$B_{s(d)} \to \mu^+\mu^-$ in the SM and beyond

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1. **Flavour-changing weak interactions and effective lagrangians**
2. $B_{s(d)} \to \ell^+\ell^-$ **in the SM**
3. **Sensitivity to new physics**
4. **Three-loop QCD corrections**
5. **Electroweak corrections**
6. **Predictions and uncertainties**
7. **Summary**

B-meson or Kaon decays occur at low energies, at scales $\mu \ll M_W$.

We pass from the full theory of electroweak interactions to an effective theory by removing the high-energy degrees of freedom, i.e. integrating out the W-boson and all the other particles with $m \sim M_W$.

$$\mathcal{L}_{\text{(full EW}\times\text{QCD)}} \longrightarrow \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}\times\text{QCD}}\begin{pmatrix}\text{quarks} \neq t \\ \text{\& leptons}\end{pmatrix} + N \sum_n C_n(\mu)\, Q_n$$

$\boldsymbol{Q_n}$ – **local interaction terms (operators),** $\boldsymbol{C_n}$ – **coupling constants (Wilson coefficients)**

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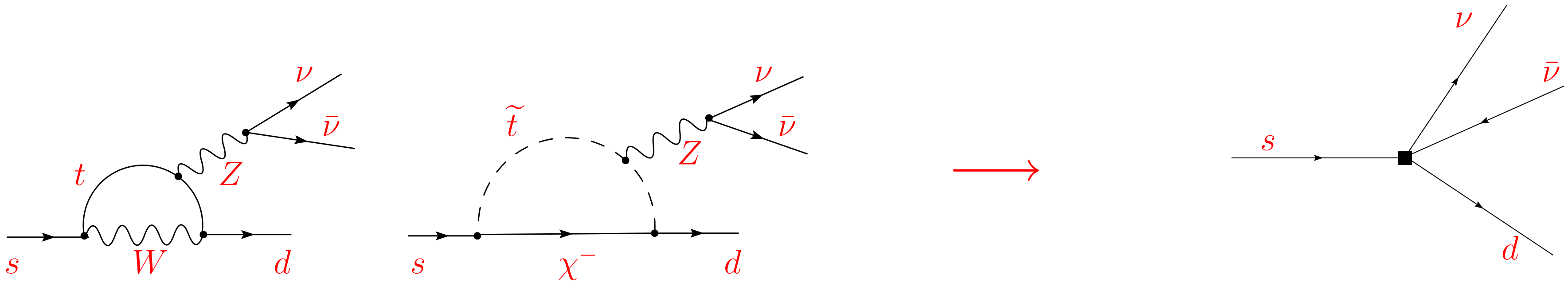
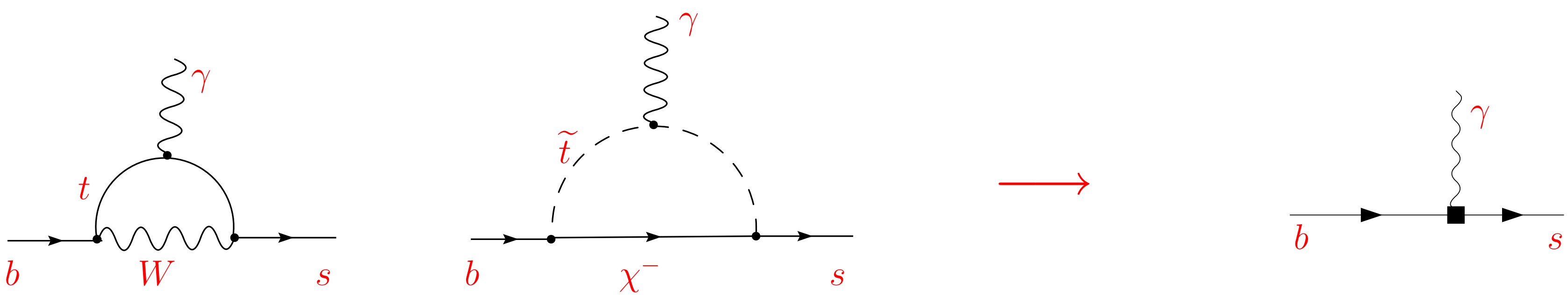
Q_n – local interaction terms (operators), C_n – coupling constants (Wilson coefficients)

Information on the electroweak-scale physics is encoded in the values of $C_i(\mu)$, e.g.,





This is a modern version of the Fermi theory for weak interactions. It is "nonrenormalizable" in the traditional sense but actually renormalizable. It is also predictive because all the C_i are calculable, and only a finite number of them is necessary at each given order in the (external momenta)/M_W expansion.

Advantages: **Resummation of $\left(\alpha_s \ln \frac{M_W^2}{\mu^2}\right)^n$ using renormalization group, easier account for symmetries.**

$B_s \to \mu^+\mu^-$ — the flavour physics highlight of the LHC

- It is a strongly suppressed, loop-generated process in the SM. Its average time-integrated branching ratio (with final-state photon bremsstrahlung included) reads:

$$\overline{\mathcal{B}}_{\rm SM} = (3.65 \pm 0.23) \times 10^{-9}$$

[C. Bobeth, M. Gorbahn, T. Hermann, MM, E. Stamou and M. Steinhauser, Phys. Rev. Lett. 112 (2014) 101801]

- It is very sensitive to new physics even in models with Minimal Flavour Violation (MFV). Enhancements by orders of magnitude are possible even when constraints from all the other measurements are taken into account.

- Recently measured branching ratios

$$\overline{\mathcal{B}}_{\rm exp} = \begin{cases} \left(2.9\,^{+1.1}_{-1.0}\right) \times 10^{-9}, & \text{LHCb} \\ \left(3.0\,^{+1.0}_{-0.9}\right) \times 10^{-9}, & \text{CMS} \end{cases}$$

LHCb: [Phys. Rev. Lett. 111 (2013) 101805]

CMS: [Phys. Rev. Lett. 111 (2013) 101804]

Combined: $\overline{\mathcal{B}}_{\rm exp} = (\,2.9 \pm 0.7\,) \times 10^{-9}$

[CMS-PAS-BPH-13-007, LHCb-CONF-2013-012]

- ATLAS: $\overline{\mathcal{B}}_{\rm exp} < 1.5 \times 10^{-8}$ @ 95%C.L.

Operators (dim 6) that matter for $B_s \to \mu^+\mu^-$ read

$$Q_A = \left(\bar{b}\gamma^\alpha\gamma_5 s\right)\left(\bar{\mu}\gamma_\alpha\gamma_5\mu\right)$$ – **the only relevant one in the SM**

$$Q_{S(P)} = \left(\bar{b}\gamma_5 s\right)\left(\bar{\mu}(\gamma_5)\mu\right) = \frac{i(\bar{b}\gamma^\alpha\gamma_5 s)\partial_\alpha(\bar{\mu}(\gamma_5)\mu)}{m_b+m_s} + \boxed{E} + \boxed{T}$$

vanishing by EOM

total derivative

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Necessary non-perturbative input: $$\langle 0|\bar{b}\gamma^\alpha\gamma_5 s|B_s(p)\rangle = ip^\alpha f_{B_s}$$

Recent lattice determinations of the B_s-meson decay constant:

$$f_{B_s} = \begin{cases} 225.0(4.0)\ \text{MeV}, & \text{HPQCD (r)}, & \text{arXiv:1110.4510} \\ 224.0(5.0)\ \text{MeV}, & \text{HPQCD (nr)}, & \text{arXiv:1302.2644} \\ 234.0(6.0)\ \text{MeV}, & \text{ROME}, & \text{arXiv:1212.0301} \\ 242.0(9.5)\ \text{MeV}, & \text{FNAL/MILC}, & \text{arXiv:1112.3051} \\ 232.0(10)\ \text{MeV}, & \text{ETM}, & \text{arXiv:1107.1441} \\ 219.0(12)\ \text{MeV}, & \text{ALPHA}, & \text{arXiv:1210.6524} \\ 235.4(12)\ \text{MeV}, & \text{RBC/UKQCD}, & \text{arXiv:1404.4670} \\ 224.0(14)\ \text{MeV}, & \text{ALPHA}, & \text{arXiv:1404.3590} \end{cases}$$

Flavour Lattice Averaging Group (FLAG), arXiv:1310.8555 gives

$$f_{B_s} = 227.7(4.5)\ \text{MeV}.$$

Average time-integrated branching ratio:

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = \frac{|N|^2 M_{B_s}^3 f_{B_s}^2}{8\pi\Gamma_H^s}\beta\left(\left|rC_A - uC_P\right|^2 F_P + \left|u\beta C_S\right|^2 F_S\right) + \mathcal{O}(\alpha_{em}),$$

where $N = \frac{V_{tb}^* V_{ts} G_F^2 M_W^2}{\pi^2}, \quad r = \frac{2m_\mu}{M_{B_s}}, \quad \beta = \sqrt{1-r^2}, \quad u = \frac{M_{B_s}}{m_b+m_s}$,

$$F_P = 1 - \frac{\Delta\Gamma^s}{\Gamma_L^s}\sin^2\left[\tfrac{1}{2}\phi_s^{\mathrm{NP}} + \arg(rC_A - uC_P)\right] \xrightarrow{\text{SM CP}} 1\ ,$$

$$F_S = 1 - \frac{\Delta\Gamma^s}{\Gamma_L^s}\cos^2\left[\tfrac{1}{2}\phi_s^{\mathrm{NP}} + \arg C_S\right] \xrightarrow{\text{SM CP}} \frac{\Gamma_H^s}{\Gamma_L^s}$$

derived following [K. de Bruyn *et al.*, Phys. Rev. Lett. 109 (2012) 041801]

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In the limit of no CP-violation, mass eigenstates are CP eigenstates:

Heavier, CP-odd: $B_s^H = \frac{1}{\sqrt{2}}(B_s + \bar{B}_s)$, annihilated by $\bar{b}\gamma_5 s + \bar{s}\gamma_5 b$, $(\tau_H = 1.615(21)\text{ ps})$

Lighter, CP-even: $B_s^L = \frac{1}{\sqrt{2}}(B_s - \bar{B}_s)$, annihilated by $\bar{b}\gamma_5 s - \bar{s}\gamma_5 b$, $(\tau_L = 1.516(11)\text{ ps})$

Our interactions in this limit are all CP-even:

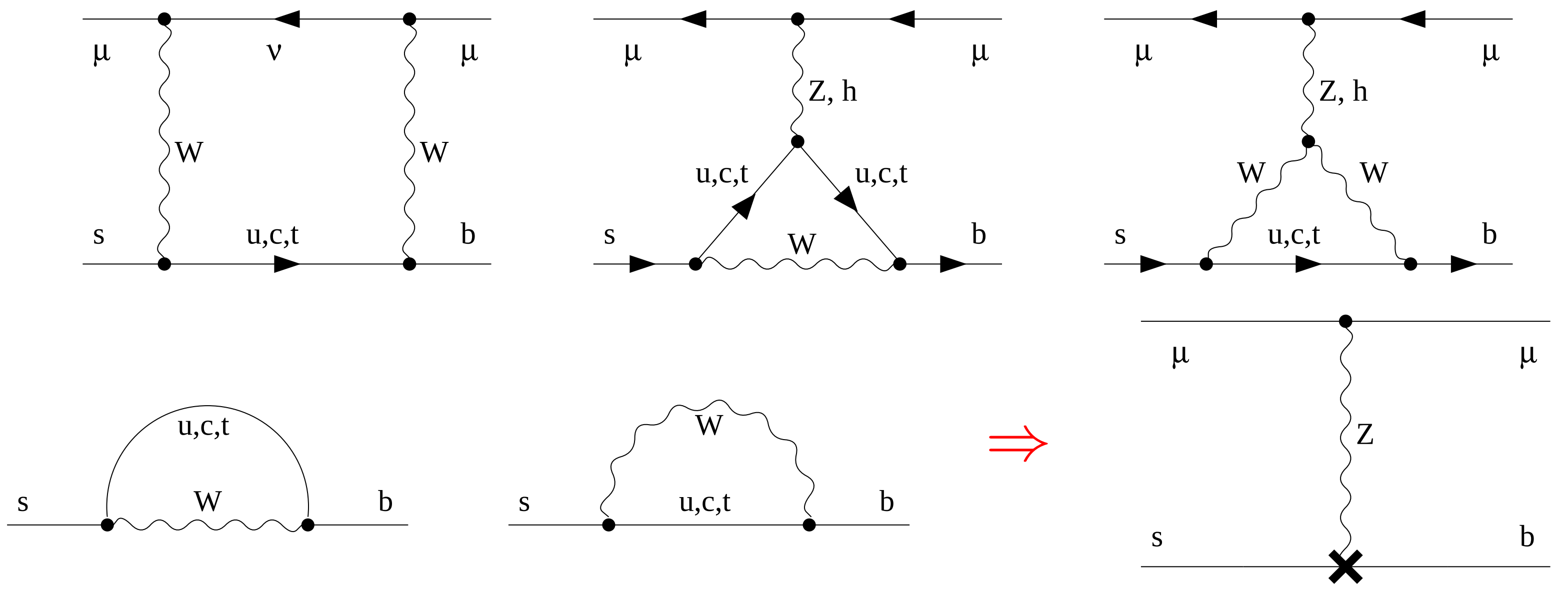
$$\left.\begin{aligned} Q_A + Q_A^\dagger &= \left[(\bar{b}\gamma^\alpha\gamma_5 s) + (\bar{s}\gamma^\alpha\gamma_5 b)\right](\bar{\mu}\gamma_\alpha\gamma_5\mu) \\ Q_P + Q_P^\dagger &= \left[(\bar{b}\gamma_5 s) + (\bar{s}\gamma_5 b)\right](\bar{\mu}\gamma_5\mu) \end{aligned}\right\} \text{annihilate } B_s^H\text{, produce CP-odd dimuons}$$

$$\left. Q_S + Q_S^\dagger = \left[(\bar{b}\gamma_5 s) - (\bar{s}\gamma_5 b)\right](\bar{\mu}\mu) \right\} \text{annihilates } B_s^L\text{, produces CP-even dimuons}$$

With SM-like CP-violation – still $Q_{A,P}$ annihilate B_s^H and Q_S annihilates B_s^L.

Beyond SM – interesting time-dependent observables, see arXiv:1303.3820, 1407.2771.

Evaluation of the LO Wilson coefficients in the SM:



$\bar{b}_L \not{D} s_L$

$$C_A^{(0)} = \tfrac{1}{2} Y_0\left(m_t^2/M_W^2\right), \qquad Y_0(x) = \frac{3x^2}{8(x-1)^2}\ln x + \frac{x^2-4x}{8(x-1)},$$

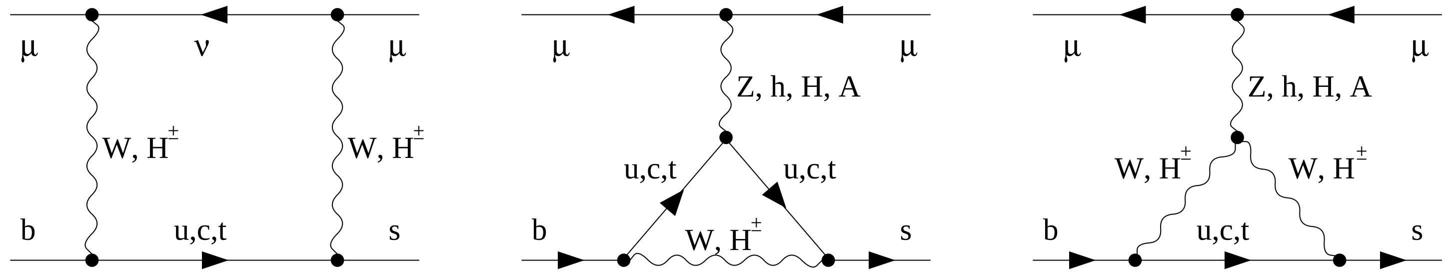
$$C_{S,P} = \mathcal{O}\left(\frac{m_\mu}{M_W}\right).$$

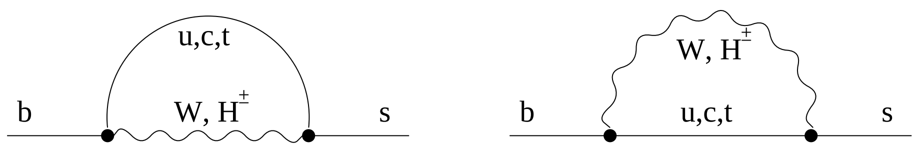
Effects of $C_{S,P}$ on the branching ratio are suppressed by $M_{B_s}^2/M_W^2 \Rightarrow$ negligible.

Thus, only C_A matters in the SM.

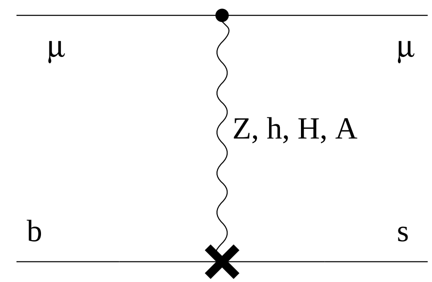
Evaluation of the Wilson coefficients beyond the SM.

Example 1: the Two-Higgs-Doublet Model II





$\Rightarrow$



$$\tan\beta = v_2/v_1, \qquad z = M^2_{H^\pm}/m_t^2,$$

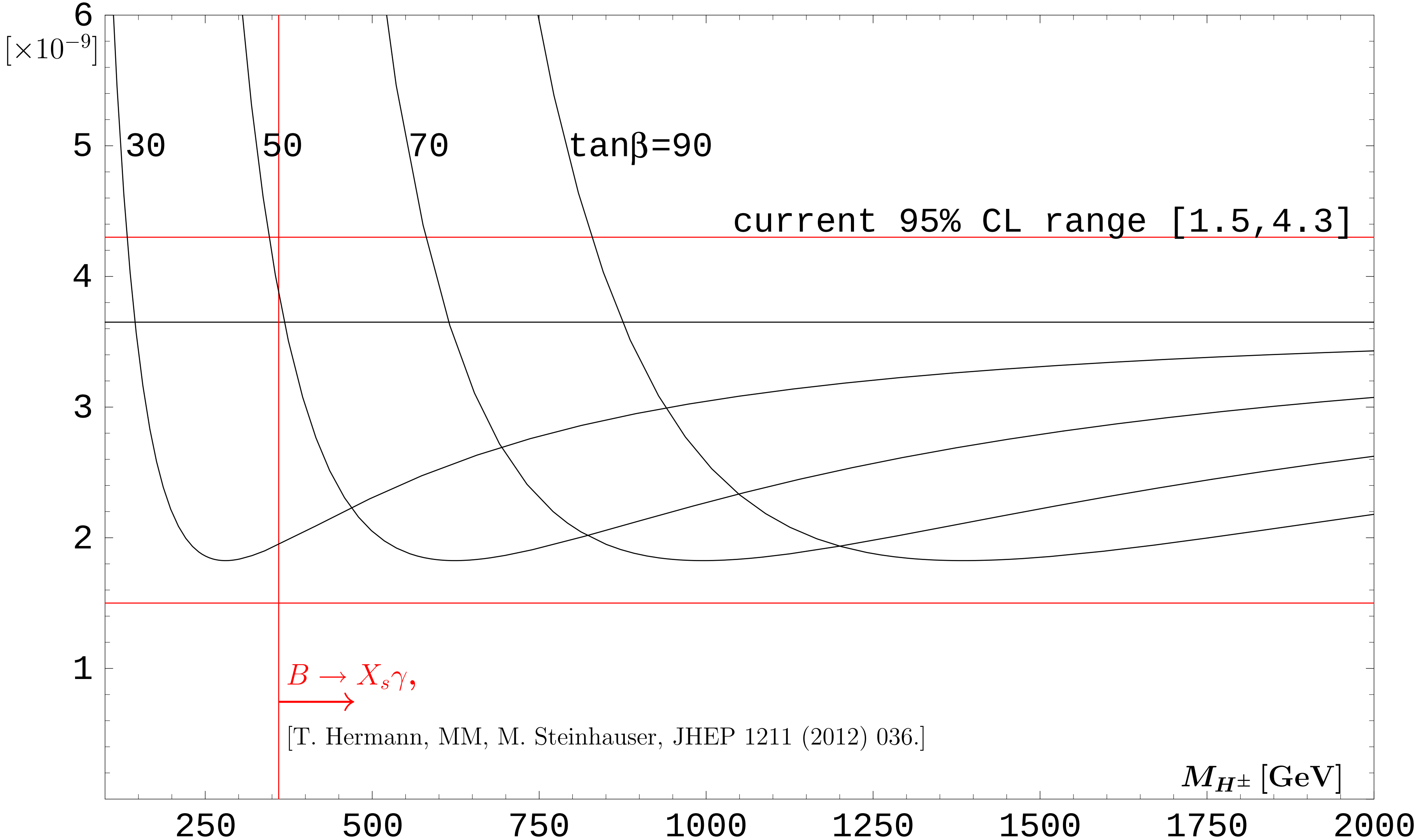
$$C_S \;\simeq\; C_P \;\simeq\; \frac{m_\mu m_b \tan^2\beta}{4M_W^2}\,\frac{\ln z}{z-1} > 0,$$

H.E. Logan and U. Nierste,
NPB 586 (2000) 39
($\mathcal{O}(\tan\beta)$ neglected)

$$\mathcal{B}(B_s \to \mu^+\mu^-) \simeq (\text{const.}) \left[\left| \frac{2m_\mu}{M_{B_s}} C_A - C_P \right|^2 + |C_S|^2 \right]$$

$$C_A = \underbrace{C_A^{\rm SM}}_{\text{positive}} + \underbrace{\Delta C_A}_{\text{small}} \qquad \Rightarrow \begin{cases} \text{suppression for moderate } C_{S,P} \\ \text{enhancement for huge } \tan\beta \text{ only} \end{cases}$$

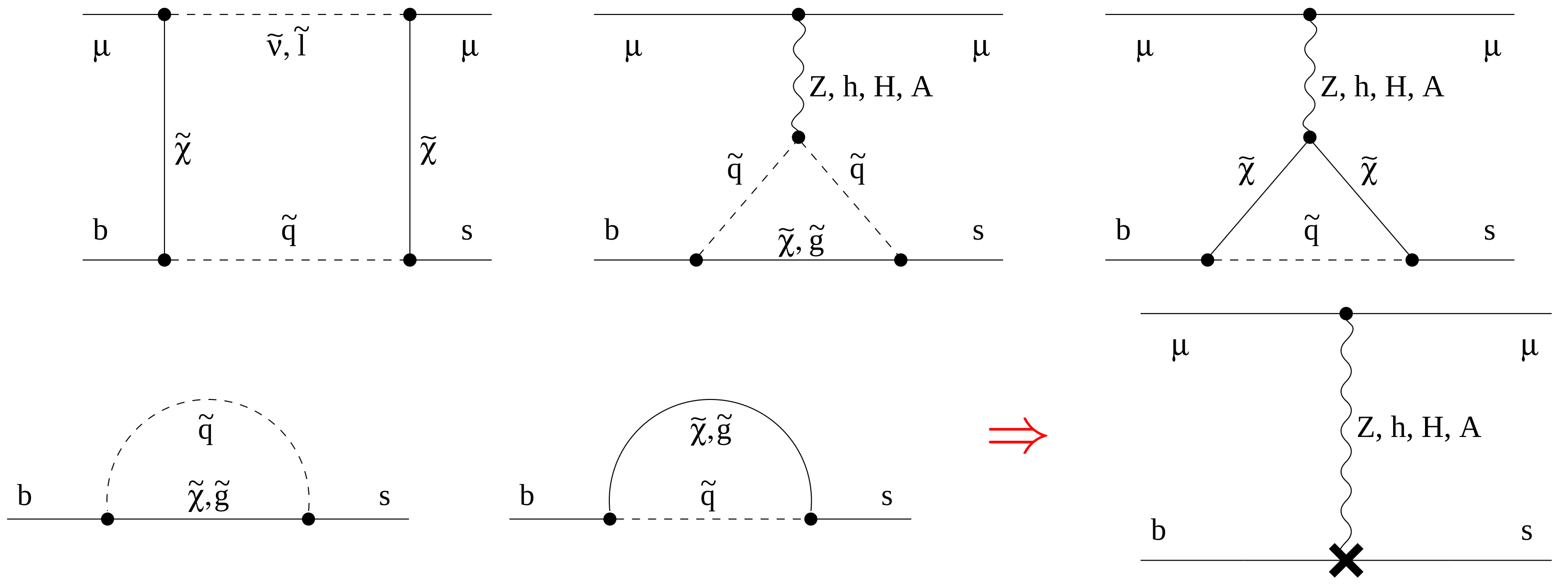
$\mathcal{B}(B_s \to \mu^+\mu^-)$ in the Two-Higgs-Doublet Model II



For $M_{H^\pm} = 600\,\text{GeV}$ and $\tan\beta = 50$: suppression by a factor of ~ 2.
Enhancement possible only for $\tan\beta > 65$.

Evaluation of the Wilson coefficients beyond the SM.

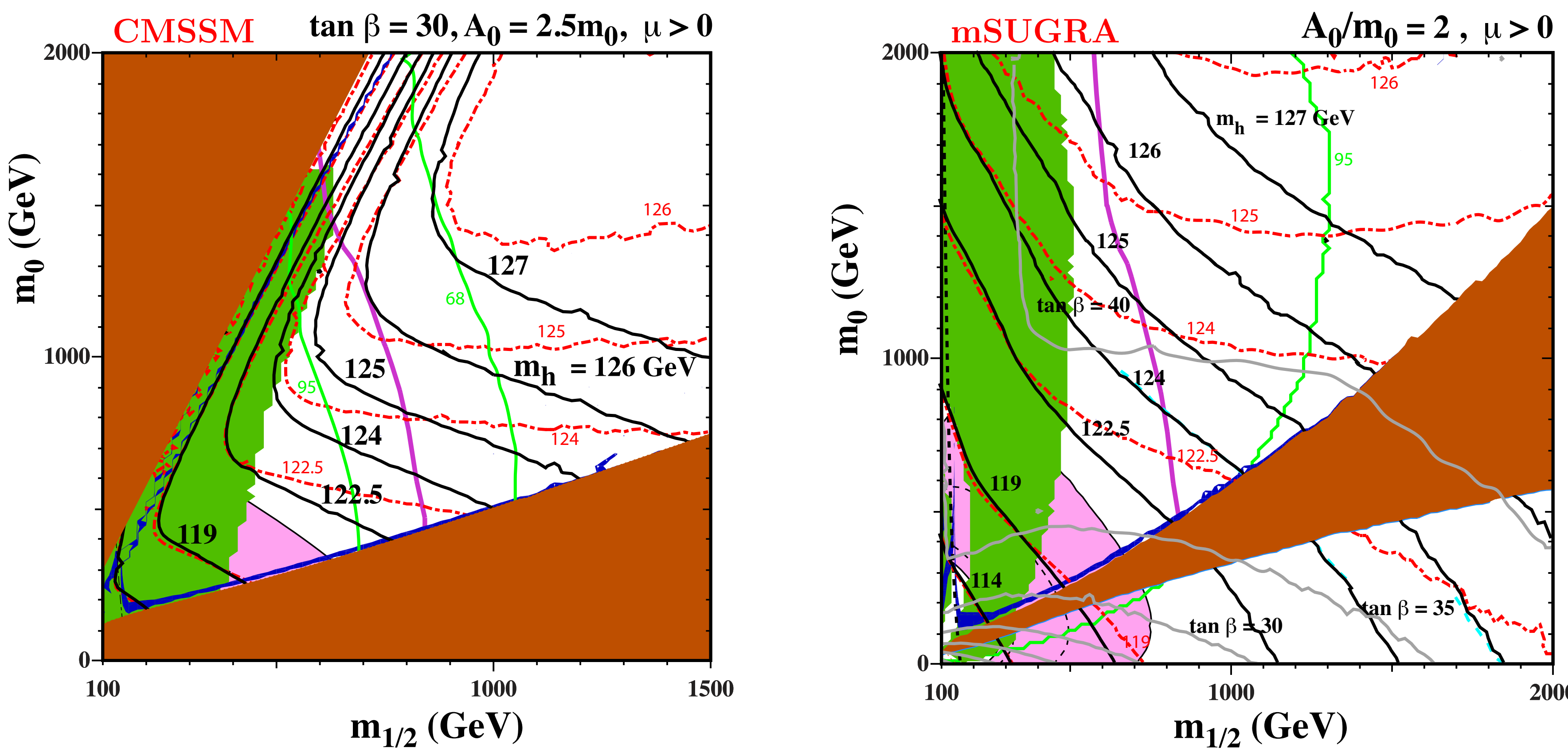
Example 2: the MSSM.



For large $\tan\beta$: $$\mathcal{B}\left(B_s \to \mu^+\mu^-\right) \sim \frac{m_b^2 m_\mu^2}{M_A^4}\tan^6\beta$$

K. S. Babu and C. F. Kolda, Phys. Rev. Lett. 84 (2000) 228.

Examples of constraints on the MSSM parameter space:



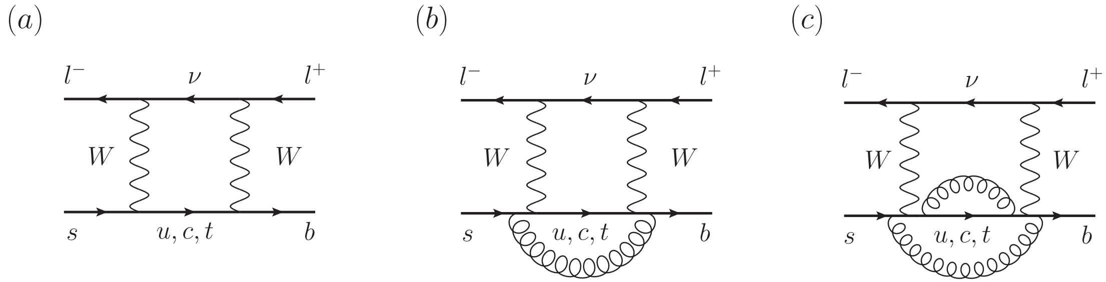
Figs. 1 and 7 from arXiv:1312.5426 by John Ellis.

green lines	– bounds from $B_s \to \mu^+\mu-$ (CMS & LHCb 2013, exclusion to the left)
purple lines	– ATLAS 95%CL bounds from $\not{E}_T+$ jets
green shaded	– excluded by $b \to s\gamma$
brown shaded	– charged LSP
pink shaded	– SUSY helps with $g-2$
blue strips	– favoured by Ω_{DM}

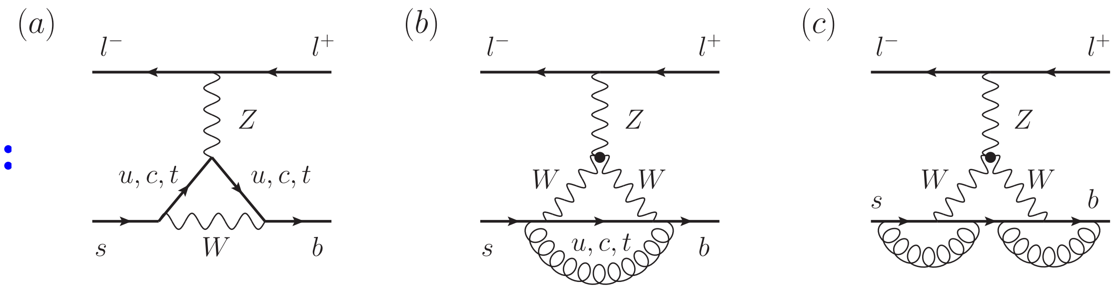
Evaluation of the NNLO QCD matching corrections in the SM

[T. Hermann, MM, M. Steinhauser, JHEP 1312 (2013) 097]

W-boxes:
(1LPI)



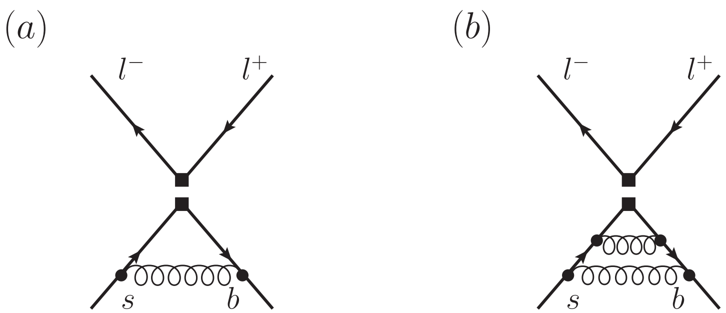
Z-penguins:
(1LPI)



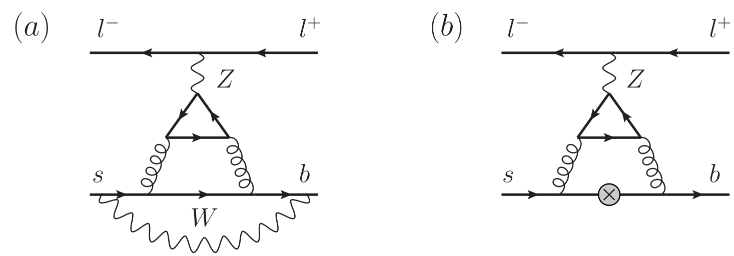
Subtleties: (i) counterterms with finite parts $\sim \bar{b}_L \not{D} s_L$

(ii) evanescent operators: $E_B = (\bar{b}\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5 s)(\bar{\mu}\gamma^\sigma\gamma^\rho\gamma^\nu\gamma_5\mu) - 4(\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\mu}\gamma^\alpha\gamma_5\mu)$

$E_T = \mathrm{Tr}\,(\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\alpha\gamma_5)(\bar{b}\gamma_\nu\gamma_\rho\gamma_\sigma s)(\bar{\mu}\gamma_\alpha\gamma_5\mu) + 24(\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\mu}\gamma^\alpha\gamma_5\mu)$



Renormalization of E_B

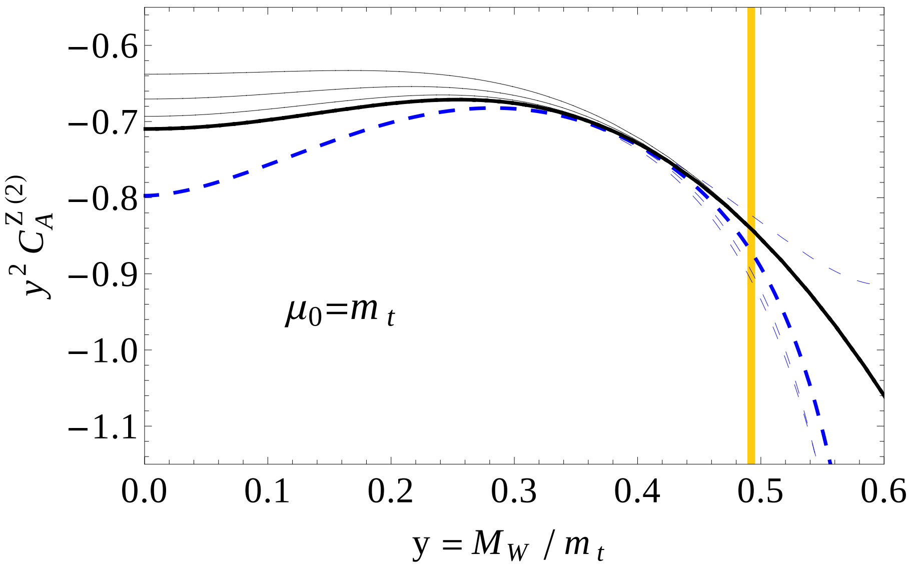


Diagrams generating E_T

Perturbative series for the Wilson coefficient at $\mu = \mu_0 \sim m_t, M_W$:

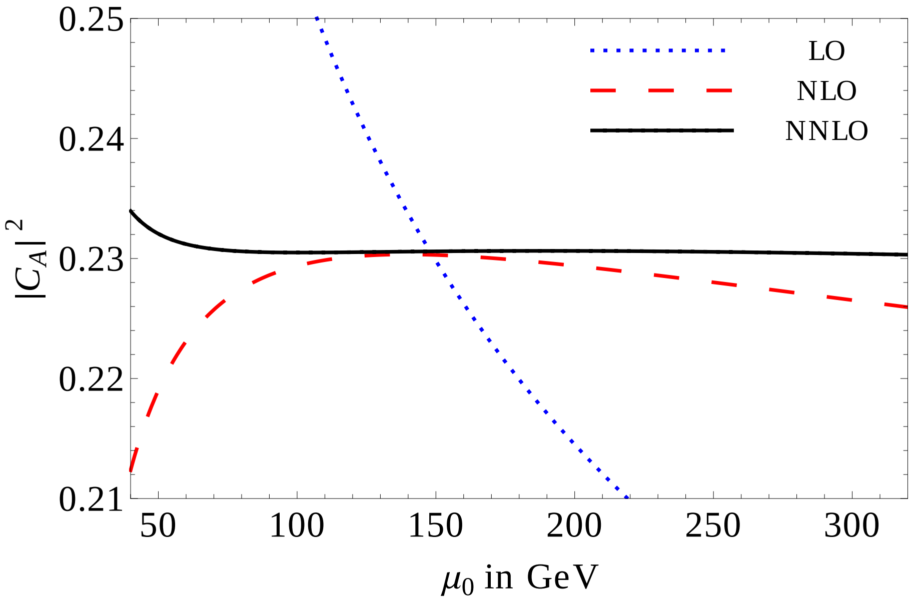
$$C_A(\mu_0) = C_A^{(0)}(\mu_0) + \frac{\alpha_s}{4\pi} C_A^{(1)}(\mu_0) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_A^{(2)}(\mu_0) + \frac{\alpha_{em}}{4\pi} \Delta_{\rm EW} C_A(\mu_0) + \ldots$$

The top quark mass is $\overline{\rm MS}$-renormalized at μ_0 with respect to QCD, and on shell with respect to the EW interactions. Both α_s and α_{em} are $\overline{\rm MS}$-renormalized at μ_0 in the effective theory.



$$C_A^{(n)} = C_A^{W,(n)} + C_A^{Z,(n)}$$

To deal with single-scale tadpole integrals, we expand around $y = 1$ (solid lines) and around $y = 0$ (dashed lines), where $y = M_W/m_t$. The expansions reach $(1-y^2)^{16}$ and y^{12}, respectively. The blue band indicates the physical region.



Matching scale dependence of $|C_A|^2$ gets significantly reduced. The plot corresponds to $\Delta_{\rm EW} C_A(\mu_0) = 0$. However, with our conventions for m_t and the global normalization, μ_0-dependence is due to QCD only.

NNLO fit (with $\Delta_{\rm EW} C_A(\mu_0) = 0$):

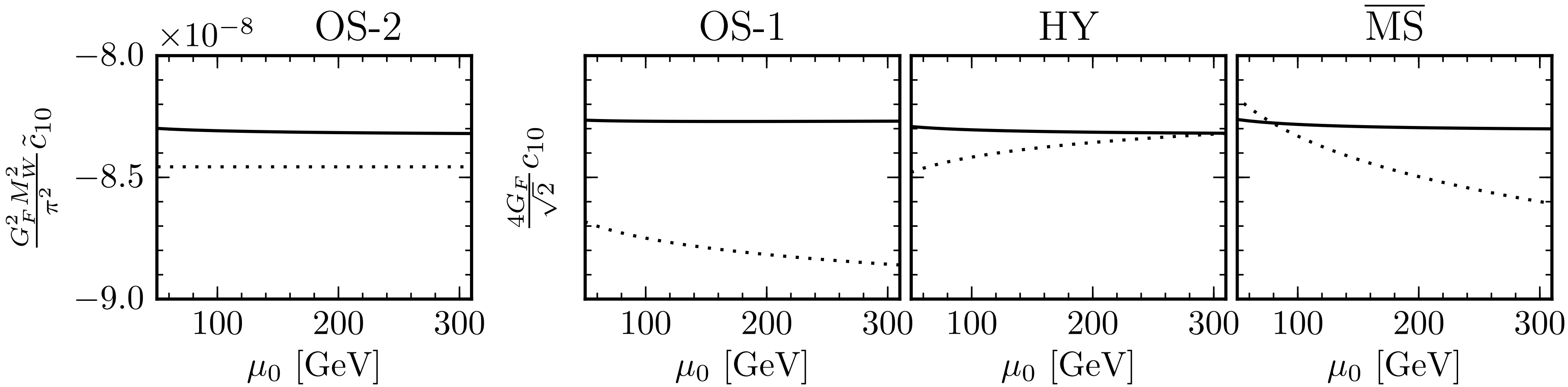
$$C_A = 0.4802 \left(\frac{M_t}{173.1}\right)^{1.52} \left(\frac{\alpha_s(M_Z)}{0.1184}\right)^{-0.09} + \mathcal{O}(\alpha_{em})$$

Evaluation of the NLO EW matching corrections in the SM

[C. Bobeth, M. Gorbahn, E. Stamou, Phys. Rev. D 89 (2014) 034023]

Method: similar to the NNLO QCD case. Two-loop integrals with three mass scales are present.

Dependence of the final result on μ_0 in various renormalization schemes (dotted – LO, solid – NLO):



In all the four plots: no QCD corrections to C_A included, $m_t(m_t)$ w.r.t. QCD used.

OS-2 scheme: Global normalization factor in $\mathcal{L}_{\rm eff}$ set to $N = V_{tb}^* V_{ts}\, G_F^2 M_W^2/\pi^2$

Masses at the LO renormalized on-shell w.r.t. EW interactions (including M_W in N)

Plotted quantity: $-2C_A\, G_F^2 M_W^2/\pi^2$ in GeV^{-2}

NLO EW matching correction to the BR: -3.7%

other schemes: Global normalization factor in $\mathcal{L}_{\rm eff}$ set to $4V_{tb}^* V_{ts}\, G_F/\sqrt{2}$

At the LO, $\alpha_{em}(\mu_0)$ used

$\overline{\mathrm{MS}}$: Masses and $\sin^2\theta_W$ renormalized at μ_0

OS-1: Masses as in OS-2, $\sin^2\theta_W$ on-shell

HY (hybrid): Masses as in OS-2, $\sin^2\theta_W$ as in $\overline{\mathrm{MS}}$.

SM predictions for all the branching ratios $\overline{\mathcal{B}}_{q\ell} \equiv \overline{\mathcal{B}}(B_q \to \ell^+\ell^-)$

[C. Bobeth, M. Gorbahn, T. Hermann, MM, E. Stamou, M. Steinhauser, PRL 112 (2014) 101801]

$$\begin{aligned}
\overline{\mathcal{B}}_{se} \times 10^{14} &= (8.54 \pm 0.13)\, R_{t\alpha}\, R_s = 8.54 \pm 0.55, \\
\overline{\mathcal{B}}_{s\mu} \times 10^{9} &= (3.65 \pm 0.06)\, R_{t\alpha}\, R_s = 3.65 \pm 0.23, \qquad (\text{LHCb \& CMS}:\ 2.9 \pm 0.7) \\
\overline{\mathcal{B}}_{s\tau} \times 10^{7} &= (7.73 \pm 0.12)\, R_{t\alpha}\, R_s = 7.73 \pm 0.49, \\
\overline{\mathcal{B}}_{de} \times 10^{15} &= (2.48 \pm 0.04)\, R_{t\alpha}\, R_d = 2.48 \pm 0.21, \\
\overline{\mathcal{B}}_{d\mu} \times 10^{10} &= (1.06 \pm 0.02)\, R_{t\alpha}\, R_d = 1.06 \pm 0.09, \qquad (\text{LHCb \& CMS}:\ 3.6^{+1.6}_{-1.4}) \\
\overline{\mathcal{B}}_{d\tau} \times 10^{8} &= (2.22 \pm 0.04)\, R_{t\alpha}\, R_d = 2.22 \pm 0.19,
\end{aligned}$$

where

$$R_{t\alpha} = \left(\frac{M_t}{173.1\ \text{GeV}}\right)^{3.06} \left(\frac{\alpha_s(M_Z)}{0.1184}\right)^{-0.18},$$

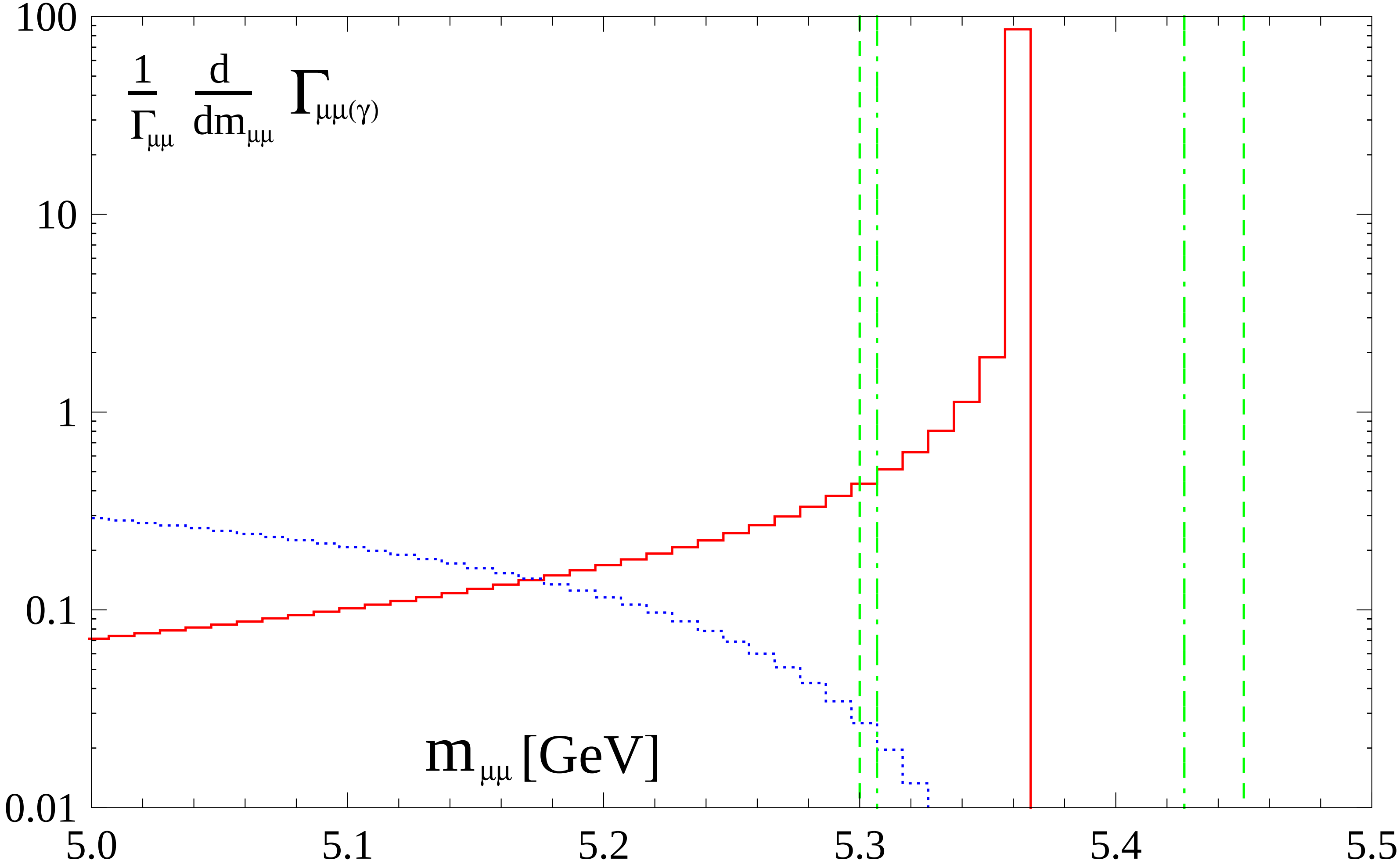
$$R_s = \left(\frac{f_{B_s}[\text{MeV}]}{227.7}\right)^2 \left(\frac{|V_{cb}|}{0.0424}\right)^2 \left(\frac{|V_{tb}^\star V_{ts}/V_{cb}|}{0.980}\right)^2 \frac{\tau_H^s\ [\text{ps}]}{1.615},$$

$$R_d = \left(\frac{f_{B_d}[\text{MeV}]}{190.5}\right)^2 \left(\frac{|V_{tb}^\star V_{td}|}{0.0088}\right)^2 \frac{\tau_d^{\text{av}}\ [\text{ps}]}{1.519}.$$

Sources of uncertainties	f_{B_q}	CKM	τ_H^q	M_t	α_s	other parametric	non-parametric	$\sum$	
$\overline{\mathcal{B}}_{s\ell}$	4.0%	4.3%	1.3%	1.6%	0.1%	$< 0.1\%$	1.5%	6.4%	$\longrightarrow$ 4.7% (?)
$\overline{\mathcal{B}}_{d\ell}$	4.5%	6.9%	0.5%	1.6%	0.1%	$< 0.1\%$	1.5%	8.5%	

In the case of $\overline{\mathcal{B}}_{s\ell}$, the main uncertainty (4.2%) originates from $|V_{cb}| = 0.0424(9)$ that comes from a recent fit to the inclusive semileptonic data [P. Gambino and C. Schwanda, arXiv:1307.4551].

Radiative tail in the dimuon invariant mass spectrum



Ggreen vertical lines – experimental windows (→ MC)

Red line – no real photon and/or radiation only from the muons. It vanishes when $m_\mu \longrightarrow 0$.

Blue line – remainder due to radiation from the quarks. IR-safe because B_s is neutral.
Phase-space suppressed but survives in the $m_\mu \longrightarrow 0$ limit.

Interference between the two contributions is negligible – suppressed both by phase-space and $m_\mu^2/M_{B_s}^2$.

Summary

- Combining the recently calculated NNLO QCD and NLO EW corrections to $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$, we find a significant reduction of the non-parametric theoretical uncertainties $(\sim 8\% \to \sim 1.5\%)$.

- The current SM result $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = (3.65 \pm 0.23) \times 10^{-9}$ is consistent with the measured value of $(2.9 \pm 0.7) \times 10^{-9}$. The main theory uncertainties are parametric $(|V_{cb}|,\ f_{B_s}, \ldots)$.

- Determination of $|V_{cb}|$ from inclusive semileptonic B decays is currently limited by theory uncertainties. Rough estimates of higher-dimensional operator matrix elements would help.