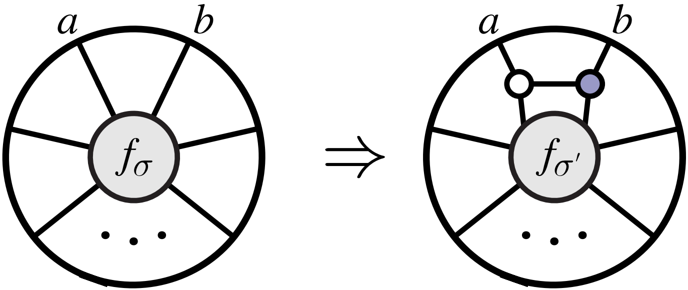

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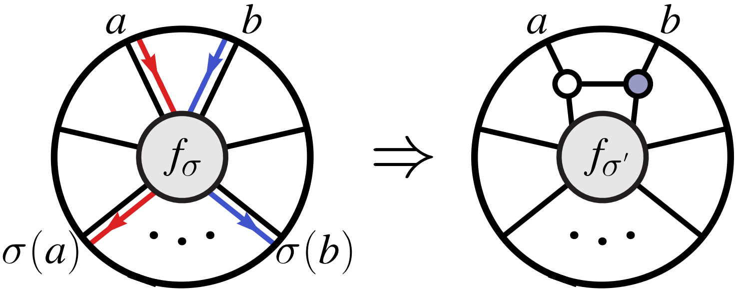

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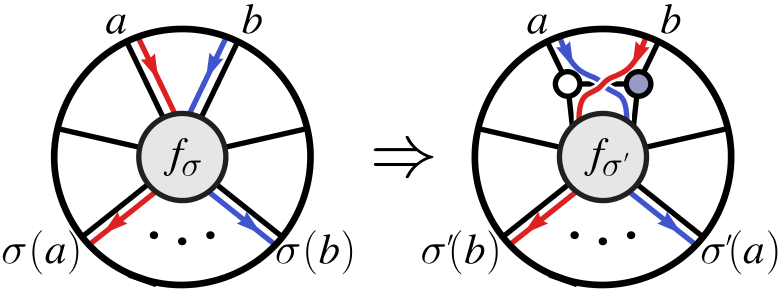

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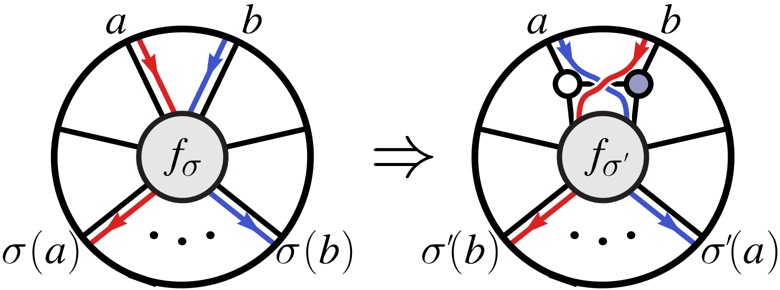

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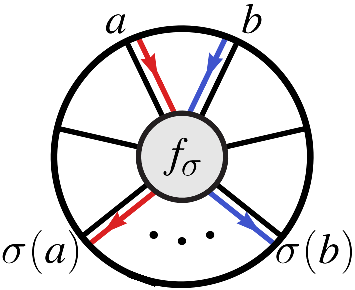

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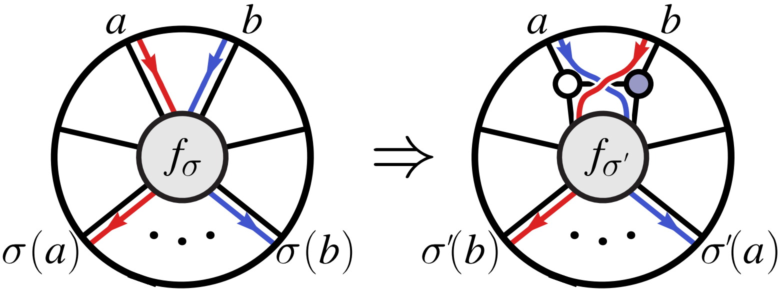

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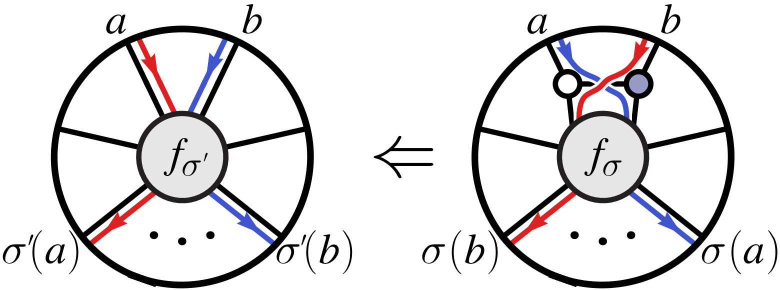

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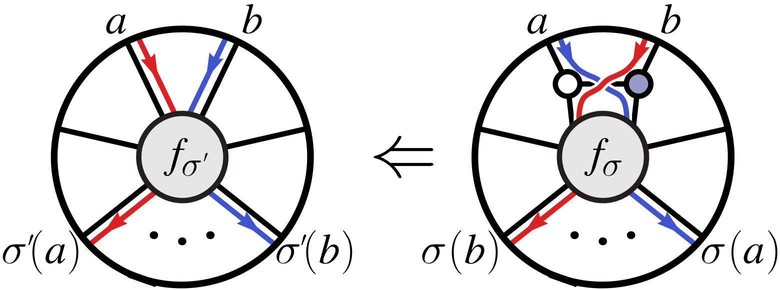

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There are many ways to decompose a permutation into transpositions

'Bridge' Decomposition

$$\sigma:\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 3 & 5 & 6 & 7 & 8 & 10\end{pmatrix}$$

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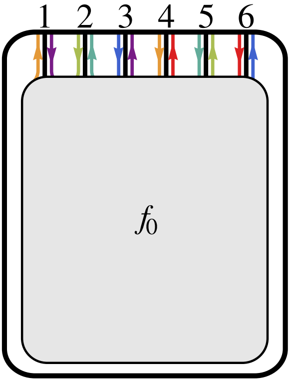

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$$\begin{array}{cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \\ f_0 & \{3 & 5 & 6 & 7 & 8 & 10\} & \end{array}$$

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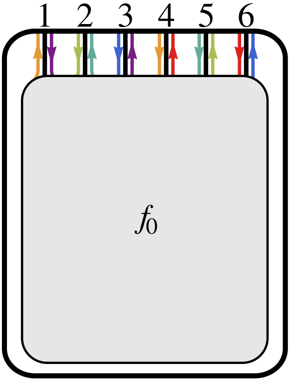

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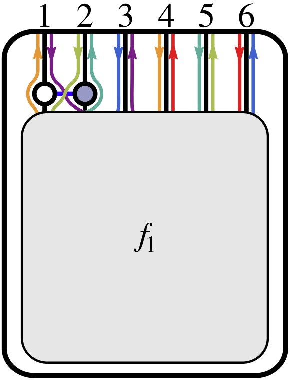

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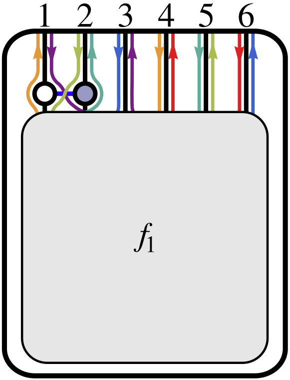

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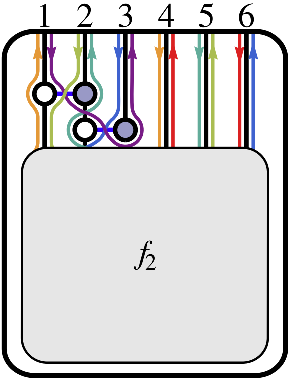

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	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	

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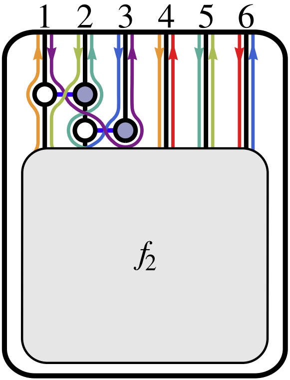

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\, b)$ such that $\sigma(a) < \sigma(b)$:

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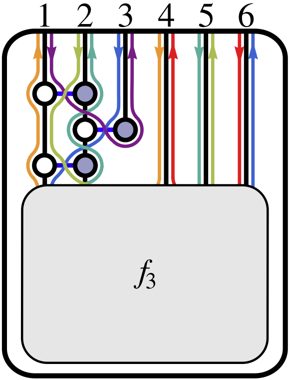

'Bridge' Decomposition

$$\begin{array}{cccccccc c}
 & 1 & 2 & 3 & 4 & 5 & 6 & & \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \tau \\
f_0 & \{3 & 5 & 6 & 7 & 8 & 10\} & & (1\,2) \\
f_1 & \{5 & 3 & 6 & 7 & 8 & 10\} & & (2\,3) \\
f_2 & \{5 & 6 & 3 & 7 & 8 & 10\} & & (1\,2) \\
f_3 & \{6 & 5 & 3 & 7 & 8 & 10\} & &
\end{array}$$

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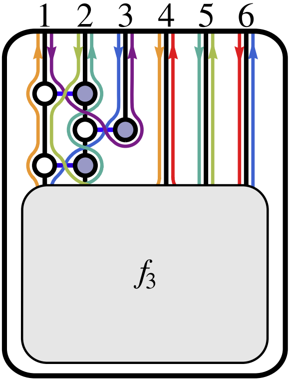

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	
							$(1\,2)$
f_1	{5	3	6	7	8	10}	
							$(2\,3)$
f_2	{5	6	3	7	8	10}	
							$(1\,2)$
f_3	{6	5	3	7	8	10}	
							$(2\,4)$

Canonical Coordinates for Computing On-Shell Functions

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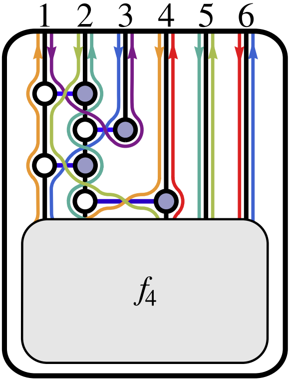

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	

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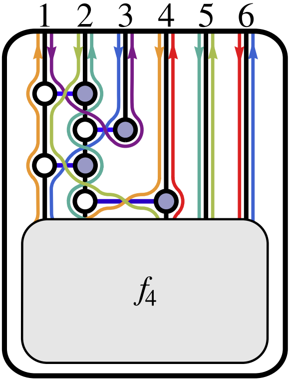

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$

Canonical Coordinates for Computing On-Shell Functions

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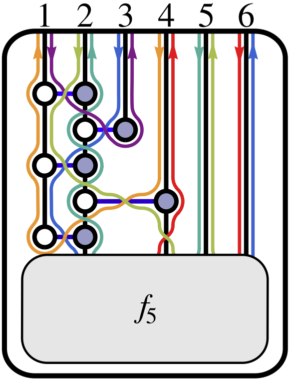

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	

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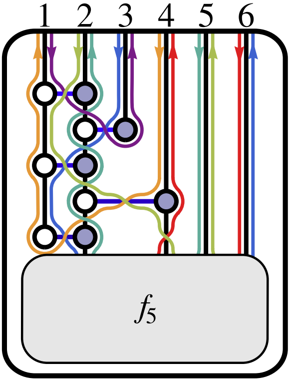

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)

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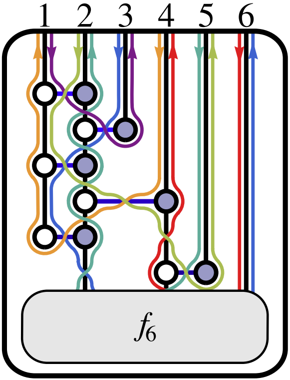
$$f_0 = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6} f_6$$



'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	

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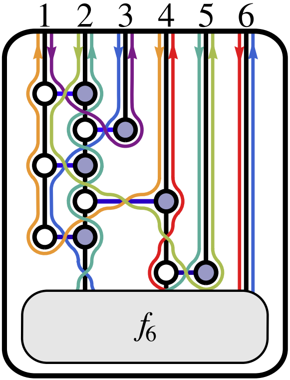
$$f_0 = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6} f_6$$



'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)

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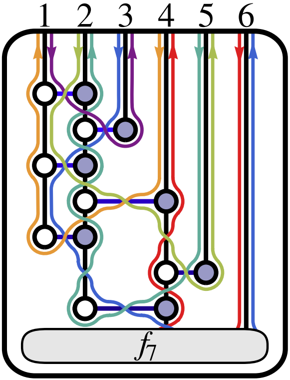
There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

$$f_0 = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}\frac{d\alpha_7}{\alpha_7} f_7$$



'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	

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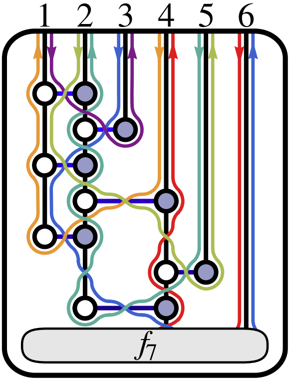

'Bridge' Decomposition

	1	2	3	4	5	6	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
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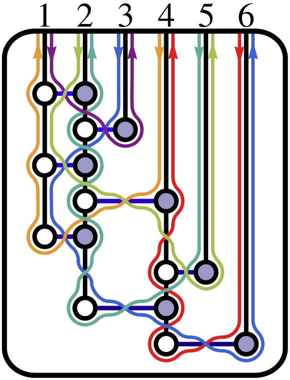

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

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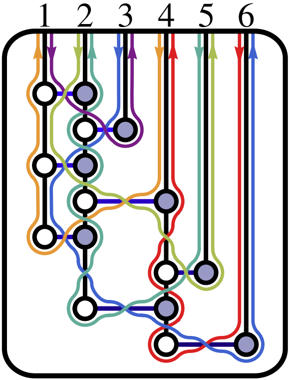

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6 }	

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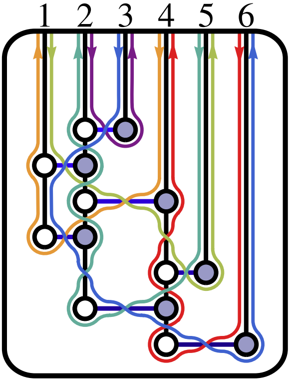

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_1	{5	3	6	7	8	10}	
							$(2\,3)$
f_2	{5	6	3	7	8	10}	
							$(1\,2)$
f_3	{6	5	3	7	8	10}	
							$(2\,4)$
f_4	{6	7	3	5	8	10}	
							$(1\,2)$
f_5	{7	6	3	5	8	10}	
							$(4\,5)$
f_6	{7	6	3	8	5	10}	
							$(2\,4)$
f_7	{7	8	3	6	5	10}	
							$(4\,6)$
f_8	{7	8	3	10	5	6 }	

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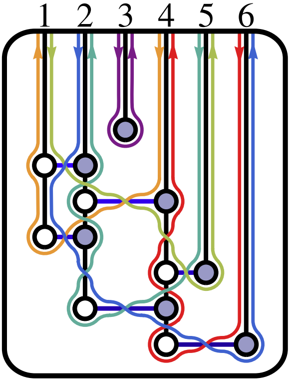

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_2	{5	6	3	7	8	10}	
							$(1\,2)$
f_3	{6	5	3	7	8	10}	
							$(2\,4)$
f_4	{6	7	3	5	8	10}	
							$(1\,2)$
f_5	{7	6	3	5	8	10}	
							$(4\,5)$
f_6	{7	6	3	8	5	10}	
							$(2\,4)$
f_7	{7	8	3	6	5	10}	
							$(4\,6)$
f_8	{7	8	3	10	5	6 }	

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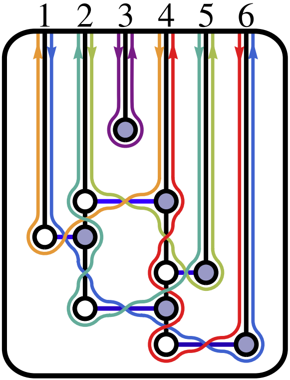

'Bridge' Decomposition

	$1\downarrow$	$2\downarrow$	$3\downarrow$	$4\downarrow$	$5\downarrow$	$6\downarrow$	τ
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
f_7	{7	8	3	6	5	10}	$(4\,6)$
f_8	{7	8	3	10	5	6 }	

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There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\, b)$ such that $\sigma(a) < \sigma(b)$:

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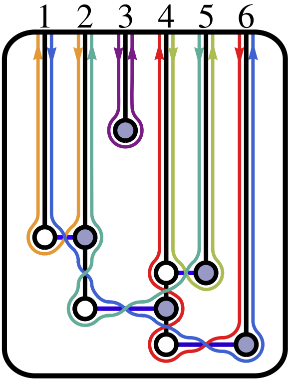

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
f_7	{7	8	3	6	5	10}	$(4\,6)$
f_8	{7	8	3	10	5	6}	

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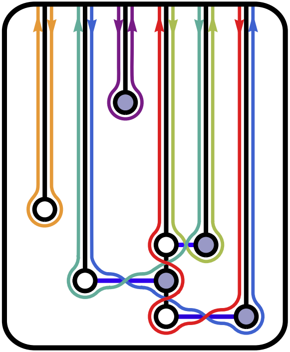

'Bridge' Decomposition

$$\begin{array}{lcccccccc}
 & & 1 & 2 & 3 & 4 & 5 & 6 & \\
 & & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \\
f_5 & \{ & 7 & 6 & 3 & 5 & 8 & 10 & \} \\
 & & & & & & & & (4\,5) \\
f_6 & \{ & 7 & 6 & 3 & 8 & 5 & 10 & \} \\
 & & & & & & & & (2\,4) \\
f_7 & \{ & 7 & 8 & 3 & 6 & 5 & 10 & \} \\
 & & & & & & & & (4\,6) \\
f_8 & \{ & 7 & 8 & 3 & 10 & 5 & 6 & \}
\end{array}$$

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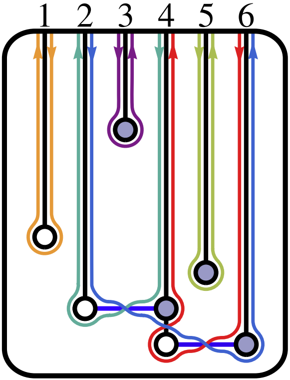
There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

$$f_0 = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}\frac{d\alpha_7}{\alpha_7}\frac{d\alpha_8}{\alpha_8} f_8$$



'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_6	{7	6	3	8	5	10}	
							(2 4)
f_7	{7	8	3	6	5	10}	
							(4 6)
f_8	{7	8	3	10	5	6}	

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

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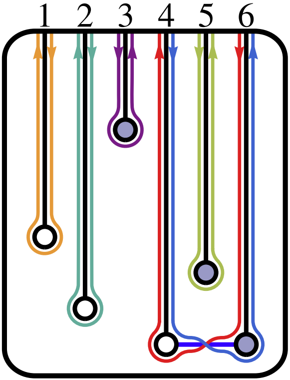

'Bridge' Decomposition

$$\begin{array}{lcccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & & \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \tau \\
f_7 & \{7 & 8 & 3 & 6 & 5 & 10\} & & \\
f_8 & \{7 & 8 & 3 & 10 & 5 & 6\} & (4\,6) &
\end{array}$$

Canonical Coordinates for Computing On-Shell Functions

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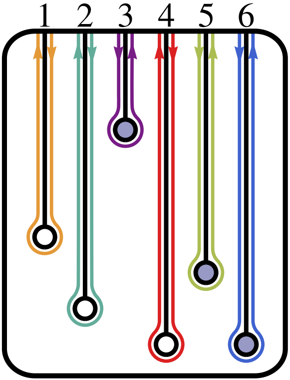

'Bridge' Decomposition

1	2	3	4	5	6	τ
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	

$f_8\ \{7\ \ 8\ \ 3\ \ 10\ \ 5\ \ 6\}$

Canonical Coordinates for Computing On-Shell Functions

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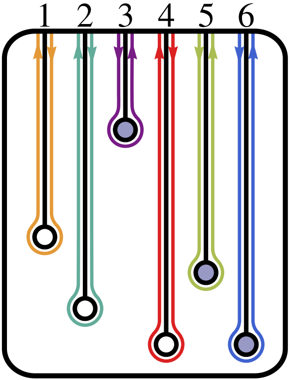

'Bridge' Decomposition

1	2	3	4	5	6	τ
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	

$f_8\ \{7\ \ 8\ \ 3\ \ 10\ \ 5\ \ 6\ \}$

Canonical Coordinates for Computing On-Shell Functions

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$$f_8 = \prod_{a=\sigma(a)+n}\left(\delta^4(\widetilde{\eta}_a)\delta^2(\widetilde{\lambda}_a)\right)\prod_{b=\sigma(b)}\left(\delta^2(\lambda_b)\right)$$

'Bridge' Decomposition

$$\begin{array}{cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \\ f_8 & \{7 & 8 & 3 & 10 & 5 & 6\,\} & \end{array}$$

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

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$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

'Bridge' Decomposition

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \tau \end{array}$$

$f_8\ \{7\ \ 8\ \ 3\ \ 10\ \ 5\ \ 6\ \}$

Canonical Coordinates for Computing On-Shell Functions

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$$f_8 = \delta^{3\times 4}\big(C\cdot\widetilde{\eta}\big)\,\delta^{3\times 2}\big(C\cdot\widetilde{\lambda}\big)\,\delta^{2\times 3}\big(\lambda\cdot C^{\perp}\big)$$

$$C \equiv \begin{array}{c} \\ \\ \end{array}\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

$$C \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

'Bridge' Decomposition

$$\begin{array}{cccccc c} 1 & 2 & 3 & 4 & 5 & 6 & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \end{array}$$

$$f_8\ \{7\ \ 8\ \ 3\ \ 10\ \ 5\ \ 6\ \}$$

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$$f_8 = \delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{c} \\ \\ \end{array}\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$
$$C \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

'Bridge' Decomposition

$$\begin{array}{cccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \end{array}$$

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$$f_7 = \frac{d\alpha_8}{\alpha_8}\delta^{3\times4}\big(C\cdot\widetilde{\eta}\big)\delta^{3\times2}\big(C\cdot\widetilde{\lambda}\big)\delta^{2\times3}\big(\lambda\cdot C^{\perp}\big)$$

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right) \end{array}$$

$$(4\,6):\quad c_6 \mapsto c_6 + \alpha_8\, c_4$$

'Bridge' Decomposition

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \end{array}$$

$$\begin{array}{ll} f_7 & \{7\;\; 8\;\; 3\;\; 6\;\; 5\;\; 10\} \\ f_8 & \{7\;\; 8\;\; 3\;\; 10\;\; 5\;\; 6\} \end{array} (4\,6)$$

Canonical Coordinates for Computing On-Shell Functions

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$$f_6 = \frac{d\alpha_7}{\alpha_7}\frac{d\alpha_8}{\alpha_8}\delta^{3\times4}(C\cdot\widetilde{\eta})\delta^{3\times2}(C\cdot\widetilde{\lambda})\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{pmatrix} \end{array}$$

$$(2\,4):\quad c_4 \mapsto c_4 + \alpha_7\, c_2$$

'Bridge' Decomposition

$$\begin{array}{lcccccccl} & 1 & 2 & 3 & 4 & 5 & 6 & & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \tau \\ f_6 & \{7 & 6 & 3 & 8 & 5 & 10\} & & (2\,4) \\ f_7 & \{7 & 8 & 3 & 6 & 5 & 10\} & & (4\,6) \\ f_8 & \{7 & 8 & 3 & 10 & 5 & 6\} & & \end{array}$$

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

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$$f_5 = \frac{d\alpha_6}{\alpha_6}\frac{d\alpha_7}{\alpha_7}\frac{d\alpha_8}{\alpha_8}\delta^{3\times4}\big(C\cdot\widetilde{\eta}\big)\delta^{3\times2}\big(C\cdot\widetilde{\lambda}\big)\delta^{2\times3}\big(\lambda\cdot C^{\perp}\big)$$

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right) \end{array}$$

$(4\,5)$: $\quad c_5 \mapsto c_5 + \alpha_6\, c_4$

'Bridge' Decomposition

$$\begin{array}{l|cccccc|c} & 1 & 2 & 3 & 4 & 5 & 6 & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \\ f_5 & \{7 & 6 & 3 & 5 & 8 & 10\} & (4\,5) \\ f_6 & \{7 & 6 & 3 & 8 & 5 & 10\} & (2\,4) \\ f_7 & \{7 & 8 & 3 & 6 & 5 & 10\} & (4\,6) \\ f_8 & \{7 & 8 & 3 & 10 & 5 & 6\} & \end{array}$$

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

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$$f_4 = \frac{d\alpha_5}{\alpha_5}\cdots\frac{d\alpha_8}{\alpha_8}\delta^{3\times4}\big(C\cdot\widetilde{\eta}\big)\delta^{3\times2}\big(C\cdot\widetilde{\lambda}\big)\delta^{2\times3}\big(\lambda\cdot C^{\perp}\big)$$

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} 1 & \alpha_5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix} \end{array}$$

$$(1\,2):\quad c_2 \mapsto c_2 + \alpha_5\, c_1$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
f_7	{7	8	3	6	5	10}	$(4\,6)$
f_8	{7	8	3	10	5	6}	

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

$$f_0 = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}\frac{d\alpha_7}{\alpha_7}\frac{d\alpha_8}{\alpha_8} f_8$$

$$f_3 = \frac{d\alpha_4}{\alpha_4}\cdots\frac{d\alpha_8}{\alpha_8}\delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} 1 & \alpha_5 & 0 & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & 0 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix} \end{array}$$

$$(2\,4):\quad c_4 \mapsto c_4 + \alpha_4\, c_2$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
f_7	{7	8	3	6	5	10}	$(4\,6)$
f_8	{7	8	3	10	5	6}	

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

$$f_0 = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}\frac{d\alpha_7}{\alpha_7}\frac{d\alpha_8}{\alpha_8} f_8$$

$$f_2 = \frac{d\alpha_3}{\alpha_3}\cdots\frac{d\alpha_8}{\alpha_8}\delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & (\alpha_3+\alpha_5) & 0 & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & 0 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right) \end{array}$$

$$(1\,2):\quad c_2 \mapsto c_2 + \alpha_3\, c_1$$

'Bridge' Decomposition

$$\begin{array}{lcccccccl} & 1 & 2 & 3 & 4 & 5 & 6 & & \tau \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ f_2 & \{5 & 6 & 3 & 7 & 8 & 10\} & & (1\,2) \\ f_3 & \{6 & 5 & 3 & 7 & 8 & 10\} & & (2\,4) \\ f_4 & \{6 & 7 & 3 & 5 & 8 & 10\} & & (1\,2) \\ f_5 & \{7 & 6 & 3 & 5 & 8 & 10\} & & (4\,5) \\ f_6 & \{7 & 6 & 3 & 8 & 5 & 10\} & & (2\,4) \\ f_7 & \{7 & 8 & 3 & 6 & 5 & 10\} & & (4\,6) \\ f_8 & \{7 & 8 & 3 & 10 & 5 & 6\} & & \end{array}$$

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

$$f_0 = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}\frac{d\alpha_7}{\alpha_7}\frac{d\alpha_8}{\alpha_8} f_8$$

$$f_1 = \frac{d\alpha_2}{\alpha_2} \cdots \frac{d\alpha_8}{\alpha_8}\delta^{3\times4}\big(C\cdot\widetilde{\eta}\big)\delta^{3\times2}\big(C\cdot\widetilde{\lambda}\big)\delta^{2\times3}\big(\lambda\cdot C^{\perp}\big)$$

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & (\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right) \end{array}$$

$$(2\,3):\quad c_3 \mapsto c_3 + \alpha_2\, c_2$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau\equiv(a\,b)$ such that $\sigma(a)<\sigma(b)$:

$$f_0=\frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}\frac{d\alpha_7}{\alpha_7}\frac{d\alpha_8}{\alpha_8}f_8$$

$$f_0=\frac{d\alpha_1}{\alpha_1}\cdots\frac{d\alpha_8}{\alpha_8}\delta^{3\times4}\big(C\cdot\widetilde{\eta}\big)\delta^{3\times2}\big(C\cdot\widetilde{\lambda}\big)\delta^{2\times3}\big(\lambda\cdot C^{\perp}\big)$$

$$C\equiv\begin{array}{c} \\ \\ \end{array}\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \end{array}$$
$$C\equiv\left(\begin{array}{cccccc} 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

$$(1\,2):\quad c_2\mapsto c_2+\alpha_1\,c_1$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

$$f_0 = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}\frac{d\alpha_7}{\alpha_7}\frac{d\alpha_8}{\alpha_8} f_8$$

$$f_0 = \frac{d\alpha_1}{\alpha_1}\cdots\frac{d\alpha_8}{\alpha_8}\,\delta^{3\times4}\big(C\cdot\widetilde{\eta}\big)\,\delta^{3\times2}\big(C\cdot\widetilde{\lambda}\big)\,\delta^{2\times3}\big(\lambda\cdot C^{\perp}\big)$$

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right) \end{array}$$

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

$$f_0 = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}\frac{d\alpha_7}{\alpha_7}\frac{d\alpha_8}{\alpha_8} f_8$$

$$f_0 = \frac{d\alpha_1}{\alpha_1}\cdots\frac{d\alpha_8}{\alpha_8}\delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{c} \\ \end{array}\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$
$$C \equiv \begin{pmatrix} 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{pmatrix}$$

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
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Parke-Taylor 'Amplitudes' and Grassmannian Residues

Recall the natural desire to generalize the Parke-Taylor formula according to:

Amplitudes with 2 negative-helicity gluons

$$\mathcal{A}_n^{(2)} = \frac{\delta^{2\times4}(\lambda\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\cdots\langle n\,1\rangle}$$

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$$\lambda \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \cdots & \lambda_n^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \cdots & \lambda_n^2 \end{pmatrix}$$

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$\widetilde{\lambda}_{\text{2-plane}}$

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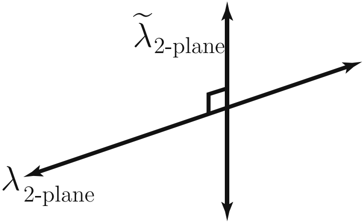

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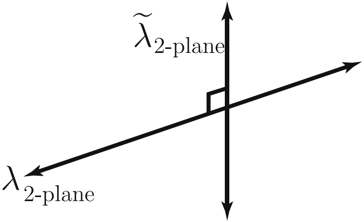

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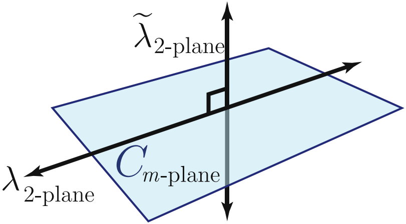

In order for momentum conservation, $\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)$, to be part of the constraints, we must have that $C \supset \lambda$

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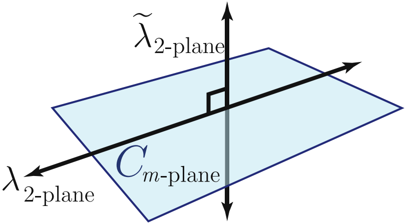

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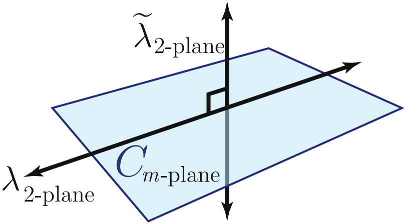

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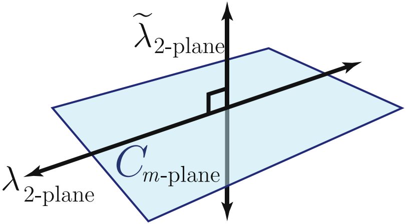

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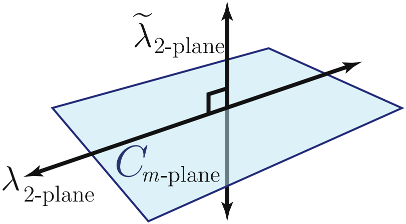
$$\dim(C) = 3\times 6 - 3\times 3 = 9$$



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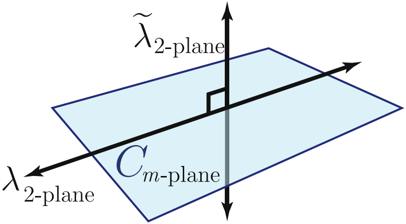
$$\dim(C) = 3\times6 - 3\times3 = 9 = 8 + 1$$



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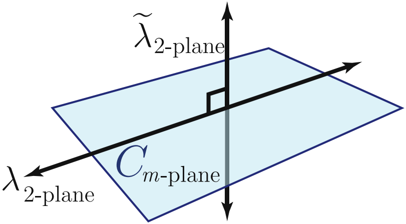

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$$\oint \frac{d\tau \qquad \delta^{3\times4}(C\cdot\widetilde{\eta}) \qquad \delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle123\rangle(\tau)\cdot\langle234\rangle(\tau)\cdot\langle345\rangle(\tau)\cdot\langle456\rangle(\tau)\cdot\langle561\rangle(\tau)\cdot\langle612\rangle(\tau)}$$

$$C \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ c_1^3 & c_2^3 & c_3^3 & c_4^3 & c_5^3 & c_6^3 \end{pmatrix}$$

$$\dim(C) = 3\times6 - 3\times3 = 9 = {\color{red}8} + {\color{blue}1}$$

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Recall the natural desire to generalize the Parke-Taylor formula according to:

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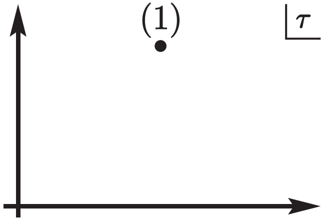
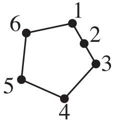
$$\oint_{\langle 123\rangle=0} \frac{d\tau}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}\,\delta^{3\times 4}(C\cdot\widetilde{\eta})\,\delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})$$

$$C \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & c_4^3 & c_5^3 & c_6^3 \end{pmatrix}$$





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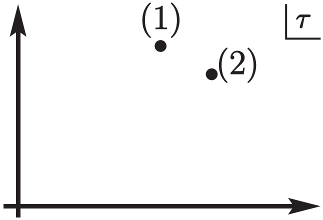

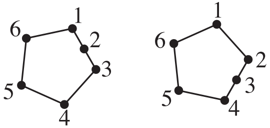

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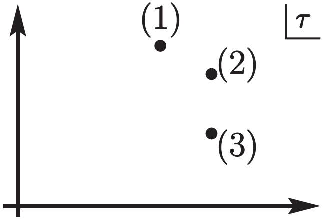

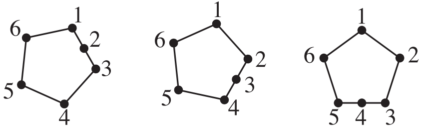

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$$\oint\limits_{\langle 456\rangle=0} \frac{d\tau \qquad \delta^{3\times4}\big(C\cdot\widetilde{\eta}\big) \qquad \delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}$$

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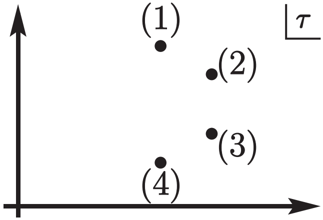

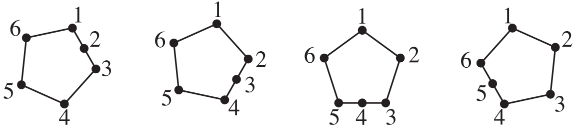

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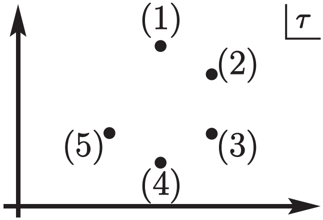
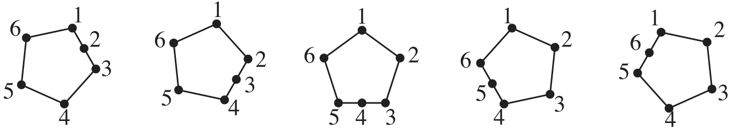
$$\oint_{\langle 561\rangle=0} \frac{d\tau \qquad \delta^{3\times4}(C\cdot\widetilde{\eta}) \qquad \delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}$$

$$C \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & c_2^3 & c_3^3 & c_4^3 & 0 & 0 \end{pmatrix}$$





Parke-Taylor 'Amplitudes' and Grassmannian Residues

Recall the natural desire to generalize the Parke-Taylor formula according to:

Amplitudes with 3 negative-helicity gluons—e.g.,

$$\oint\limits_{\langle 612\rangle=0} \frac{d\tau \qquad \delta^{3\times4}\big(C\cdot\widetilde{\eta}\big) \qquad \delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}$$

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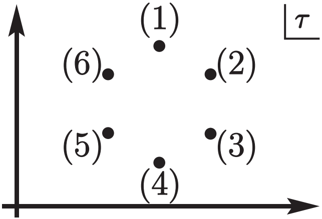

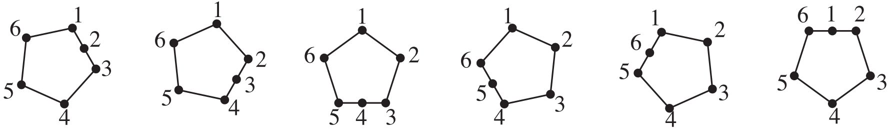

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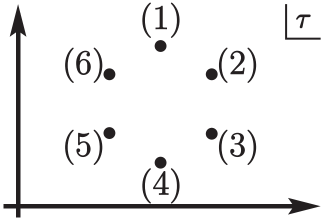

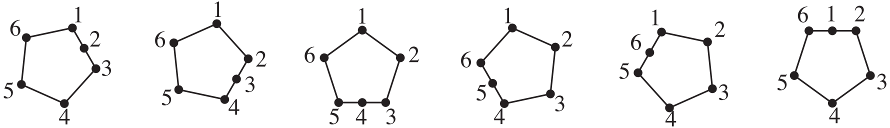

Parke-Taylor 'Amplitudes' and Grassmannian Residues

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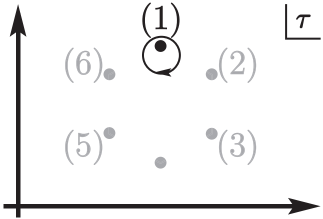

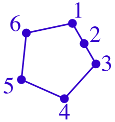

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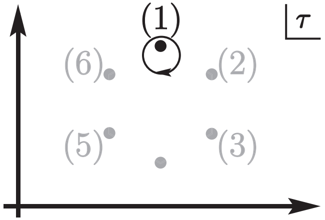

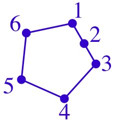

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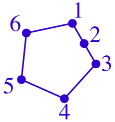
$$\frac{1}{\qquad\qquad}$$



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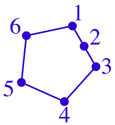
$$\frac{1}{\langle 2\,3\rangle[5\,6]}$$



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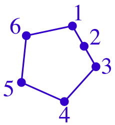

$$\frac{1}{\langle 2\,3\rangle[5\,6][6|(5+4)|3\rangle}$$

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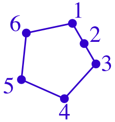
$$\frac{1}{\langle2\,3\rangle[5\,6][6|(5+4)|3\rangle s_{4\,5\,6}}$$



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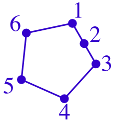
$$\frac{1}{\langle 2\,3\rangle[5\,6][6|(5+4)|3\rangle s_{4\,5\,6}\langle 1|(6+5)|4]}$$



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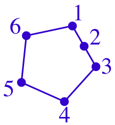

$$\frac{1}{\langle 2\,3\rangle[5\,6][6|(5+4)|3\rangle s_{456}\langle 1|(6+5)|4][4\,5]\langle 1\,2\rangle}$$

Parke-Taylor ‘Amplitudes’ and Grassmannian Residues

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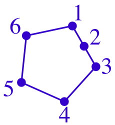
$$\frac{1}{\langle 2\,3\rangle[5\,6][6|(5+4)|3\rangle s_{4\,5\,6}\langle 1|(6+5)|4][4\,5]\langle 1\,2\rangle}$$



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$$\oint_{\langle 123\rangle = 0} \frac{d\tau \quad \left(\langle 2\,4\,6\rangle^4\, \widetilde{\eta}_2^4\widetilde{\eta}_4^4\widetilde{\eta}_6^4 + \ldots\right)\delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}$$

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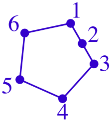

$$\frac{\langle 2|(4+6)|5]^4}{\langle 2\,3\rangle[5\,6][6|(5+4)|3\rangle s_{4\,5\,6}\langle 1|(6+5)|4][4\,5]\langle 1\,2\rangle}$$

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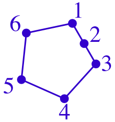
$$\mathcal{A}_6^{(3)}(+,-,+,-,+,-) = (1+r^2+r^4)\frac{\langle 2|(4+6)|5]^4}{\langle 2\,3\rangle[5\,6][6|(5+4)|3\rangle s_{4\,5\,6}\langle 1|(6+5)|4][4\,5]\langle 1\,2\rangle}$$



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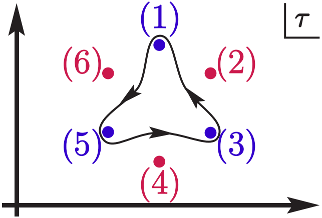
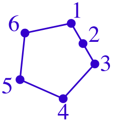
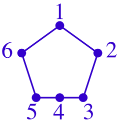
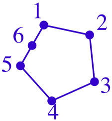


$$\mathcal{A}_6^{(3)}(+,-,+,-,+,-) = (1)+(3)+(5)$$







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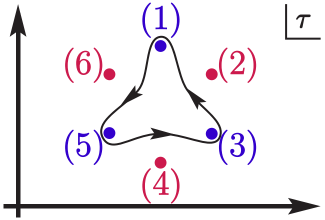

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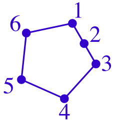

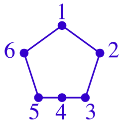

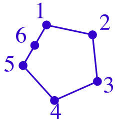

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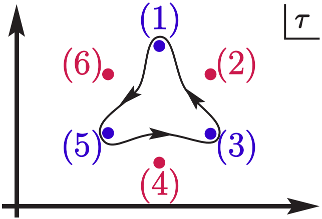
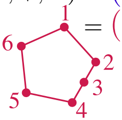
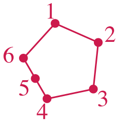
$$\mathcal{A}_6^{(3)} = \oint \frac{d\tau \qquad \delta^{3\times4}(C\cdot\widetilde{\eta}) \qquad \delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle123\rangle(\tau)\cdot\langle234\rangle(\tau)\cdot\langle345\rangle(\tau)\cdot\langle456\rangle(\tau)\cdot\langle561\rangle(\tau)\cdot\langle612\rangle(\tau)}$$

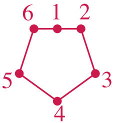
$$(1) \Leftrightarrow \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [5\,6] & [6\,4] & [4\,5] \end{pmatrix}$$



$$\begin{aligned}\mathcal{A}_6^{(3)}(+,-,+,-,+,-) &= (1)+(3)+(5)\\ &= (2)+(4)+(6)\end{aligned}$$







Parke-Taylor 'Amplitudes' and Grassmannian Residues

Recall the natural desire to generalize the Parke-Taylor formula according to:

Amplitudes with m negative-helicity gluons:

$$\mathcal{A}_n^{(m)} = \oint \frac{d^{m\times n}C}{\mathrm{vol}(GL(m))} \frac{\delta^{m\times 4}\big(C\cdot\widetilde{\eta}\big)\delta^{m\times 2}\big(C\cdot\widetilde{\lambda}\big)\delta^{2\times(n-m)}\big(\lambda\cdot C^{\perp}\big)}{\langle 1\,2\,\cdots\,m\rangle\langle 2\,3\,\cdots\,m{+}1\rangle\cdots\langle n\;1\cdots\,m{-}1\rangle}$$

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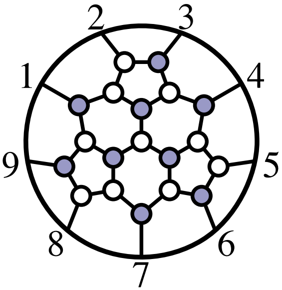
The Combinatorics and Geometry of On-Shell Physics



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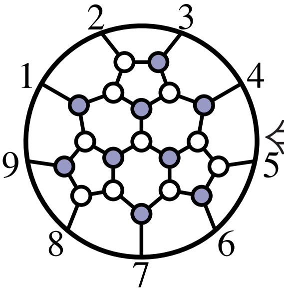

$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ & & & & & & & & \end{pmatrix}$$

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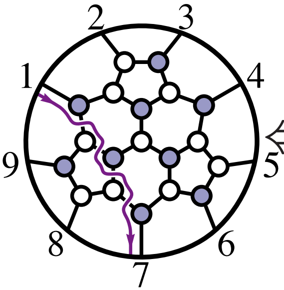

$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & & & & & & & & \\ 7 & & & & & & & & \end{pmatrix}$$

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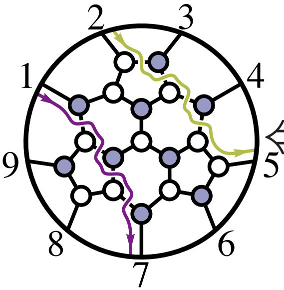

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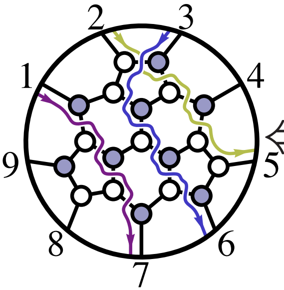

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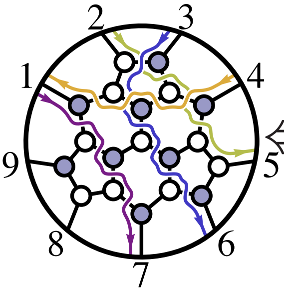

$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & & & & & \\ 7 & 5 & 6 & 10 & & & & & \end{pmatrix}$$

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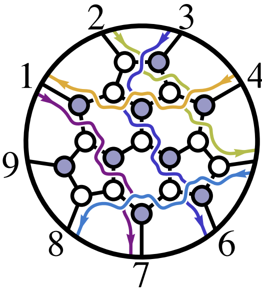

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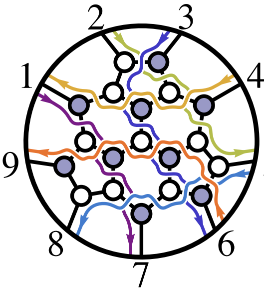

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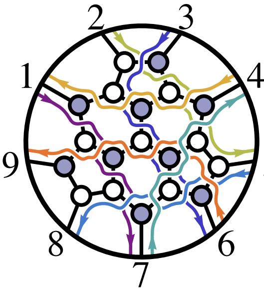

$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & & \end{pmatrix}$$

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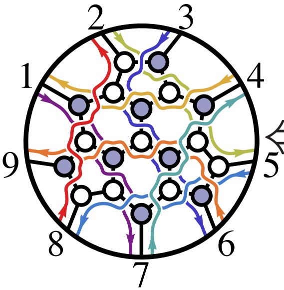

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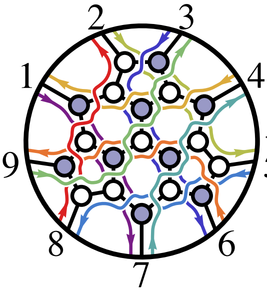

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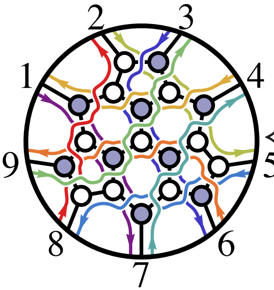
The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha) \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \in G_+(4,9)$$



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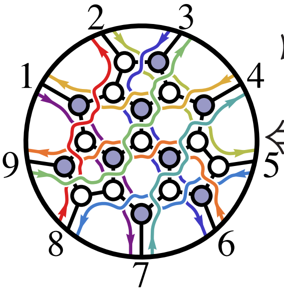
$$\Updownarrow$$



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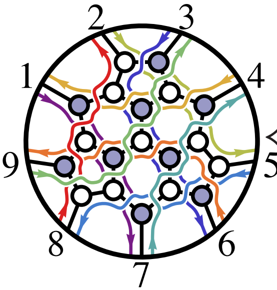

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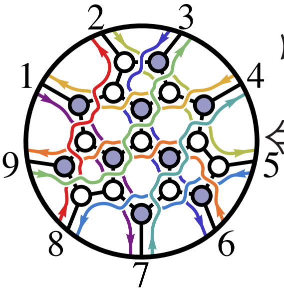

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$$C(\alpha) \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \alpha_3 \\ 0 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & 0 & 0 & 1 \end{pmatrix} \in G_+(4,9)$$

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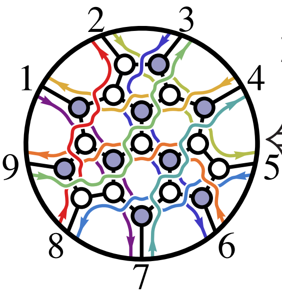

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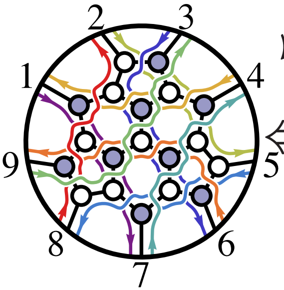
$$C(\alpha) \equiv \begin{pmatrix} 1 & 0 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \alpha_3 \\ 0 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & 0 & 0 & 1 \end{pmatrix} \in G_+(4,9)$$

$$\Updownarrow$$

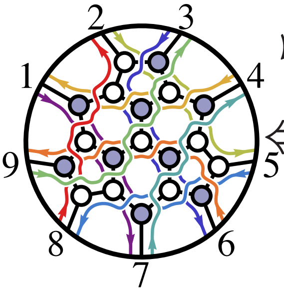


$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & 0 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & \alpha_3 \\ 0 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & 0 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

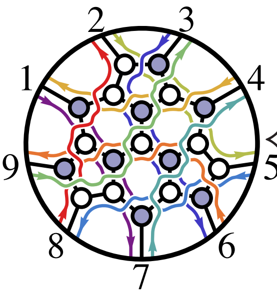


$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha) \equiv \begin{pmatrix} 1 & 0 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_4 & \alpha_7\,\alpha_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & \alpha_3 \\ 0 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix} \in G_+(4,9)$$

$$\Updownarrow$$

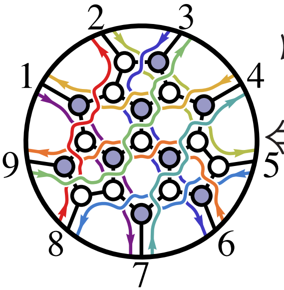


$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_4 & \alpha_7\,\alpha_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & \alpha_3 \\ 0 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

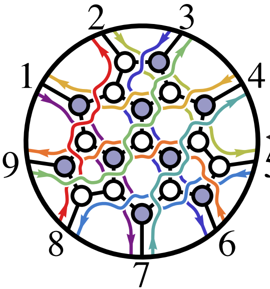


$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha) \equiv \begin{pmatrix} 1 & \alpha_8 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_4 & \alpha_7\,\alpha_4 & 0 & 0 \\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & \alpha_3 \\ -\alpha_9 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix} \in G_+(4,9)$$

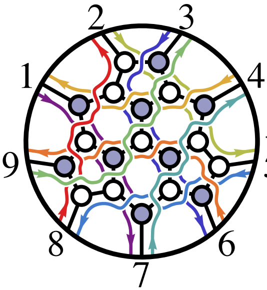
$$\Updownarrow$$



$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha) \equiv \begin{pmatrix} 1 & \alpha_8 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \alpha_{10} & \alpha_4 & \alpha_7\,\alpha_4 & 0 & 0 \\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & \alpha_3 \\ -\alpha_9 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix} \in G_+(4,9)$$

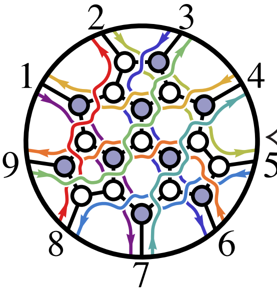


$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha) \equiv \begin{pmatrix} 1 & \alpha_8 & \alpha_5 & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \alpha_{10} & \alpha_4 & \alpha_7\,\alpha_4 & 0 & 0 \\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & \alpha_3 \\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix} \in G_+(4,9)$$

$$\Updownarrow$$

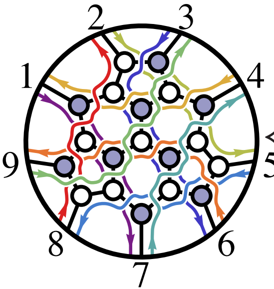


$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha) \equiv \begin{pmatrix} 1 & \alpha_8 & \alpha_5 & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \alpha_{10} & \alpha_4 & \alpha_7\,\alpha_4 & 0 & 0 \\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6) \\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix} \in G_+(4,9)$$

$$\Updownarrow$$

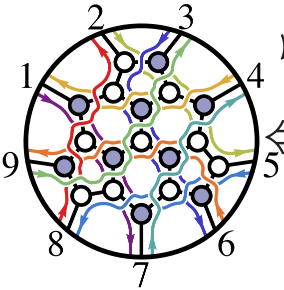


$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha) \equiv \begin{pmatrix} 1 & \alpha_8 & \alpha_5 & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0 \\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6) \\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix} \in G_+(4,9)$$

$$\Updownarrow$$

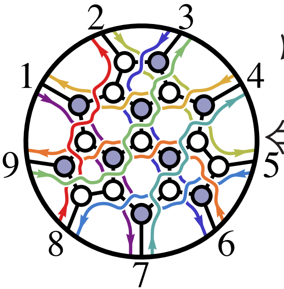


$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix}1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8)\,\alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10}\,(\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6\,(\alpha_3+\alpha_{12}\,\alpha_6) & 0\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1\end{pmatrix}\in G_+(4,9)$$

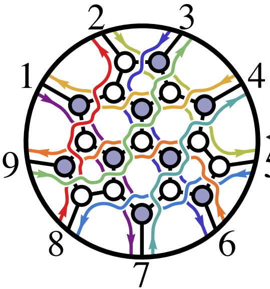
$$\Updownarrow$$



$$\Leftrightarrow\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12\end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha) \equiv \begin{pmatrix} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0 \\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6) \\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix} \in G_+(4,9)$$

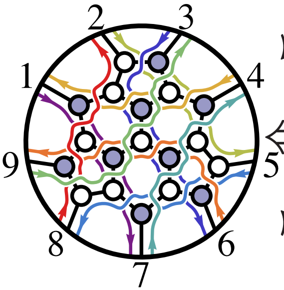


$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

$$f_\sigma \equiv \int \frac{d\alpha_1}{\alpha_1} \wedge \cdots \wedge \frac{d\alpha_{14}}{\alpha_{14}} \; \delta^{k\times 4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\,\delta^{k\times 2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\,\delta^{2\times(n-k)}\big(\lambda\cdot C(\alpha)^{\perp}\big)$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha) \equiv \begin{pmatrix} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0 \\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6) \\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix} \in G_+(4,9)$$

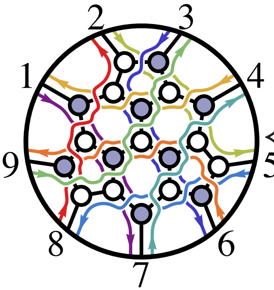


$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

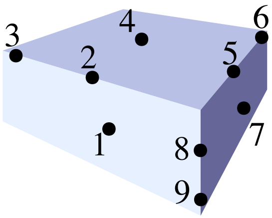
$$f_\sigma \equiv \int \frac{d\alpha_1}{\alpha_1} \wedge \cdots \wedge \frac{d\alpha_{14}}{\alpha_{14}} \;\; \delta^{k\times 4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\,\delta^{k\times 2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\,\delta^{2\times(n-k)}\big(\lambda\cdot C(\alpha)^{\perp}\big)$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6)\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$



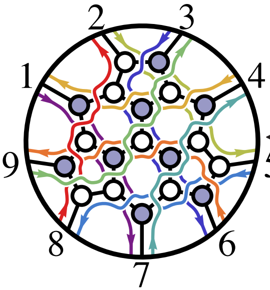
$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$



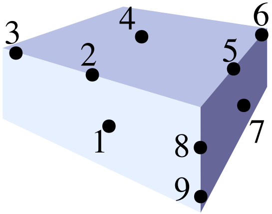
$$f_\sigma\equiv\int\frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_{14}}{\alpha_{14}}\;\delta^{k\times 4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\,\delta^{k\times 2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\,\delta^{2\times(n-k)}\big(\lambda\cdot C(\alpha)^{\perp}\big)$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6)\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$



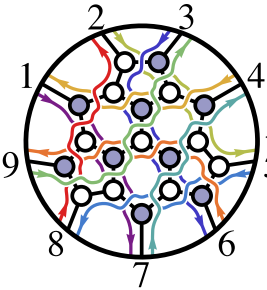
$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}\Leftrightarrow$$



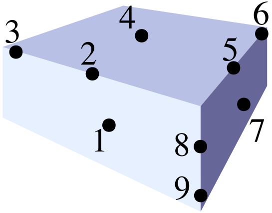
$$f_\sigma\equiv\int\frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_{14}}{\alpha_{14}}\quad\delta^{k\times4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\,\delta^{k\times2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\,\delta^{2\times(n-k)}\big(\lambda\cdot C(\alpha)^{\perp}\big)$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6)\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$



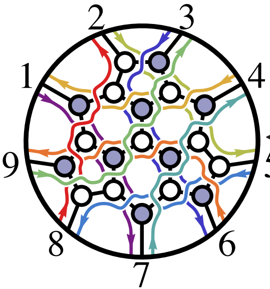
$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}\Leftrightarrow$$



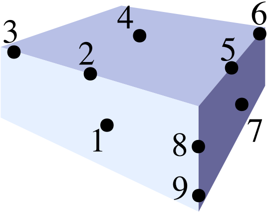
$$f_\sigma\equiv\int\frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_{14}}{\alpha_{14}}\;\delta^{k\times4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\delta^{k\times2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\delta^{2\times(n-k)}\big(\lambda\cdot C(\alpha)^{\perp}\big)$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6)\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$



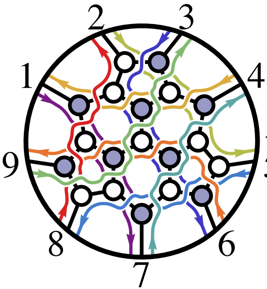
$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}\Leftrightarrow$$



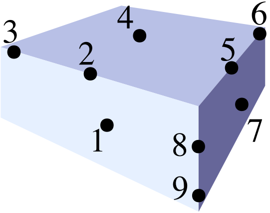
$$f_\sigma\equiv\oint_\sigma\frac{d^{4\times9}C}{GL(4)}\frac{\delta^{k\times4}\big(C\cdot\widetilde{\eta}\big)\,\delta^{k\times2}\big(C\cdot\widetilde{\lambda}\big)\,\delta^{2\times(n-k)}\big(\lambda\cdot C^{\perp}\big)}{\langle1234\rangle\langle2345\rangle\langle3456\rangle\langle4567\rangle\langle5678\rangle\langle6789\rangle\langle7891\rangle\langle8912\rangle\langle9123\rangle}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6)\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$



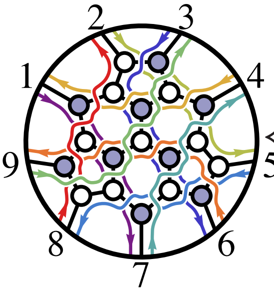
$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}\Leftrightarrow$$



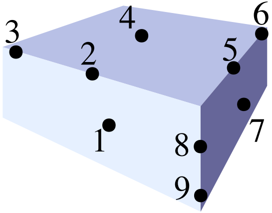
$$f_\sigma\equiv\oint_\sigma\frac{d^{4\times9}C}{GL(4)}\,\frac{\delta^{k\times4}\big(C\cdot\widetilde{\eta}\big)\,\delta^{k\times2}\big(C\cdot\widetilde{\lambda}\big)\,\delta^{2\times(n-k)}\big(\lambda\cdot C^{\perp}\big)}{\langle1234\rangle\langle2345\rangle\langle3456\rangle\langle4567\rangle\langle5678\rangle\langle6789\rangle\langle7891\rangle\langle8912\rangle\langle9123\rangle}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6)\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$



$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}\Leftrightarrow$$



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The Combinatorics and Geometry of On-Shell Physics

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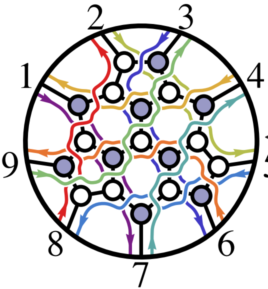
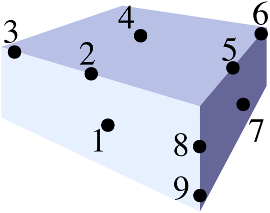


$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}\Leftrightarrow$$



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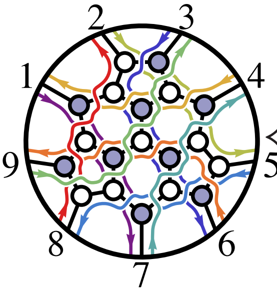

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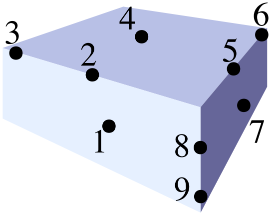

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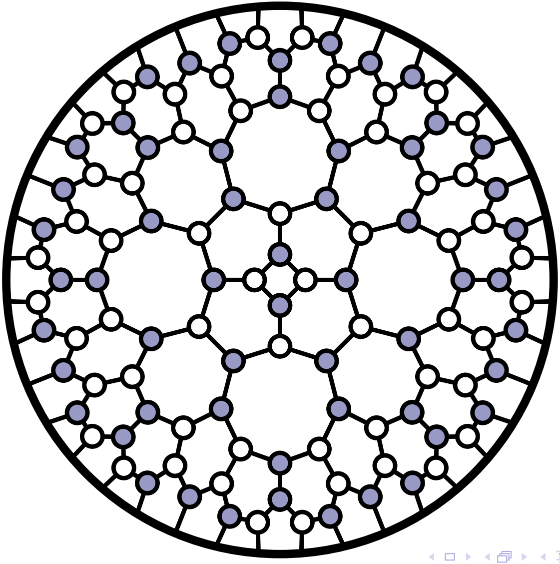

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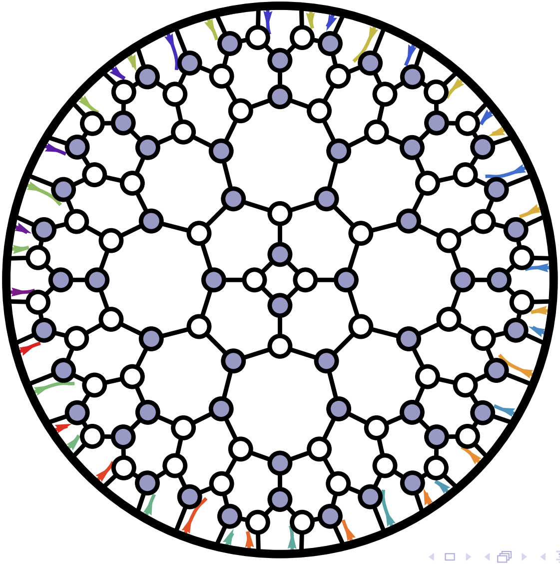

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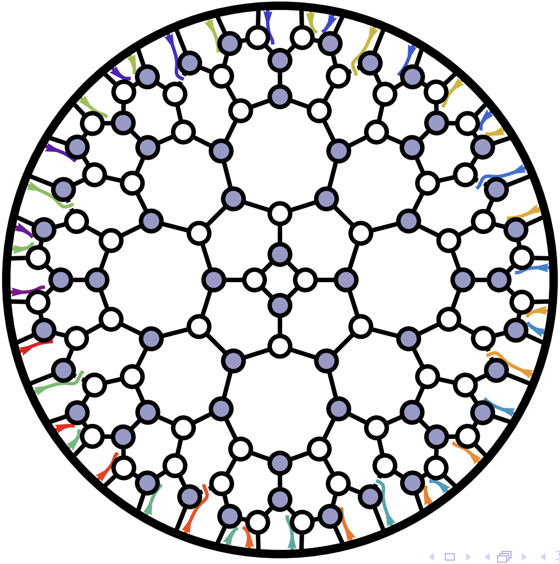

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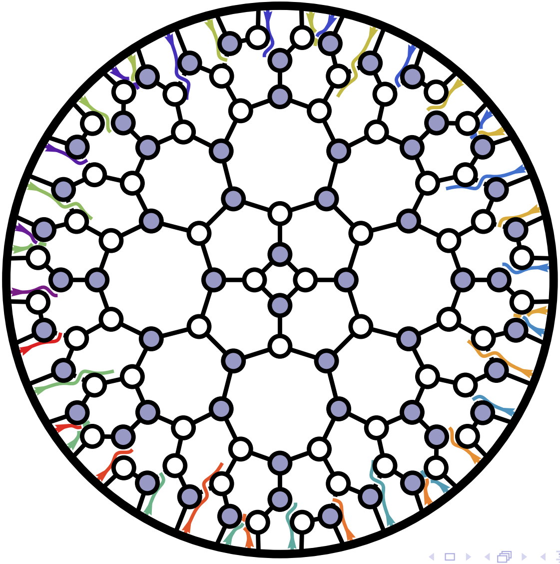

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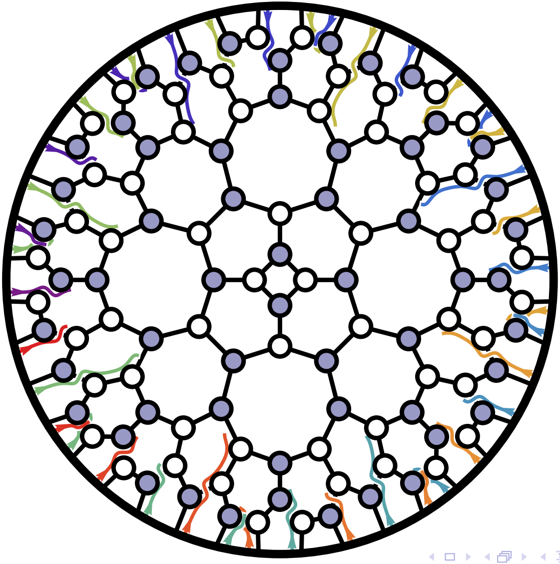

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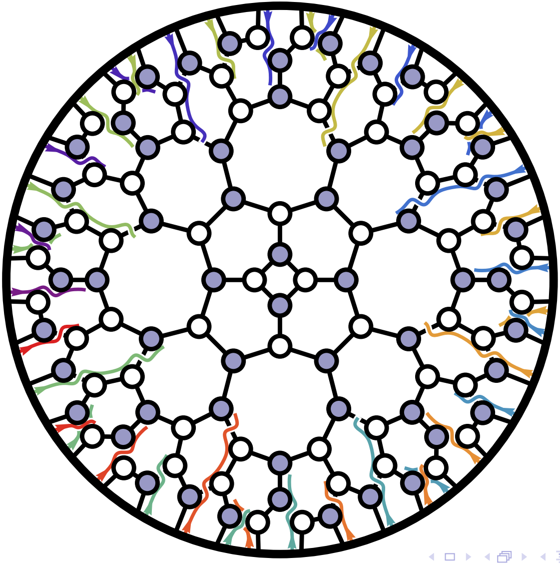

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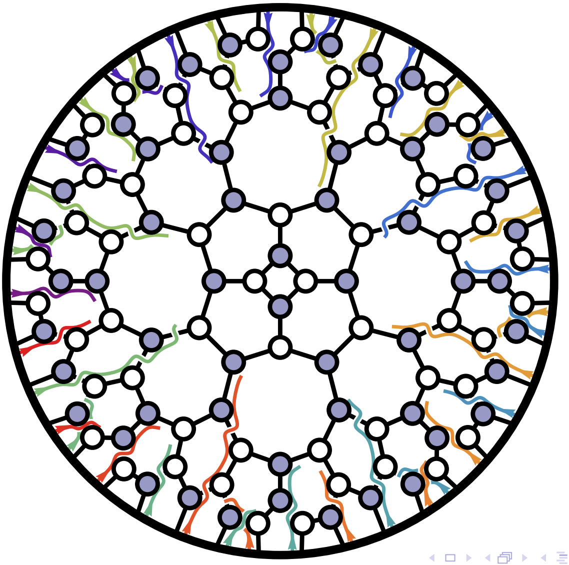

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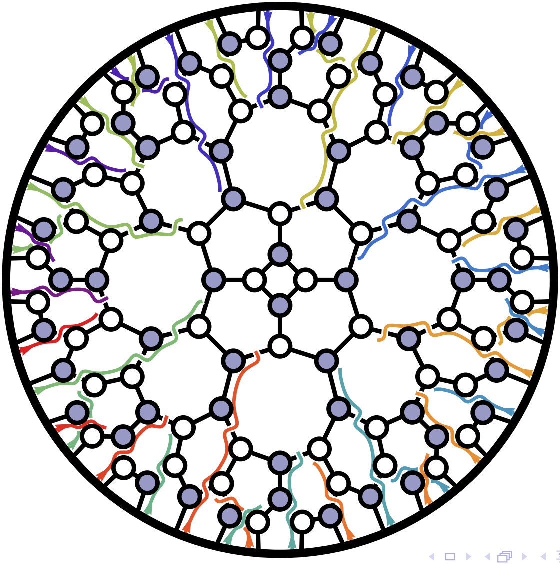

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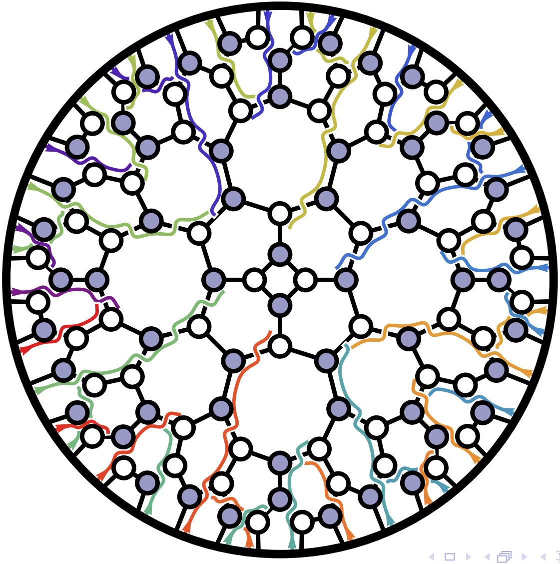

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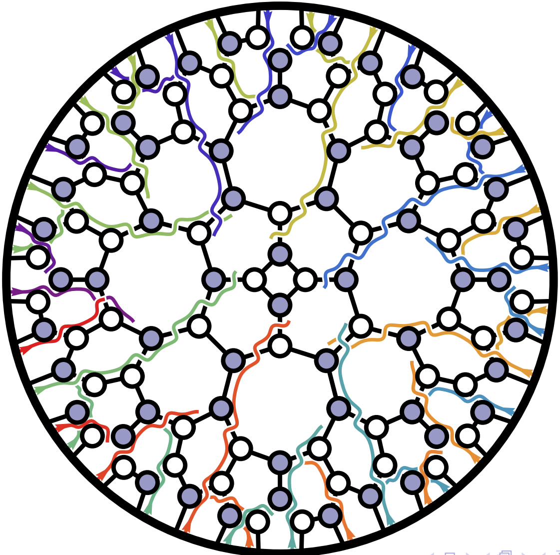

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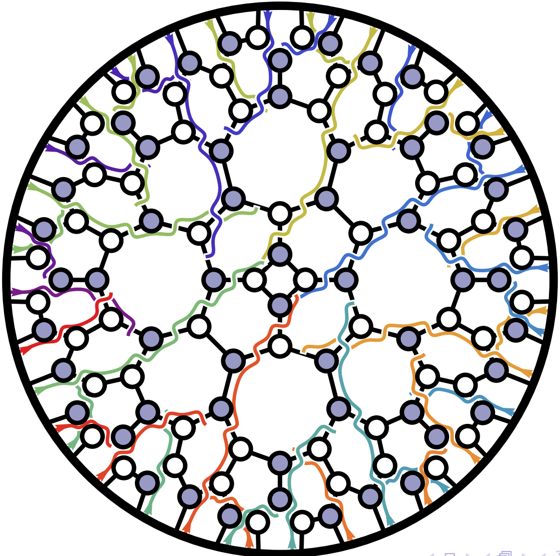

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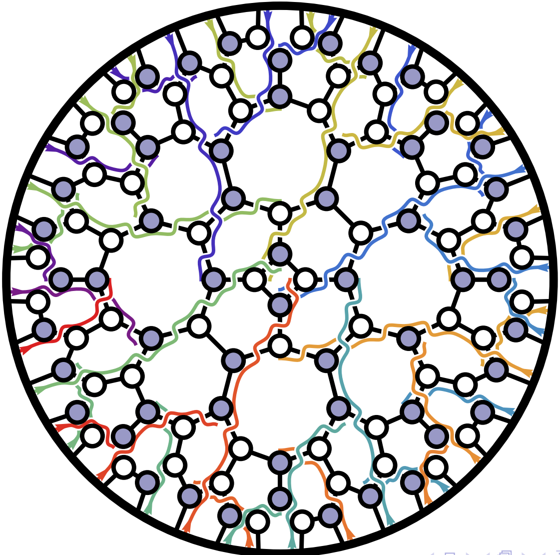

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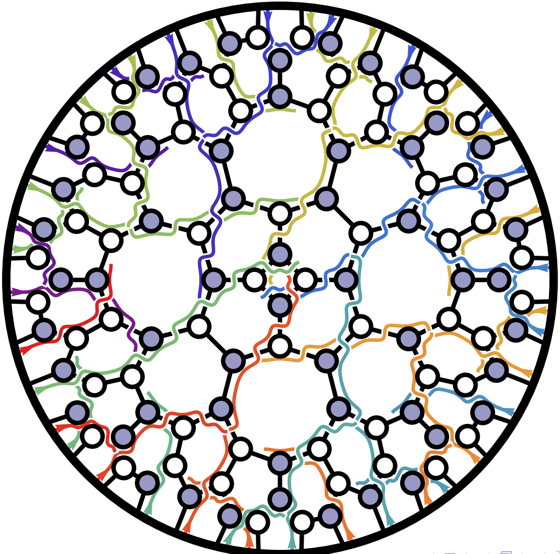

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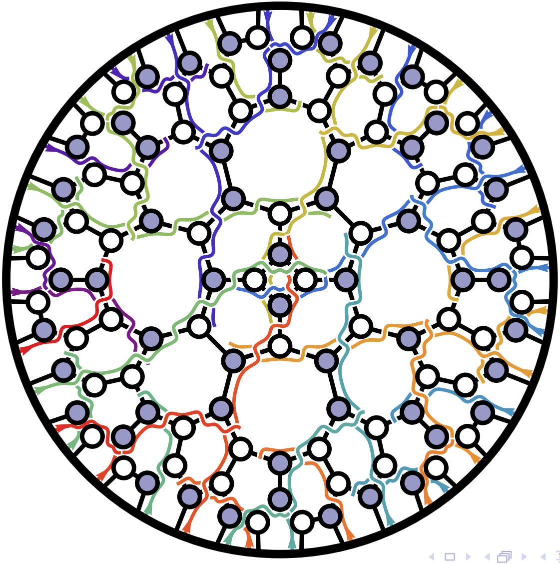

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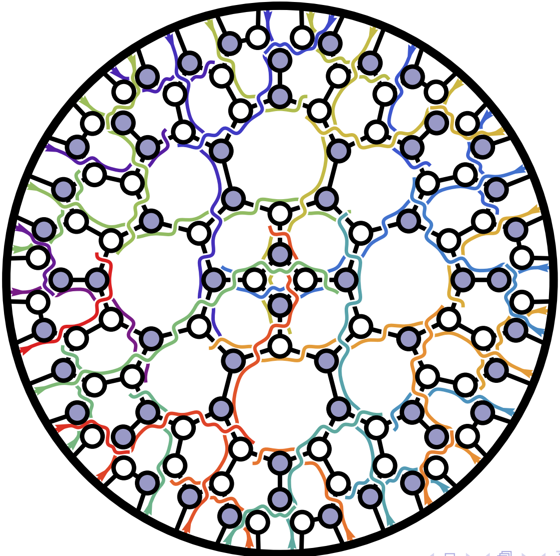

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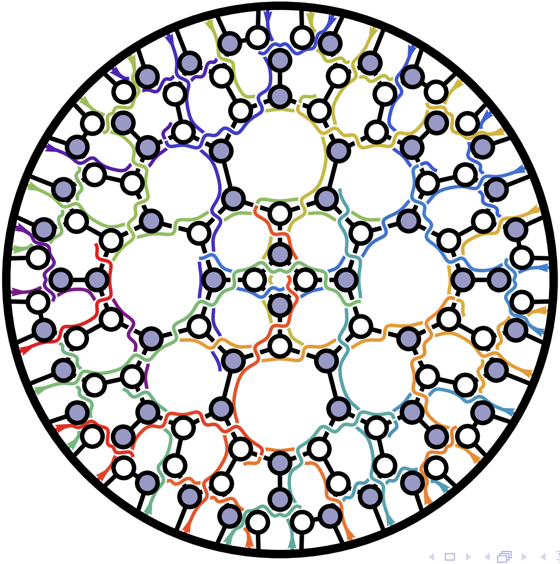

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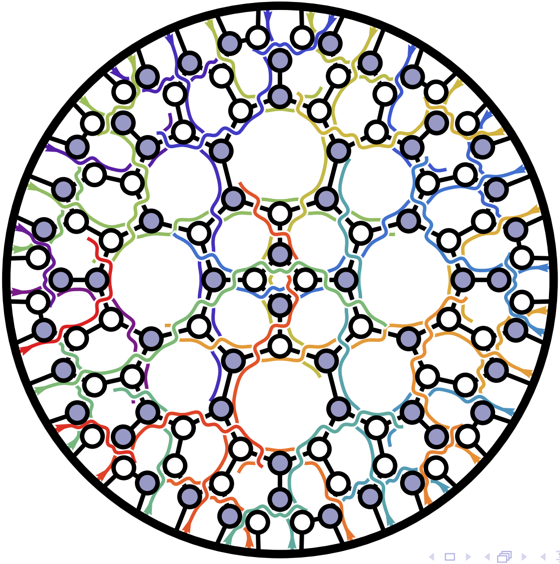

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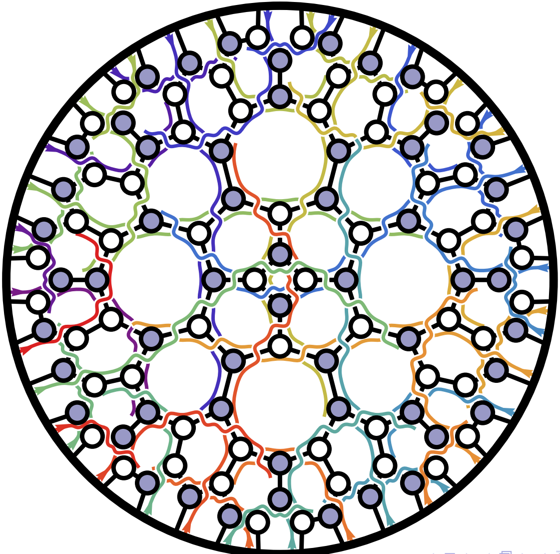

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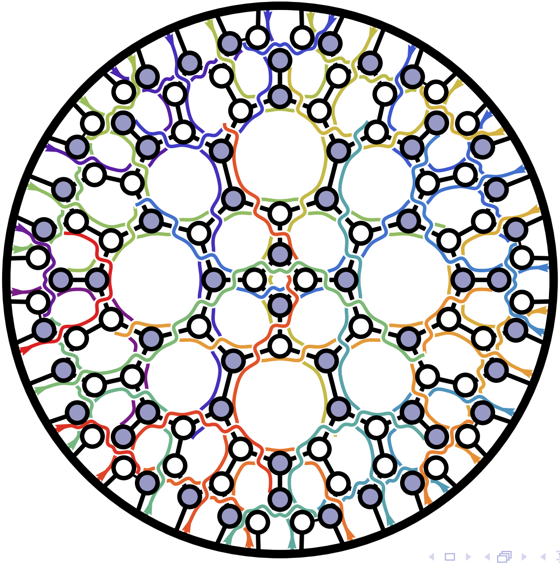

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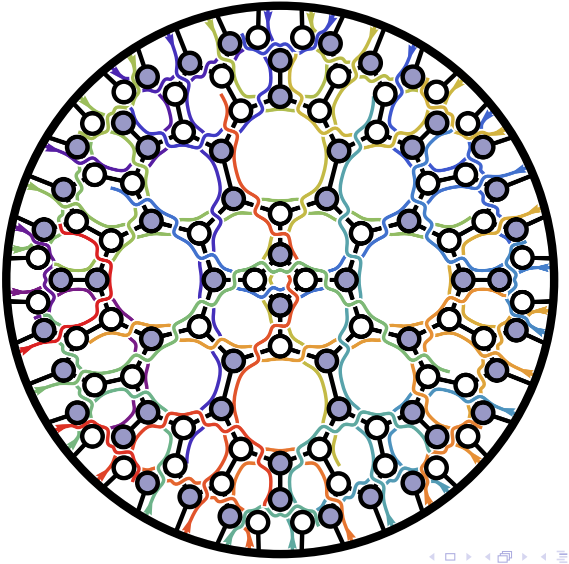

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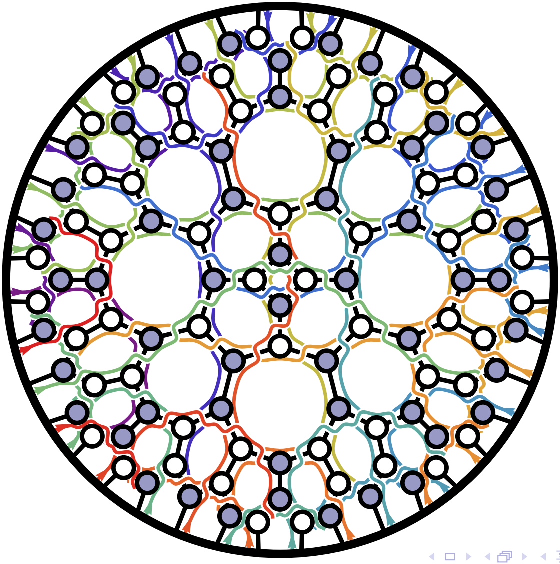

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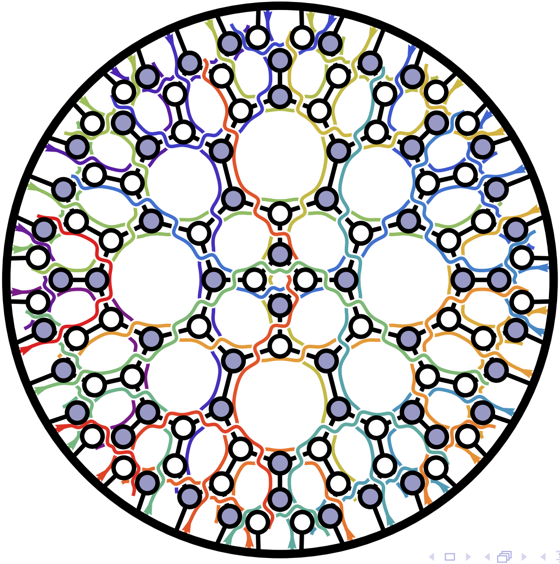

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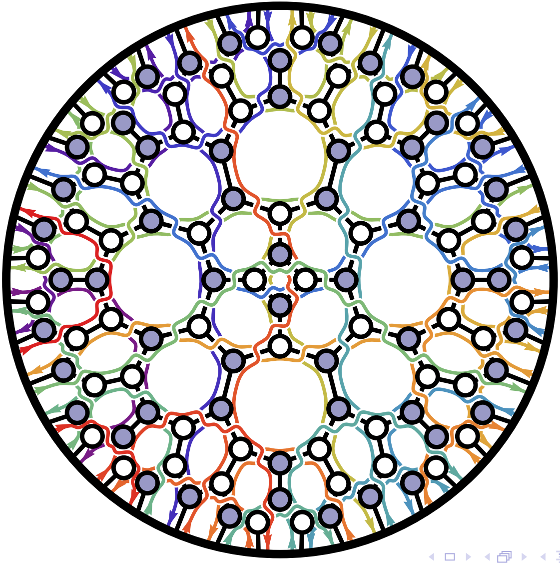

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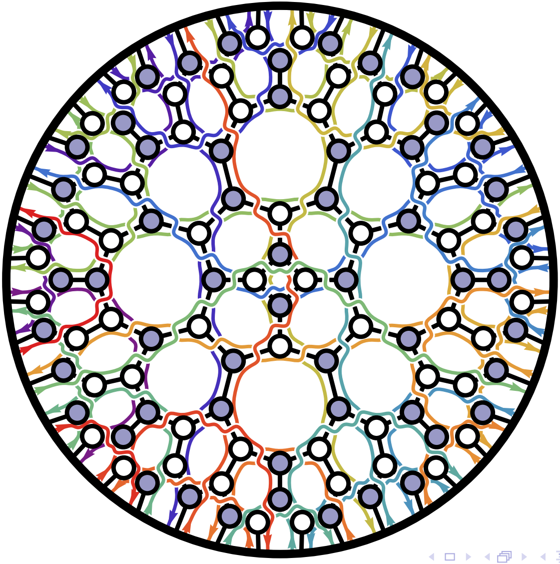

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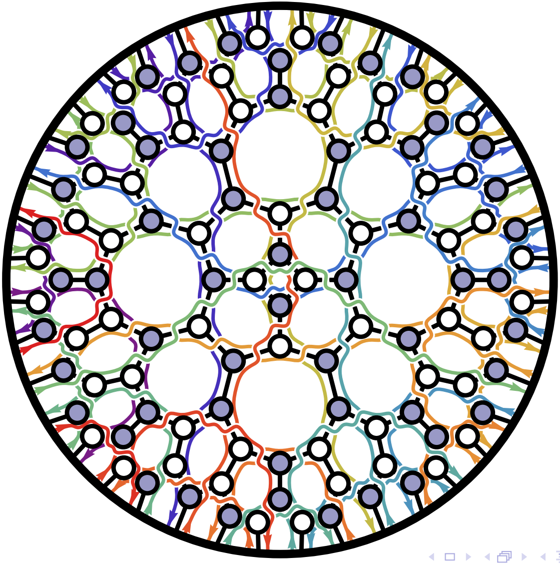

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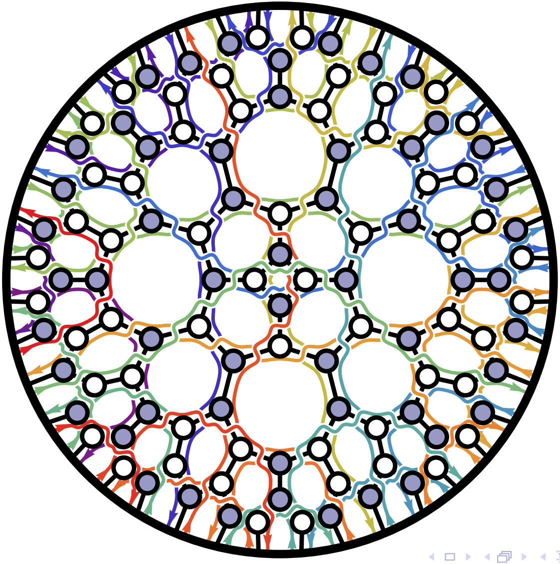

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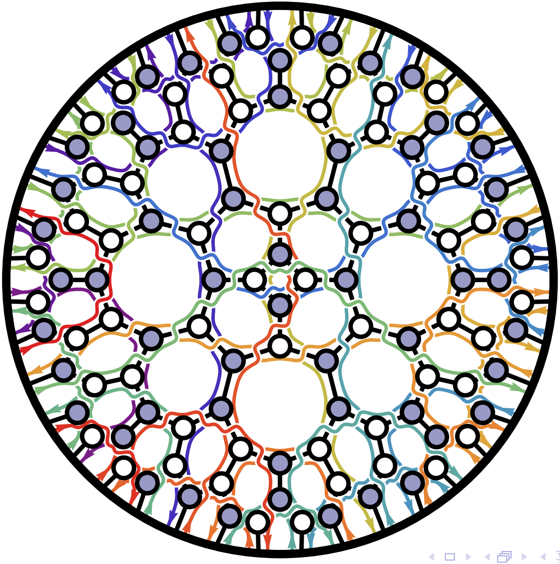

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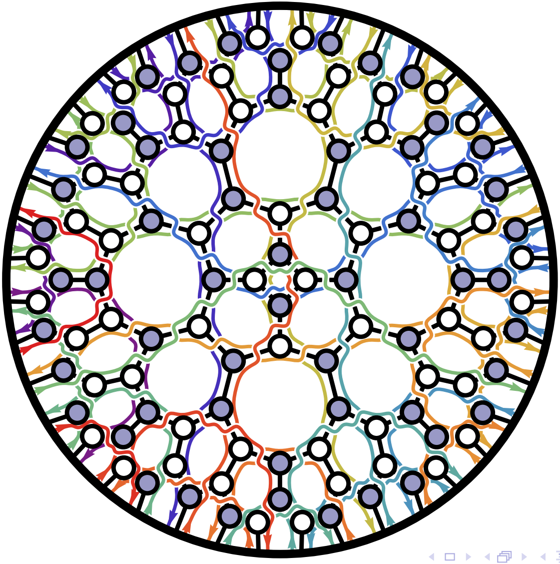

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