Heterotic Line Bundle Models on Smooth Calabi-Yau Manifolds

Andrei Constantin (University of Oxford)

Joint work with: Lara Anderson, Evgeny Buchbinder, James Gray, Andre Lukas, Challenger Mishra and Eran Palti

Based on: arXiv: 1404.2767, arXiv: 1311.1941 and arXiv: 1307.4787

SUSY 2014, Manchester

A Wish List for (Heterotic) String Pheno

Four-dimensional EFT with:

- $\mathcal{N} = 1$ SUSY
- SM gauge interactions
- massless spectrum containing 3 chiral generations of quarks and leptons and no extra fields charged under the SM gauge group; uncharged fields (moduli) allowed for the moment
- massless spectrum containing Higgs doublets throughout the moduli space
- stable proton
- hierarchy of holomorphic Yukawa couplings consistent with a heavy top
- stable moduli; broken SUSY
- compute physical Yukawa couplings

A Heterotic Setup

In 10d, the heterotic string is specified by a metric and a non-abelian gauge field. To compactify: (X, V).

Constraints:

$\mathrm{ch}_2(V) - \mathrm{ch}_2(TX) = [W]$
(Green-Schwarz anomaly cancellation)

$F_{ij} = F_{\bar{i}\bar{j}} = 0$ (V holomorphic)

$g^{i\bar{j}} F_{i\bar{j}} = 0$ DUY theorem guarantees the HYM equation is satisfied provided that V is polystable and has slope zero.

A Heterotic Setup - continued

The simplest and best understood situation: X Calabi-Yau three-fold.

In this case, the possible bundles can be divided into two classes:

- $V = TX$
  standard embedding: corresponds to $(2,2)$ worldsheet susy
- $V \neq TX$
  general embeddings: correspond to $(0,2)$ worldsheet susy

In the following, I will refer to the $E_8 \times E_8$ heterotic string.

The Heterotic Line Bundle Setup

Simplest choice for V (for, e.g. stability checks and cohomology computations): sum of line bundles

$$V = \bigoplus_{a=1}^{\mathrm{rk}(V)} \mathcal{L}_a = \bigoplus_{a=1}^{\mathrm{rk}(V)} \mathcal{O}(\vec{k}_a)$$

where $\vec{k}_a = c_1(\mathcal{L}_a)$.

E_6-models are obtained for $\mathrm{rk}(V) = 3$, $SO(10)$-models for $\mathrm{rk}(V) = 4$ and $SU(5)$-models for $\mathrm{rk}(V) = 5$.

The (intermediate) GUT group contains 2,3 and respectively 4 extra $U(1)$ symmetries. These are phono OK and can greatly constrain the superpotential.

The Heterotic Line Bundle Setup – continued

Topological constraints on V:

- $c_1(V) = 0$
- $c_2(TX) - c_2(V) \geq 0$
- $\text{ind}(V) = -3$

In addition, impose poly-stability and slope zero:

$$\mu(\mathcal{L}_a) = \int_X c_1(\mathcal{L}_a) \wedge J^2 = d_{ijk}\, \vec{k}_a^i\, t^j\, t^k = 0$$

simultaneously for all $a = 1, \ldots, \text{rk}(V)$

Result: intermediate GUT with a bunch of (effectively global) $U(1)$ symmetries, and 3 chiral families of matter.

Heterotic Line Bundle Models – continued

$$V = \mathcal{L}_1 \oplus \ldots \oplus \mathcal{L}_5$$

with $c_1(V) = 0$, s.t. the structure group is $S(U(1)^5) \subset SU(5) \subset E_8$

The result: Effective field theory: $\mathcal{N} = 1$, 4-dimensional GUT with gauge group $SU(5) \times S(U(1)^5)$ and matter in $\mathbf{10}$, $\overline{\mathbf{10}}$, $\mathbf{5}$, $\overline{\mathbf{5}}$, $\mathbf{1}$

GUT $\longrightarrow$ Standard Model. The required geometric data consists of:

- a freely-acting discrete symmetry Γ, such that X/Γ is non-simply connected;
- an equivariant structure on V, such that $V \longrightarrow X$ descends to a bundle $\widetilde{V} \longrightarrow X/\Gamma$
- complete the bundle $\widetilde{V}$ with a discrete Wilson line to $\widetilde{V} \oplus W$ in order to break the GUT group

The result: Standard-like model with gauge group $G_{\mathrm{SM}} \times S(U(1)^5)$

The 4D Effective Field Theory

Gauge group: $SU(5) \times S(U(1)^5)$. Extra $U(1)$s G-S anomalous in general.

Matter multiplets: $\mathbf{10}_a$, $\overline{\mathbf{10}}_a$, $\mathbf{5}_{a,b}$, $\overline{\mathbf{5}}_{a,b}$, $\mathbf{1}_{a,b}$

multiplet	$S(U(1)^5)$ charge	bundle	total number	required
$\mathbf{10}_a$	e_a	V	$\sum_a h^1(X, L_a)$	$3\lvert\Gamma\rvert$
$\overline{\mathbf{10}}_a$	$-\mathrm{e}_a$	V^*	$\sum_a h^1(X, L_a^*)$	0
$\overline{\mathbf{5}}_{a,b}$	$\mathrm{e}_a + \mathrm{e}_b$	$\wedge^2 V$	$\sum_{a<b} h^1(X, L_a \otimes L_b)$	$3\lvert\Gamma\rvert + n_H$
$\mathbf{5}_{a,b}$	$-\mathrm{e}_a - \mathrm{e}_b$	$\wedge^2 V^*$	$\sum_{a<b} h^1(X, L_a^* \otimes L_b^*)$	n_H
$\mathbf{1}_{a,b}$	$\mathrm{e}_a - \mathrm{e}_b$	$V \otimes V^*$	$\sum_{a,b} h^1(X, L_a \otimes L_b^*)$	n_H

$\mathbf{1}_{a,b}$: singlets under $SU(5)$ (G_{SM} after quotienting); bundle moduli

$\langle \mathbf{1}_{a,b} \rangle = 0$: line bundle sum; $\langle \mathbf{1}_{a,b} \rangle \neq 0$: non-Abelian bundle

Also: explore the moduli space of non-Abelian bundles by explicitly constructing bundles which split into a sum of line bundles

The 4D Effective Field Theory - continued

The $U(1)$ symmetries constrain the superpotential

$$\begin{aligned} W = {} & \mu\, H\bar{H} + Y^{(d)}_{pq}\, H\,\bar{\mathbf{5}}^p\mathbf{10}^q + Y^{(u)}_{pq}\,\bar{H}\,\mathbf{10}^p\mathbf{10}^q + \\ & + \rho_p\,\bar{H}L^p + \lambda_{pqr}\,\bar{\mathbf{5}}^q\bar{\mathbf{5}}^q\mathbf{10}^r + \\ & + \lambda'_{pqrs}\,\bar{\mathbf{5}}^p\mathbf{10}^q\mathbf{10}^r\mathbf{10}^s + \dots \end{aligned}$$

Example: $\mu = \mu_0 + \mu_{1,\alpha}\mathbf{1}^{\alpha}_{a,b} + \mu_{2,\alpha,\beta}\mathbf{1}^{\alpha}_{a,b}\mathbf{1}^{\beta}_{c,d} + \dots + \mu_{\mathrm{np}}$

$\mu_0 = 0$ by construction; $\mu_1 = 0$ due to the $U(1)$s

A Comprehensive Scan

The manifolds: complete intersection CY threefolds (CICYs) – common zero locus of homogeneous polynomials in products of projective spaces (Candelas, Green, Hübsch, Lütken)

Select those that are known to admit a freely-acting discrete symm (Braun) and are favourable: 68 CICYs with $h^{1,1}(X) < 7$.

The bundles: Line bundles are classified by their first Chern class:

$$c_1(L) = k^i J_i$$

with $1 \leq i \leq h^{1,1}(X)$ and $k^i \in \mathbb{Z}$. Describe a rank 5 line bundle sum

$$V = \bigoplus_{a=1}^{5} L_a = \bigoplus_{a=1}^{5} \mathcal{O}(\vec{k}_a), \qquad \text{where} \quad \vec{k}_a = (k_a^1, \ldots, k_a^{h^{1,1}(X)})$$

by $5 \times h^{1,1}(X)$ integers. For $-k_{\text{max}} \leq k_a^i \leq k_{\text{max}}$, one has many choices:

$$(2k_{\text{max}} + 1)^{h^{1,1}(X)}$$

A Comprehensive Scan – Results

We have scanned over $\sim 10^{40}$ bundles. This was possible only to the fact that many constraints (e.g.: from stability, index constraints from the spectrum) can be imposed along the way, before even constructing the whole line bundle sum.

Imposing the constraints for a consistent susy string vacuum and the index constraints that lead to a correct chiral asymmetry we found:

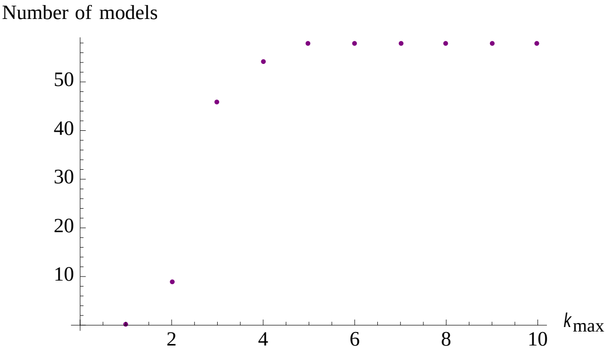
$h^{1,1}(X)$	1	2	3	4	5	6	All
No. models	0	0	6	552	21731	41036	63325

In addition, requiring the absence of $\overline{\mathbf{10}}$-multiplets and the presence of at least one $H - \overline{H}$ pair, led to:

$$34,989 \text{ models}$$

Roughly, the number of models per CY increases by one order of magnitude for each additional Kähler parameter.

Finiteness



A Theoretical Bound

$$\sum_a \mathbf{k}_a^T \widetilde{G}\, \mathbf{k}_a \leq |c_{2i}(TX)|$$

$$\widetilde{G} = \kappa\, G/(6|\mathbf{t}|)$$

$$G_{ij} = \frac{1}{2\,\mathrm{Vol}(X)} \int_X J_i \wedge \star J_j = -3\left(\frac{\kappa_{ik}}{\kappa} - \frac{2\kappa_i \kappa_j}{3\kappa^2}\right)$$

where $\mathrm{Vol}(X) = \kappa/6$ is the Calabi-Yau volume with respect to the Ricci-flat metric, $\kappa = d_{ijk}\, t^i\, t^j\, t^k$, $\kappa_i = d_{ijk}\, t^j\, t^k$ and $\kappa_{ij} = d_{ijk}\, t^k$.

$$\boxed{\sum_a |\mathbf{k}_a|^2 \leq \frac{\text{num factor}}{\lambda_{\min}}}$$

To derive this, we used the slope-zero conditions and the bound on $c_2(V)$ from the anomaly cancellation. The bound is not sensitive to the number of line bundles involved in the sum, nor to the index of V. [Buchbinder, AC, Lukas]

Position in the Kähler Moduli Space

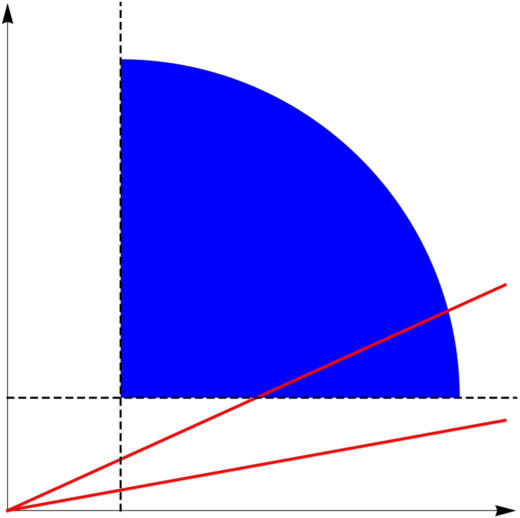
The constraints imposed by poly-stability $\mu(\mathcal{L}_1) = 0$ define a certain locus in the Kähler moduli space. The slope zero equations are homogeneous in the t^i. Thus $t^i \to \lambda t^i$ leaves this locus invariant.

There is a physically allowed region in the Kähler moduli space:

Supergravity limit: $t^i > 1$

finiteness of low-energy coupling constants: $\mathrm{Vol}(X) \lesssim V_{\mathrm{max}}$.

$$\mathrm{Vol}(X) = \frac{1}{6} d_{ijk}\, t^i\, t^j\, t^k$$



Conclusions and Outlook

- Interesting phenomenology can be achieved with line bundle models. These also provide an accessible window in the larger moduli space of non-Abelian bundles. $U(1)$ symmetries constrain the Lagrangian.
- The scan exhausted the class of line bundle models with an underlying $SU(5)$ GUT: $35,000$ models. We expect a much larger number of SMs.
- Much work remains to be done. Technical difficulties related to computing line bundle cohomology and enumerating all possible equivariant structures for a given (X, V, Γ).
- Why is the number of poly-stable bundles with $c_1(V) = 0$, $c_2(V)$ constrained by the anomaly cancellation condition and $c_3(V)$ fixed by the number of families, finite? Can this be related to some type of Donaldson-Thomas invariants?

Thank you!