DETERMINATION OF THE INFLATIONARY PARAMETERS BY THE DIRECT DETECTION OF THE PRIMORDIAL GRAVITATIONAL WAVES

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based on arXiv:1406:1666

in collaboration with
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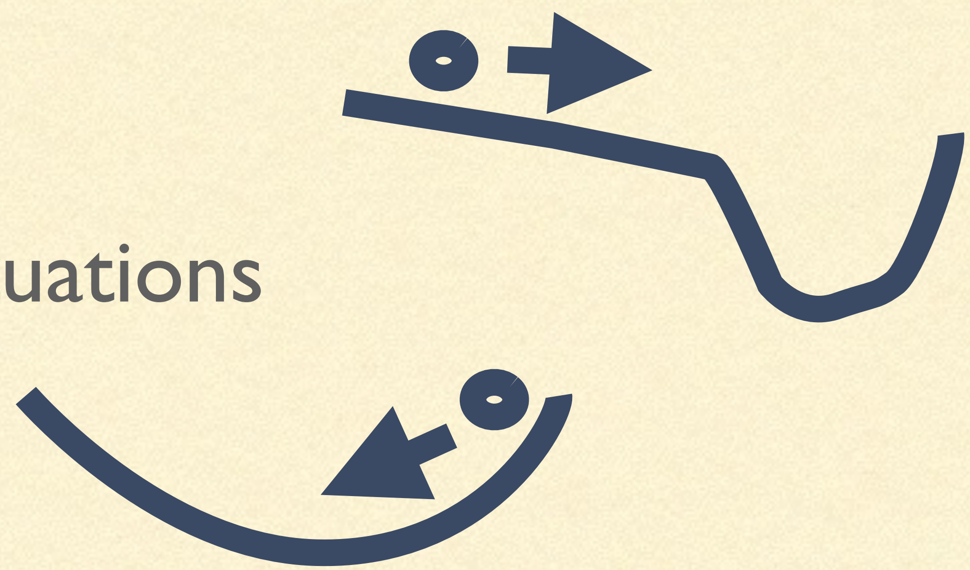
INFLATION

- Accelerating expansion of the universe

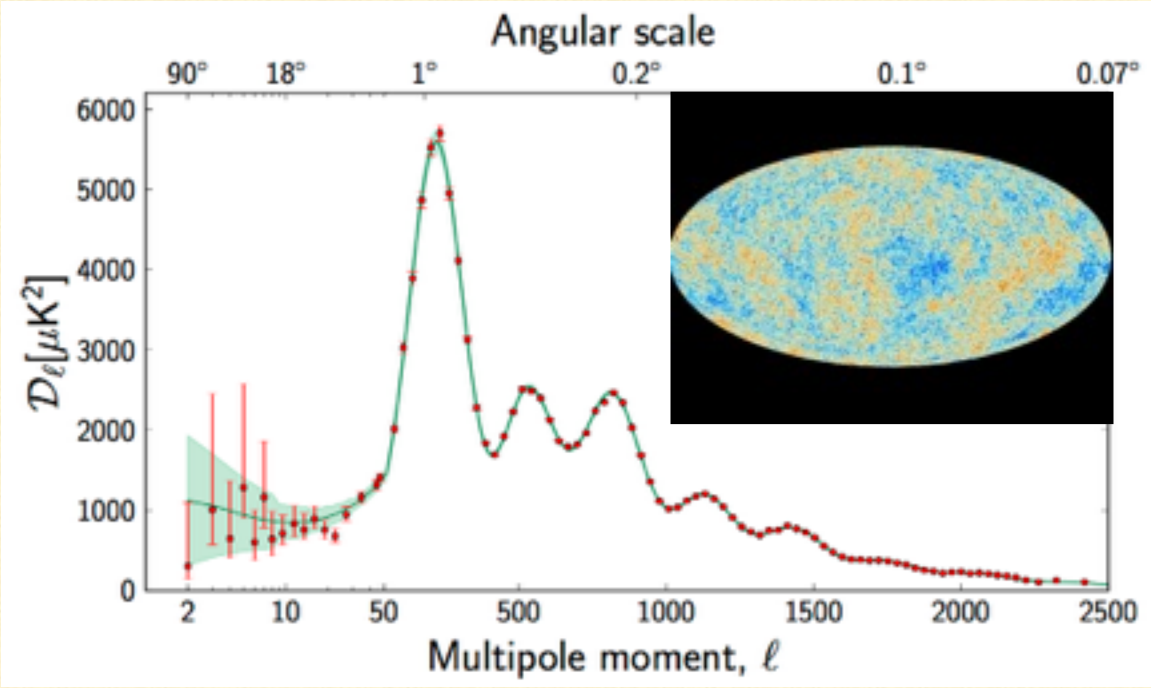


- Predicts primordial scalar & tensor fluctuations

$\mathcal{P}_S \sim V/\epsilon \quad \mathcal{P}_T \sim V$

- Potential/decay rate of inflaton

  are still unknown

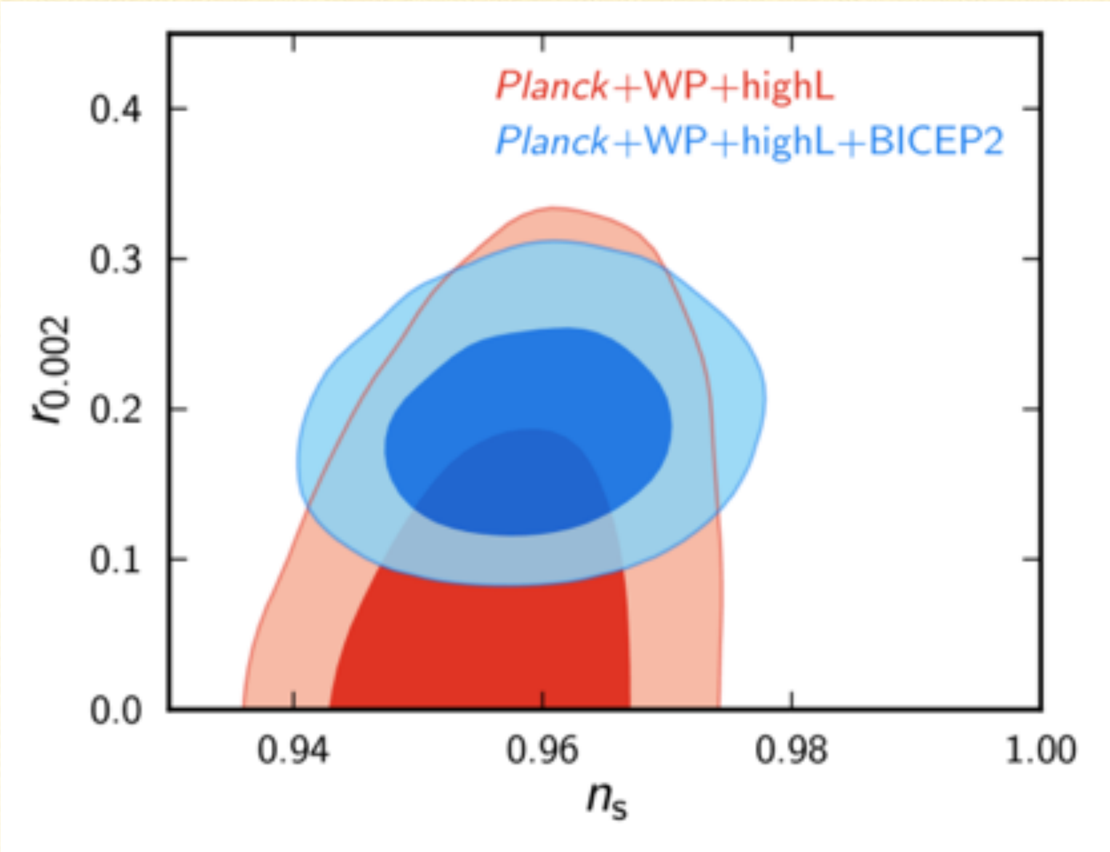


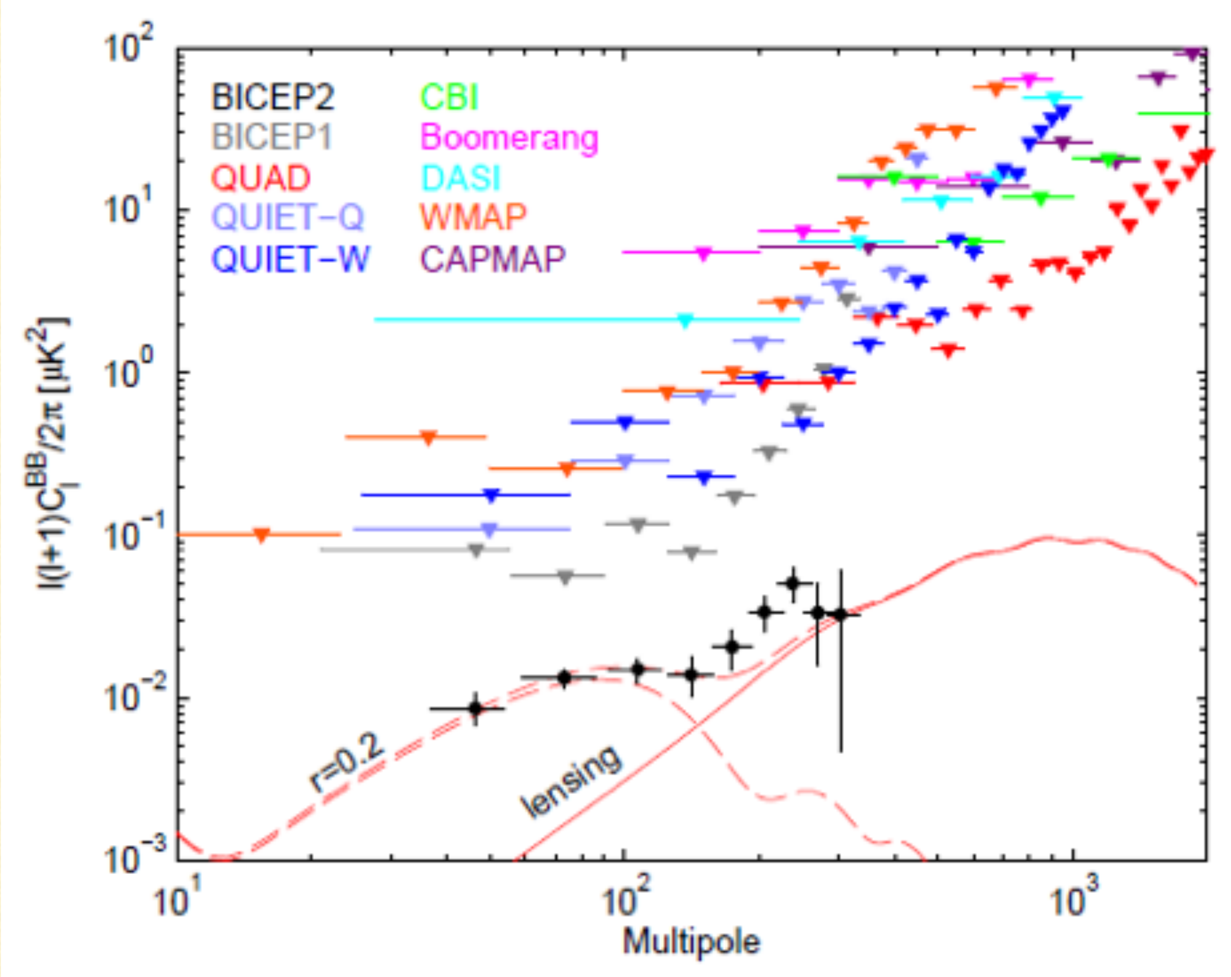
From Planck

TENSOR AMPLITUDE

- Planck&BICEP2

From BICEP2





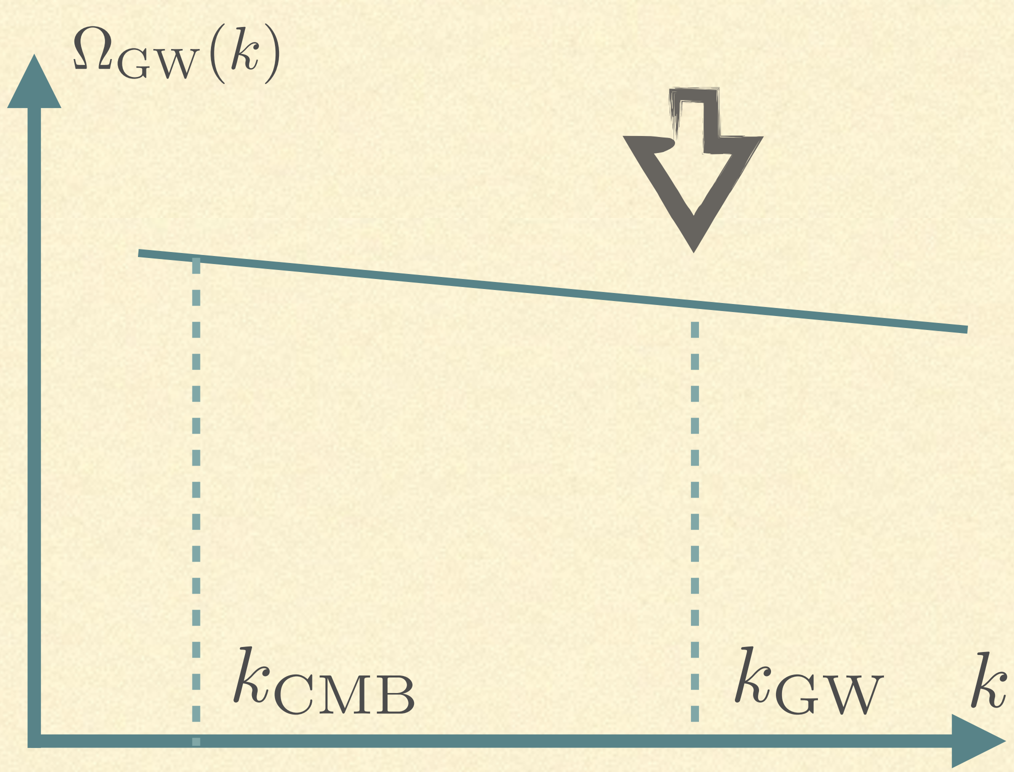
$$r \equiv \mathcal{P}_T/\mathcal{P}_S \begin{cases} < 0.13 \ (2\sigma) & \text{(Planck)} \\ = 0.2^{+0.07}_{-0.05} \ (1\sigma) & \text{(BICEP2)} \end{cases} \Rightarrow$$

$r \sim 0.1$

may be realized

DIRECT DETECTION OF GWS

- If $r \sim 0.1$, direct detection of GWs may be possible at $f = 2\pi k \simeq 1\mathrm{Hz}$ by space interferometers (BBO, DECIGO etc.)
- Both the information on inflationary parameters and reheating may be imprinted



TALK PLAN

1. Introduction
2. Properties of inflationary GWs
3. χ^2 analysis & result
4. Summary

PROPERTIES OF IGWS

PROPERTIES OF IGWS

- Definition

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij})dx^i dx^j$$

- EOM

EH action → $\ddot{h} + 3H\dot{h} + \frac{k^2}{a^2}h = 0$

- Production by quantum fluctuation during inflation

$$\mathcal{P}_{T,\mathrm{prim}}(k) = 64\pi G \left(\frac{H_{\mathrm{inf}}}{2\pi}\right)^2$$

Proportional to the height of the inflaton potential

$$\left(\langle h_{ij}(x)^2\rangle = \int d\ln k\, \mathcal{P}_{T,\mathrm{prim}}(k)\right)$$

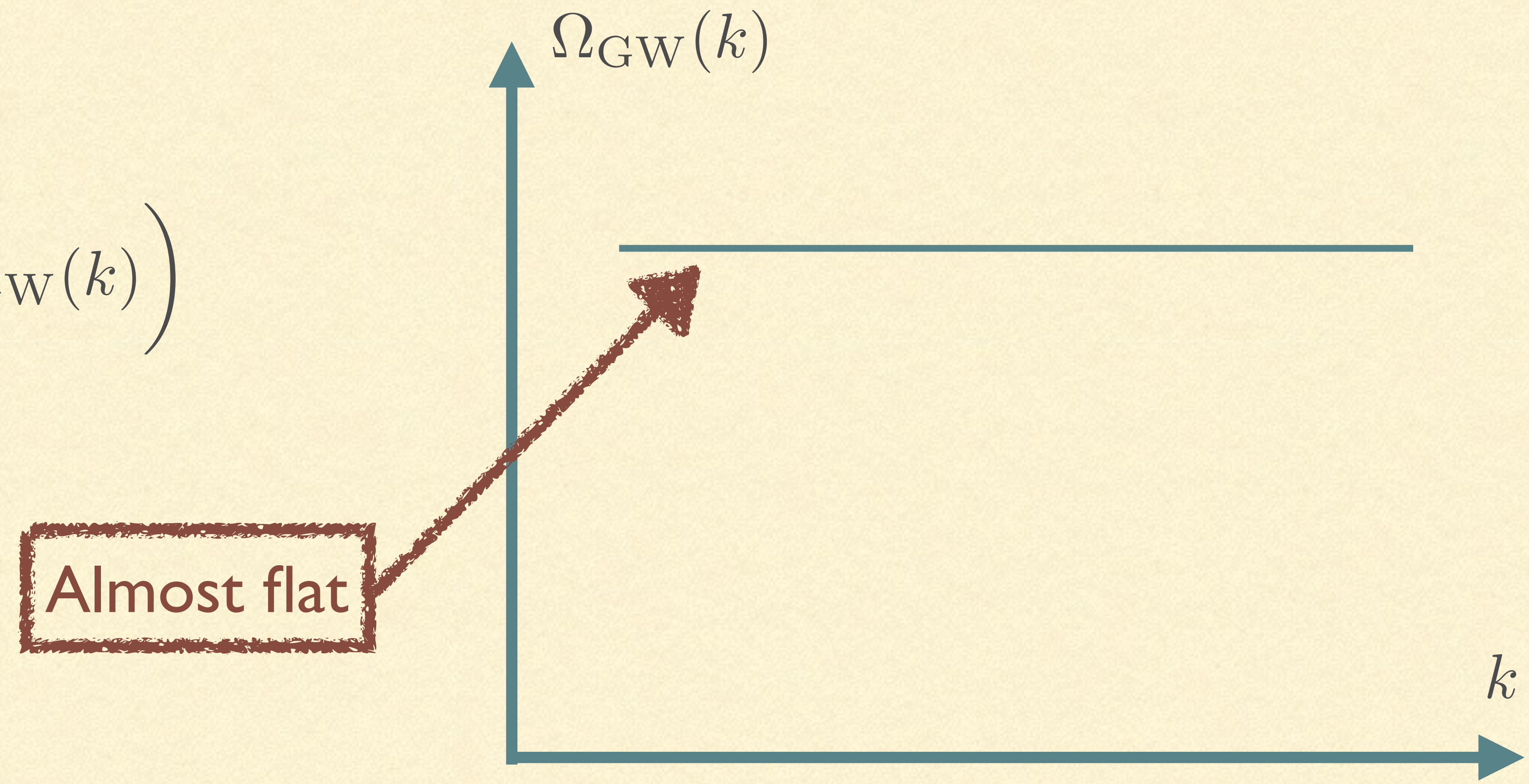
PROPERTIES OF IGWS

- Present GW amplitude

GW amplitude per logarithmic wavenumber

$$\Omega_{\rm GW}(k) \equiv \frac{\rho_{\rm GW}(k)}{\rho_{\rm cr}}$$

$$\left(\rho_{\rm GW} = \int d\ln k\ \rho_{\rm GW}(k)\right)$$



PROPERTIES OF IGWS

- Present GW amplitude

$$\ddot{h} + 3H\dot{h} + \frac{k^2}{a^2}h = 0$$

$$\rightarrow \; h \; \propto \begin{cases} a^0 & (H > k/a) \\ a^{-2} & (H < k/a) \end{cases}$$

GWs tend to decrease inside the horizon

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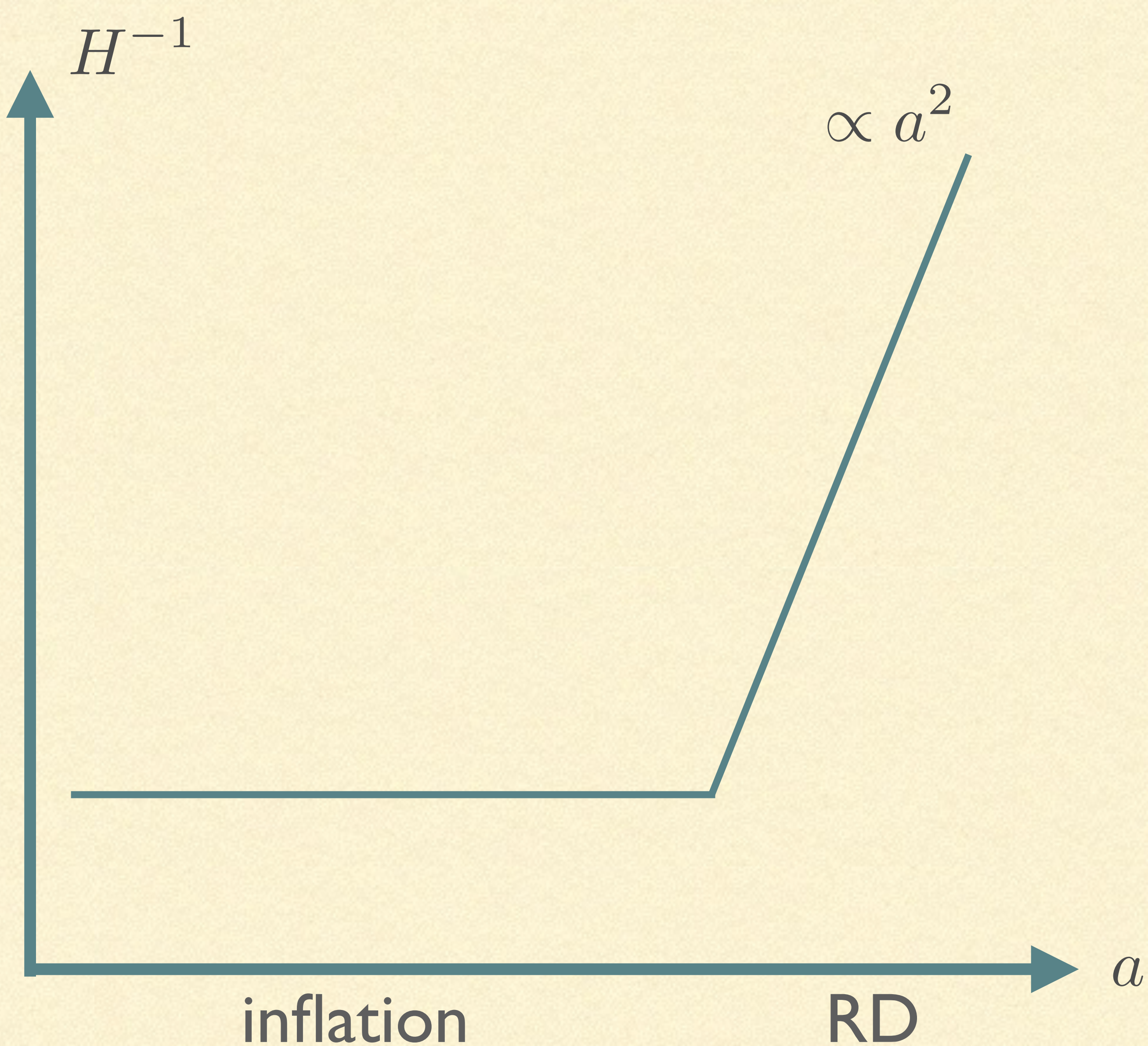

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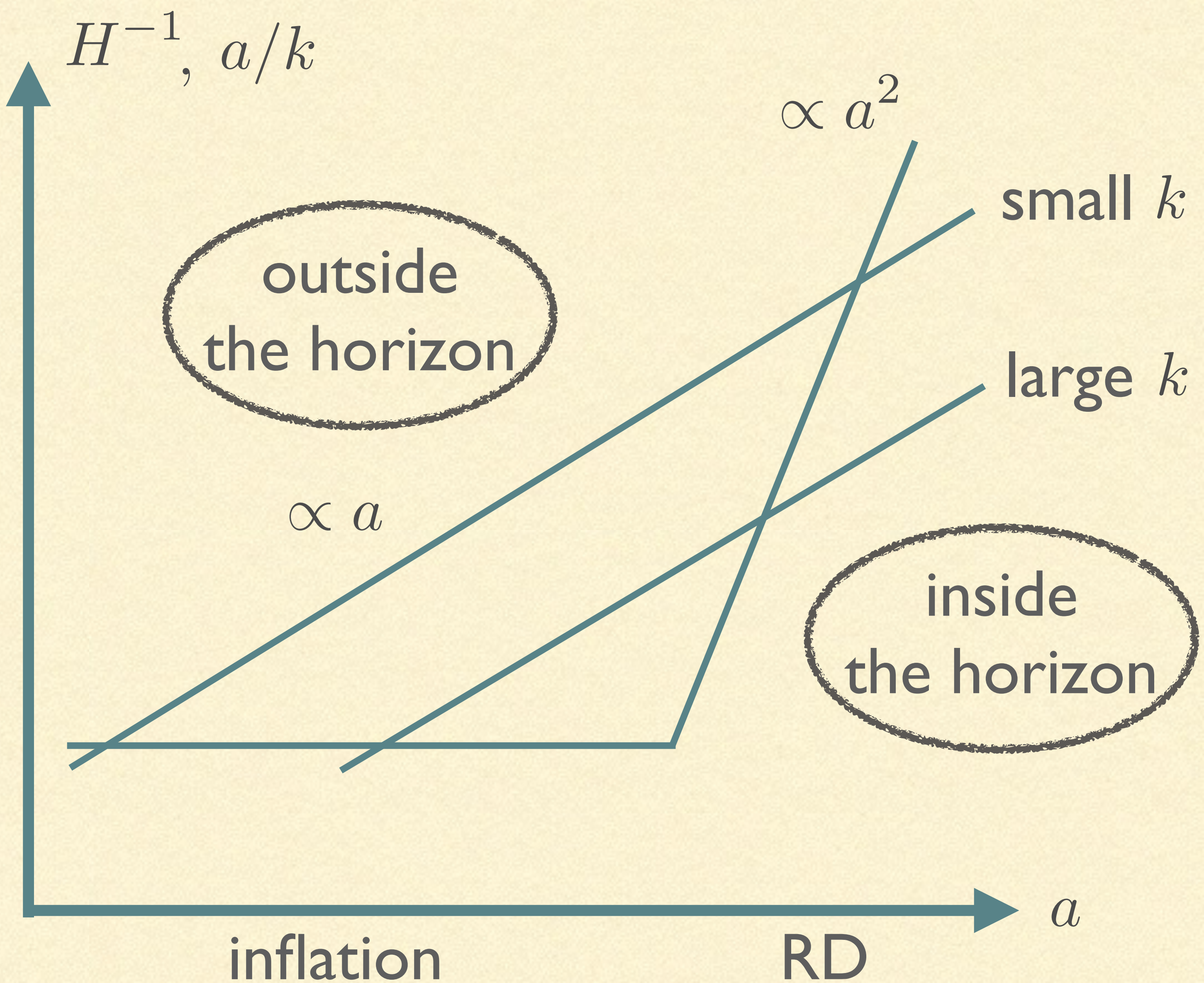

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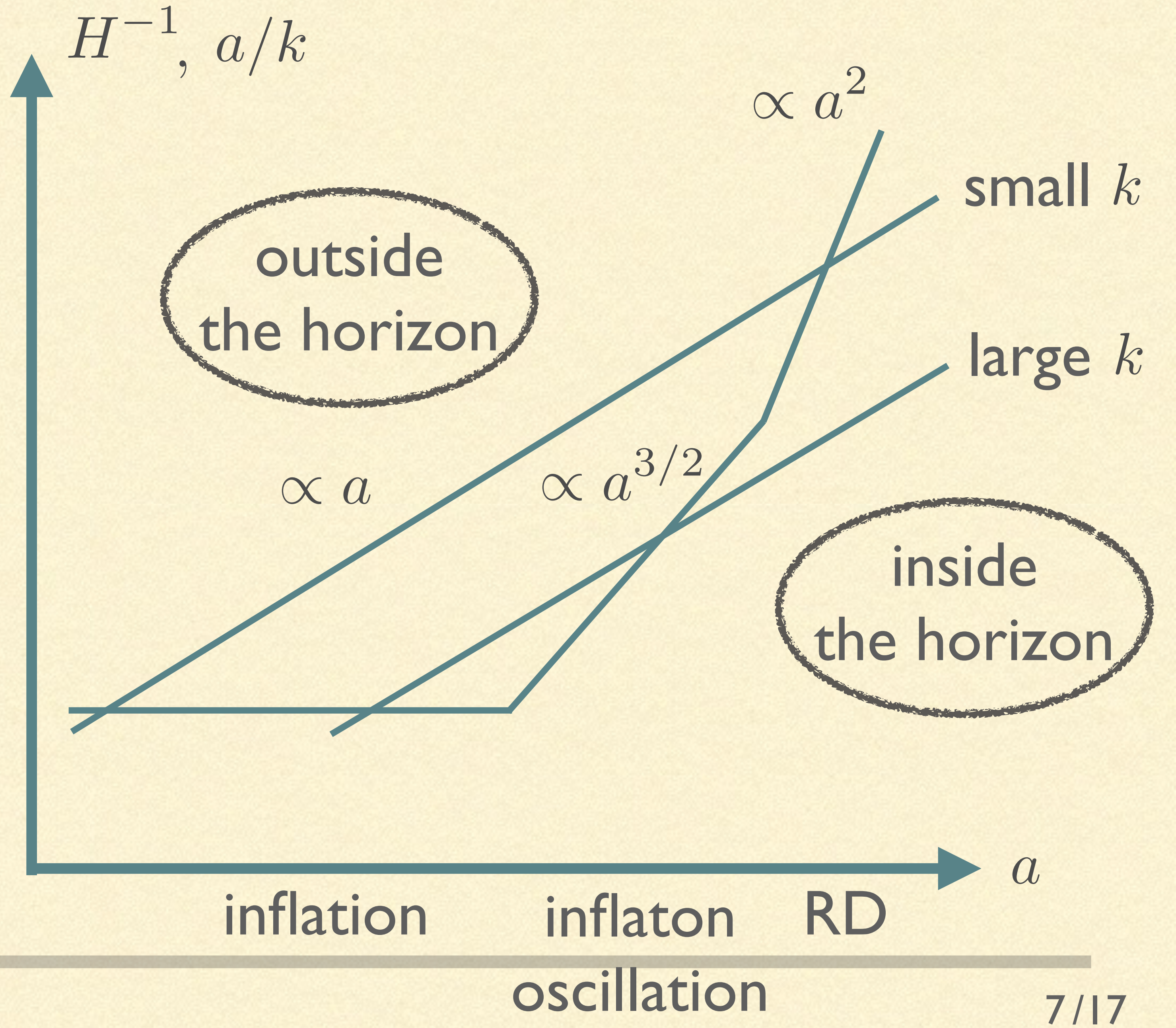


PROPERTIES OF IGWS

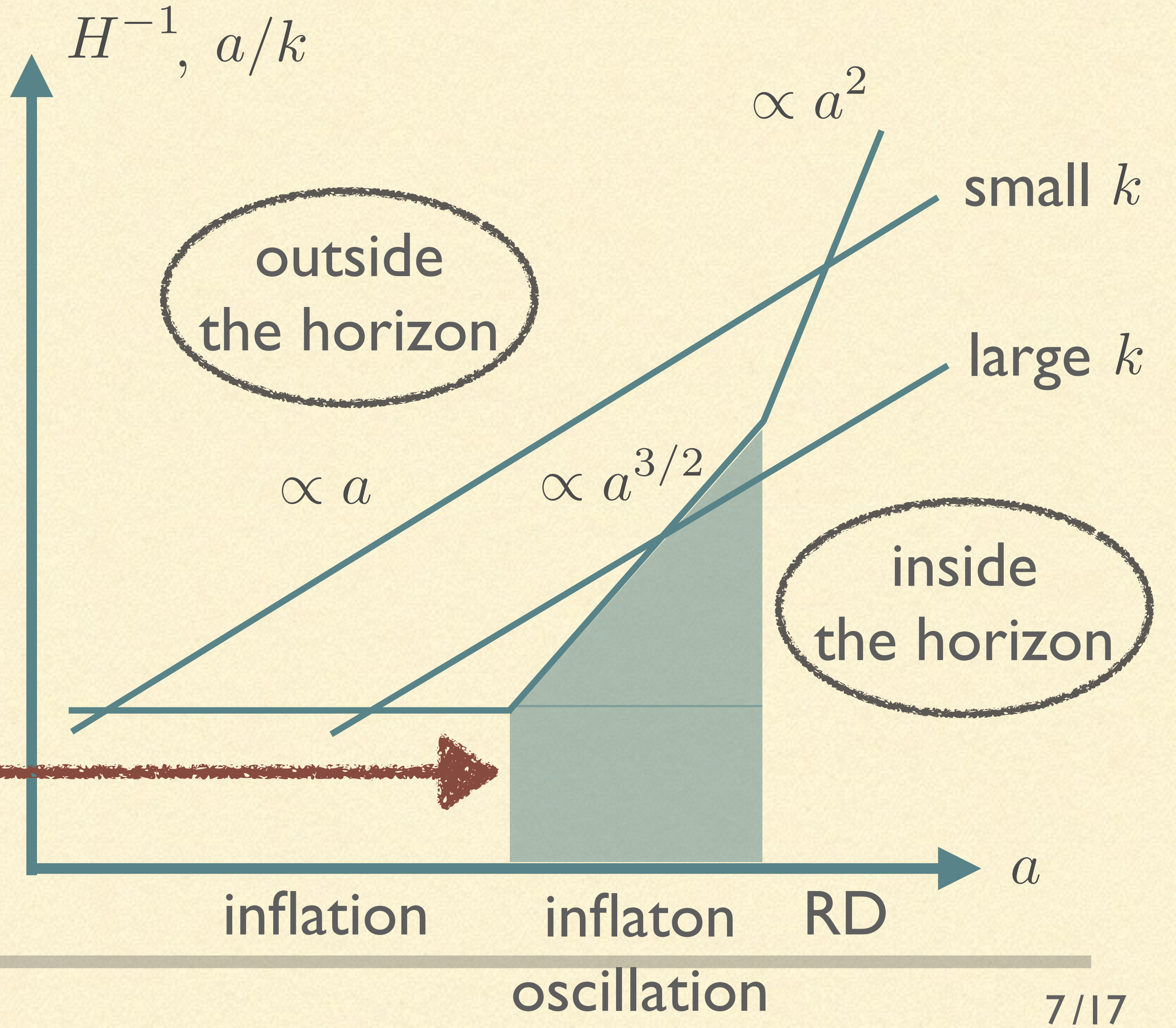
- Present GW amplitude

$$\ddot{h} + 3H\dot{h} + \frac{k^2}{a^2}h = 0$$

$$\rightarrow \quad h \;\propto \begin{cases} a^0 & (H > k/a) \\ a^{-2} & (H < k/a) \end{cases}$$

GWs tend to decrease inside the horizon

GWs with large k stay longer inside the horizon



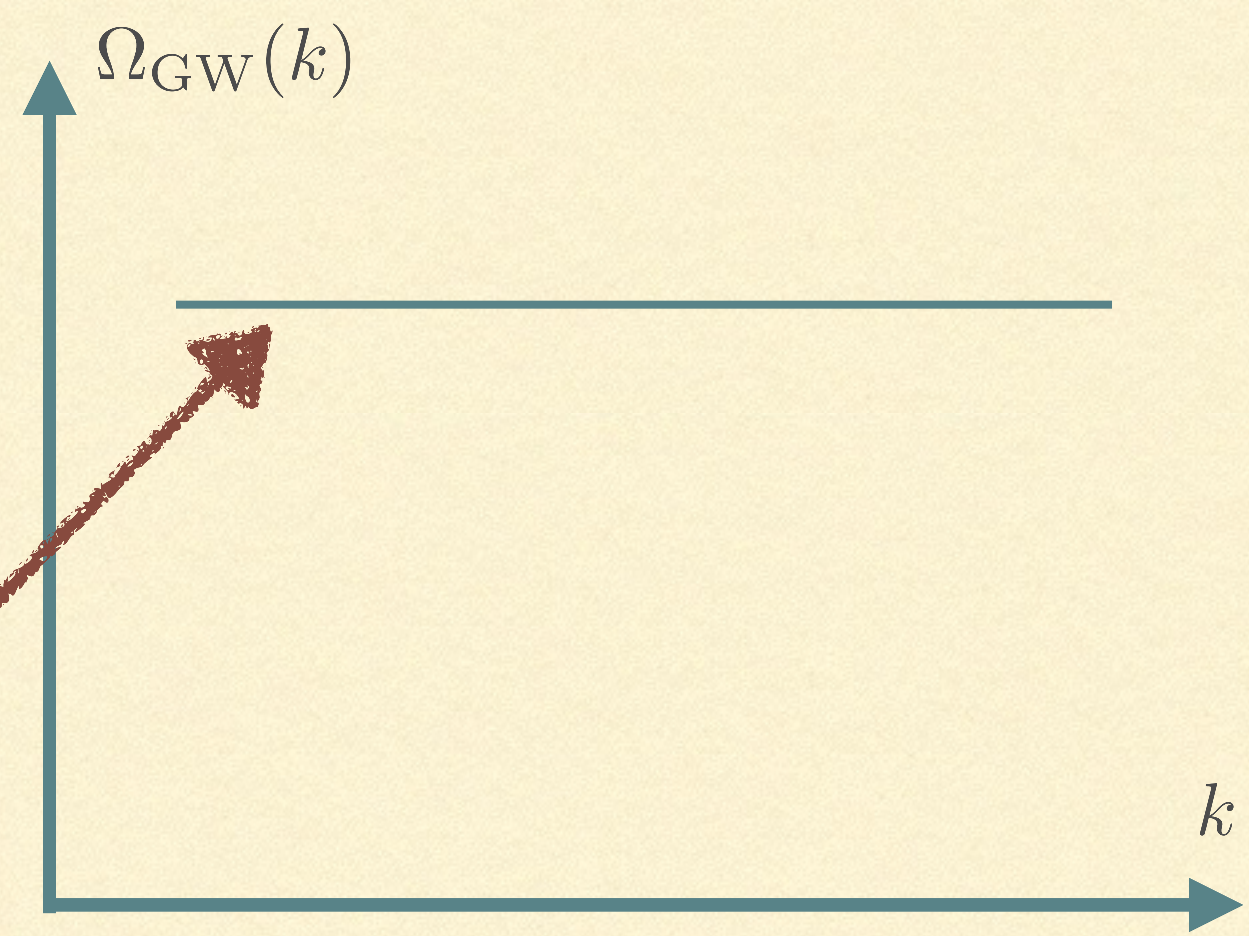
PROPERTIES OF IGWS

- Present GW amplitude

GW amplitude per logarithmic wavenumber

$$\Omega_{\mathrm{GW}}(k) \equiv \frac{\rho_{\mathrm{GW}}(k)}{\rho_{\mathrm{cr}}}$$

$$\left(\rho_{\mathrm{GW}} = \int d\ln k \; \rho_{\mathrm{GW}}(k) \right)$$



Almost flat

PROPERTIES OF IGWS

- Present GW amplitude

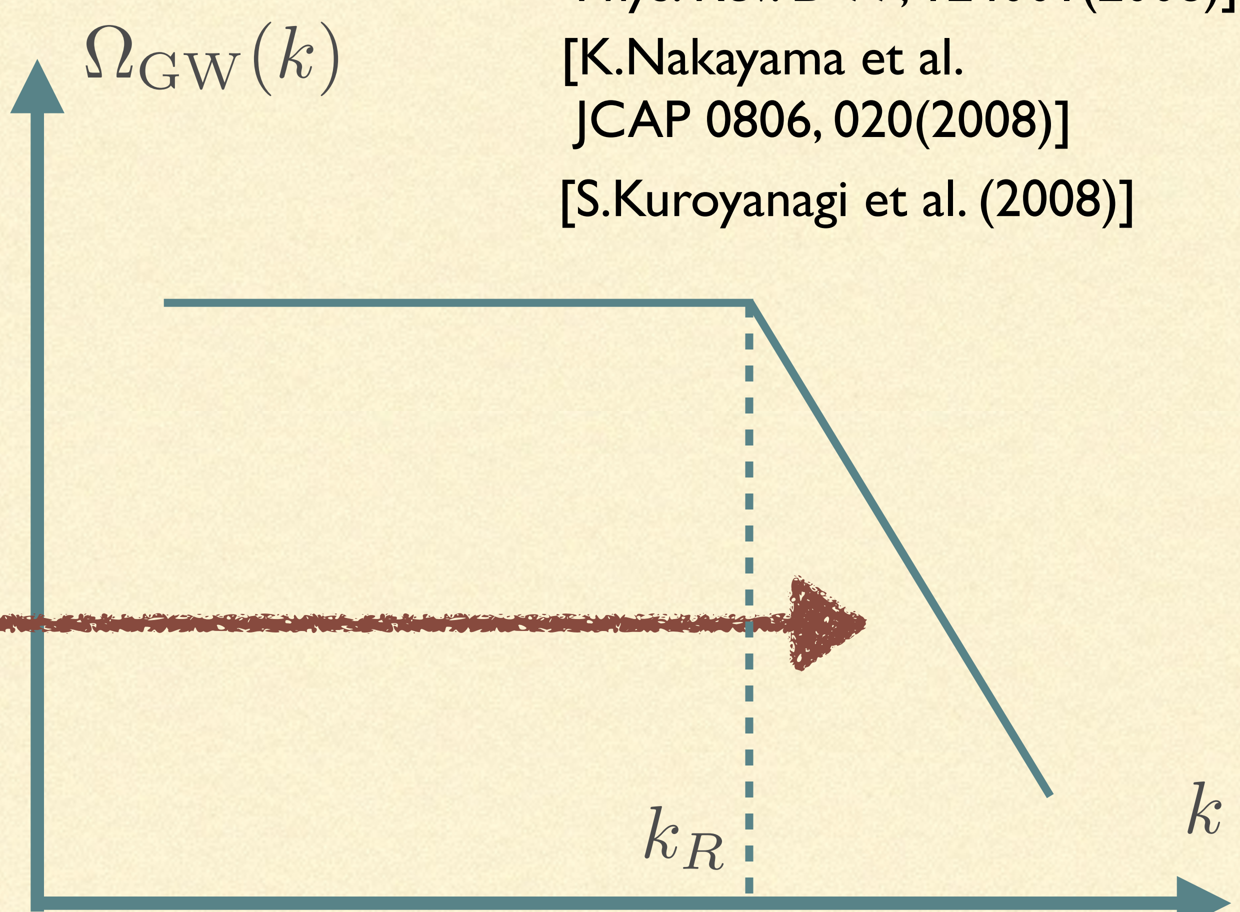
GW amplitude per logarithmic wavenumber

$$\Omega_{\mathrm{GW}}(k) \equiv \frac{\rho_{\mathrm{GW}}(k)}{\rho_{\mathrm{cr}}}$$

$$\left(\rho_{\mathrm{GW}} = \int d\ln k \; \rho_{\mathrm{GW}}(k) \right)$$

[K.Nakayama et al.
Phys. Rev. D 77, 124001(2008)]
[K.Nakayama et al.
JCAP 0806, 020(2008)]
[S.Kuroyanagi et al. (2008)]

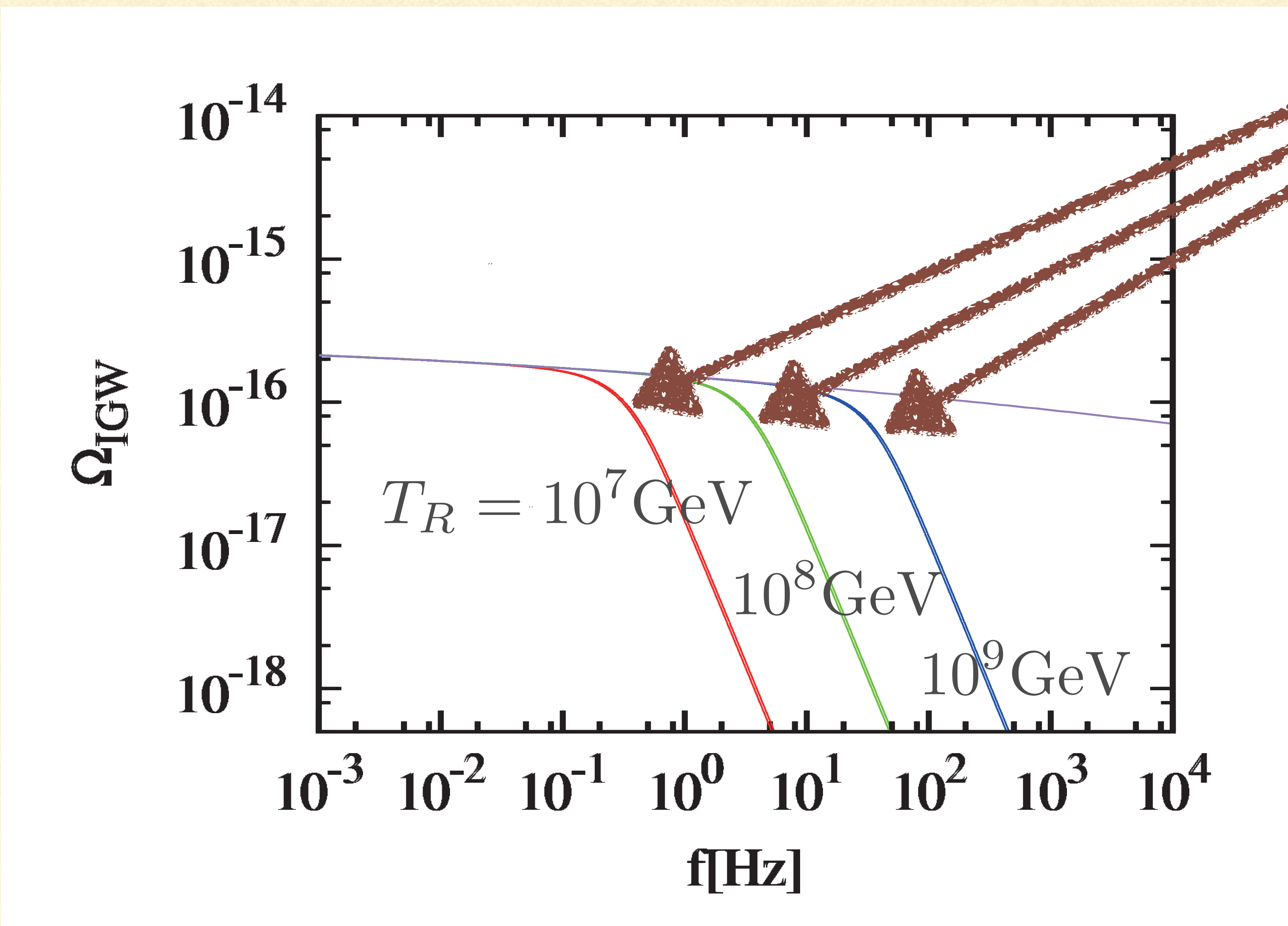
GWs with large k
get suppressed



$$f_R = 2\pi k_R \simeq 0.3\mathrm{Hz}\frac{T_R}{10^8\mathrm{GeV}}$$

PROPERTIES OF IGWS

- Numerically-calculated spectrum



Effect of reheating

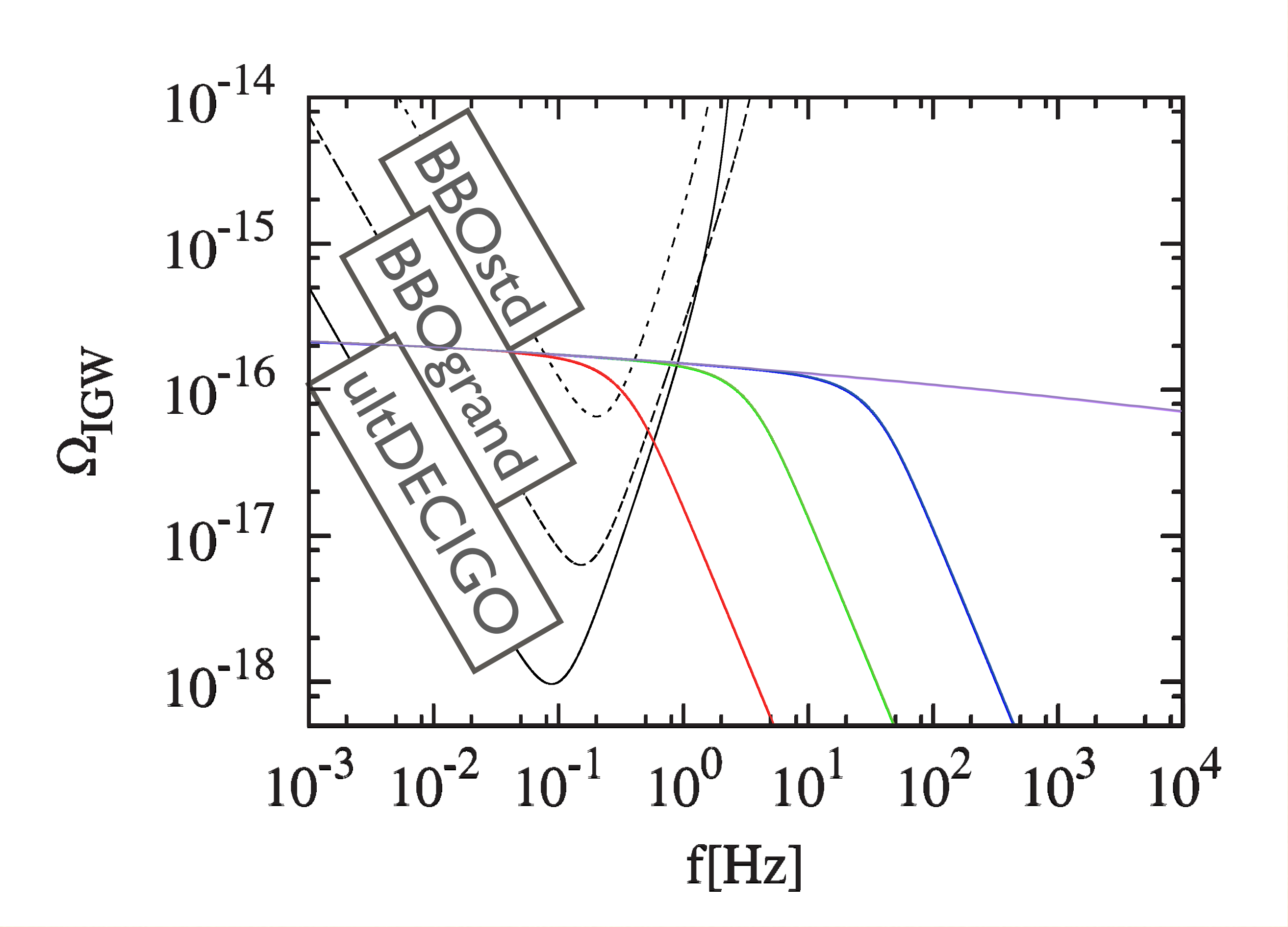
χ^2 ANALYSIS

χ^2 ANALYSIS

- Noise

  BBO standard / BBO grand / ultimate DECIGO

[G. M. Harry et al.(2006)]
[N. Seto et al.(2011)]
[E.S.Phinney et al.
The Big Bang Observer,
NASA Mission Concept Study (2003)]



- Signal

  Fundamental parameters

  : $\Omega_{\rm GW}(f_*),\ T_R$

  Fiducial values

  : predictions of ϕ^2chaotic inflation

  ($r \simeq 0.15$ at CMB scale)

- Expression for χ^2 [H.Kudoh et al.(2006)]

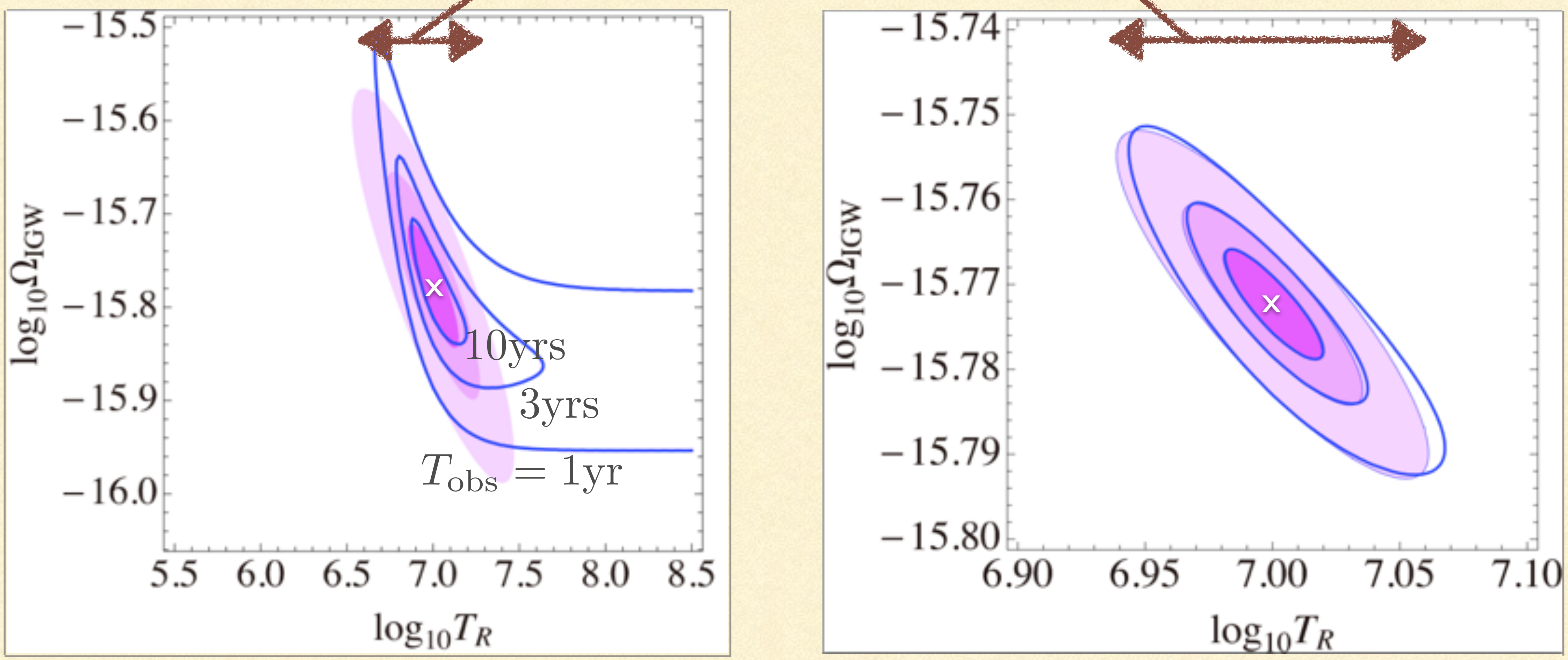
$$\chi^2 \simeq \sum_f \frac{(\Omega_{\rm GW,postulated} - \Omega_{\rm GW,true})^2}{\Delta\Omega^2_{\rm GW}}$$

RESULT

RESULT

- $T_R = 10^7\,\mathrm{GeV}$ (contours for $\delta\chi^2 = 5.99$)

BBO std T_R is well-constrained BBO grand

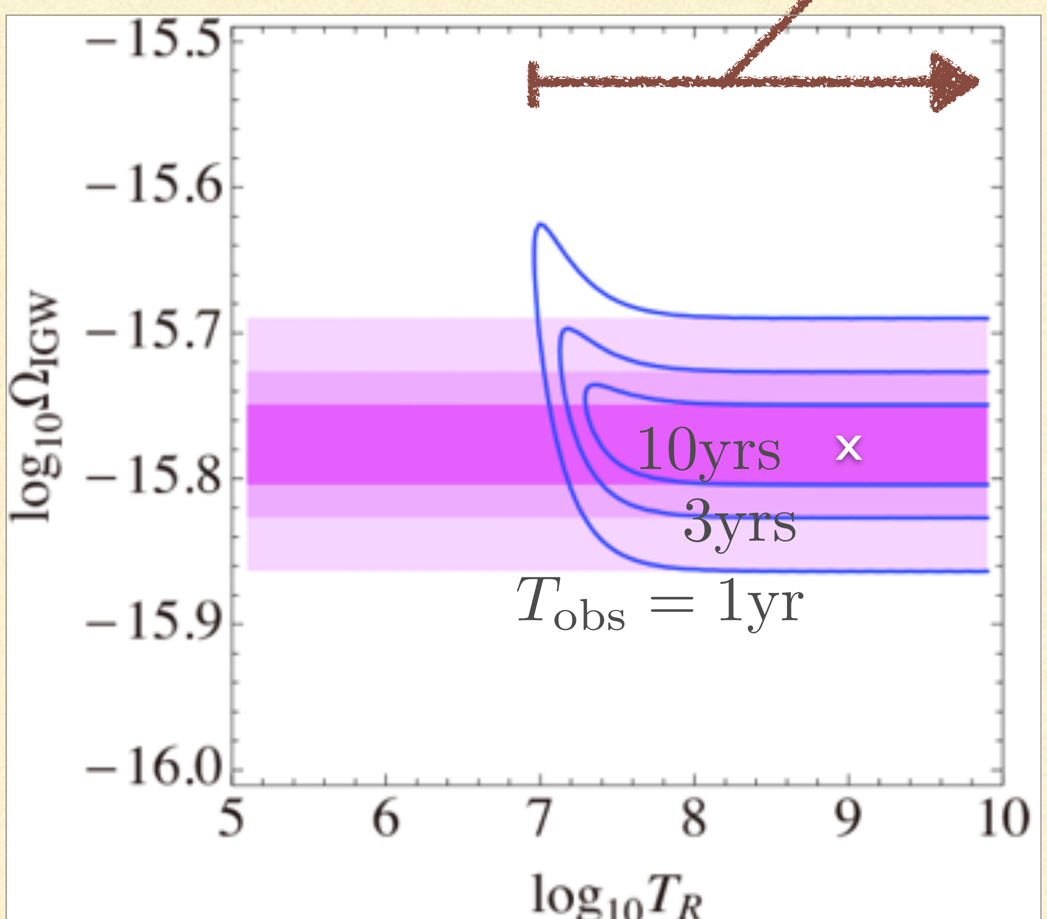


RESULT

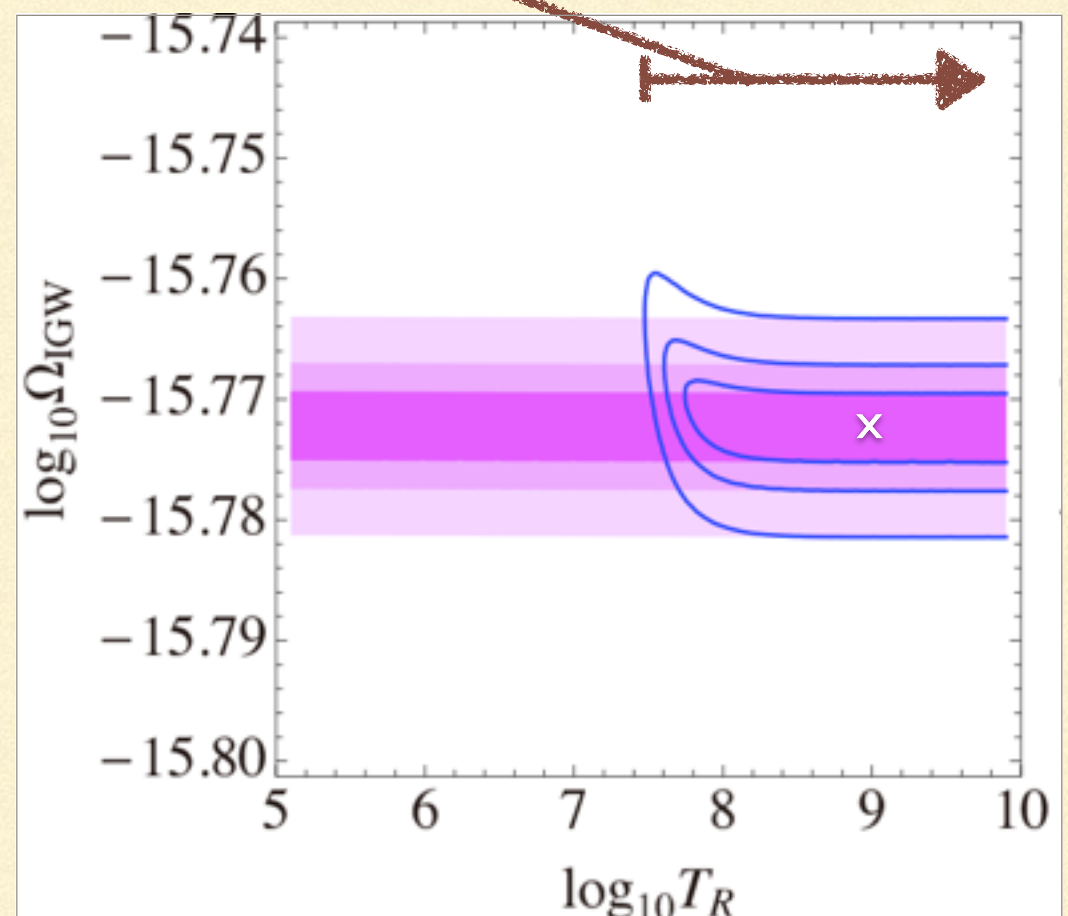
- $T_R = 10^9\,\mathrm{GeV}$ (contours for $\delta\chi^2 = 5.99$)

T_R is only bounded below

BBO std



BBO grand



RESULT

- Upper/lower bounds on T_R

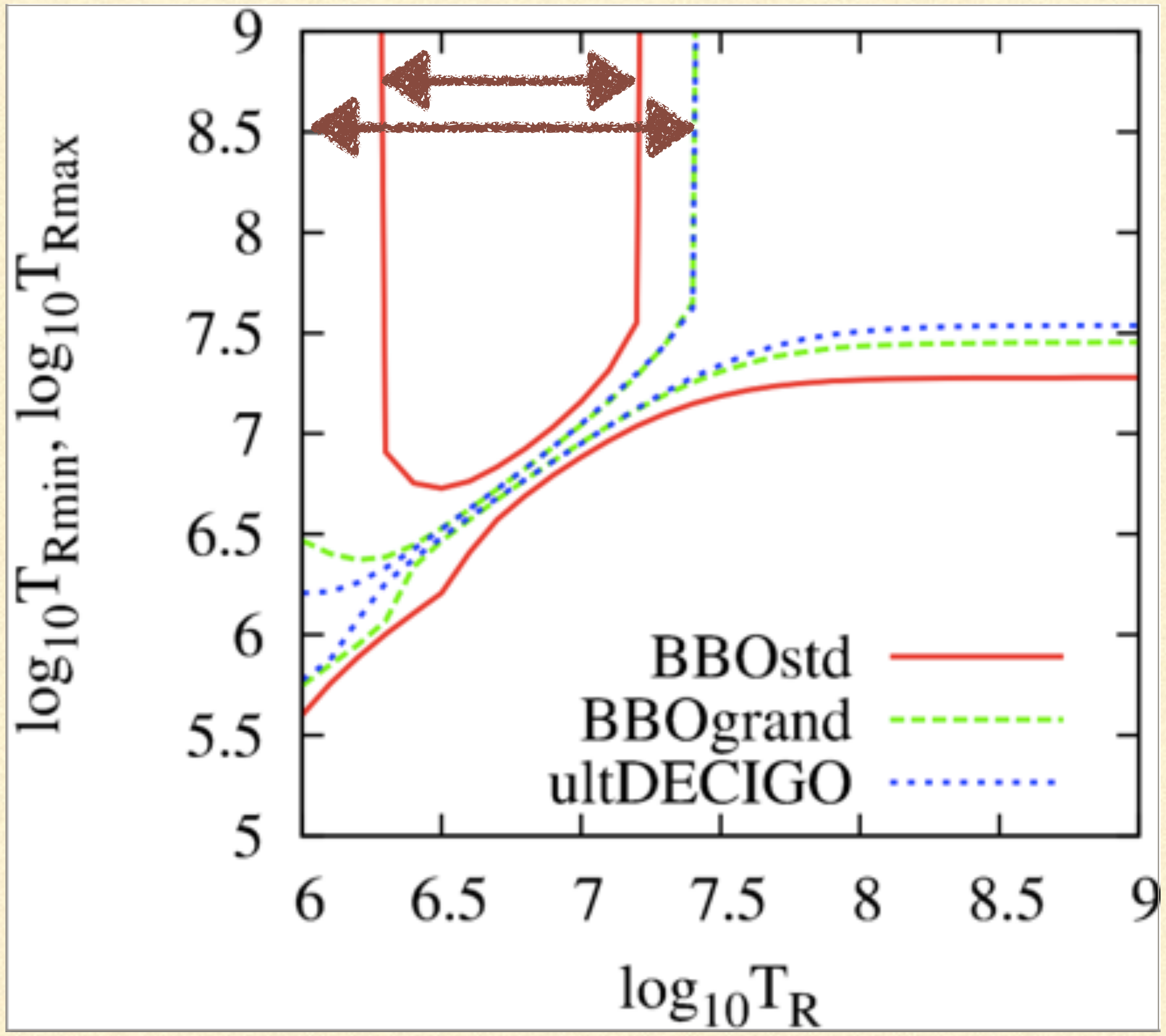
Both upper & lower bounds are obtained for

$$T_R = 10^{6.3-7.2}\mathrm{GeV}$$

(BBO standard)

$$T_R = 10^{6-7.4}\mathrm{GeV}$$

(BBO grand)



PROPERTIES OF IGWS

- Definition

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij})dx^i dx^j$$

Information on inflaton potential
& its slow-roll

- EOM

EH action → $\ddot{h} + 3H\dot{h} + \frac{k^2}{a^2}h = 0$

- Production by quantum fluctuation during inflation

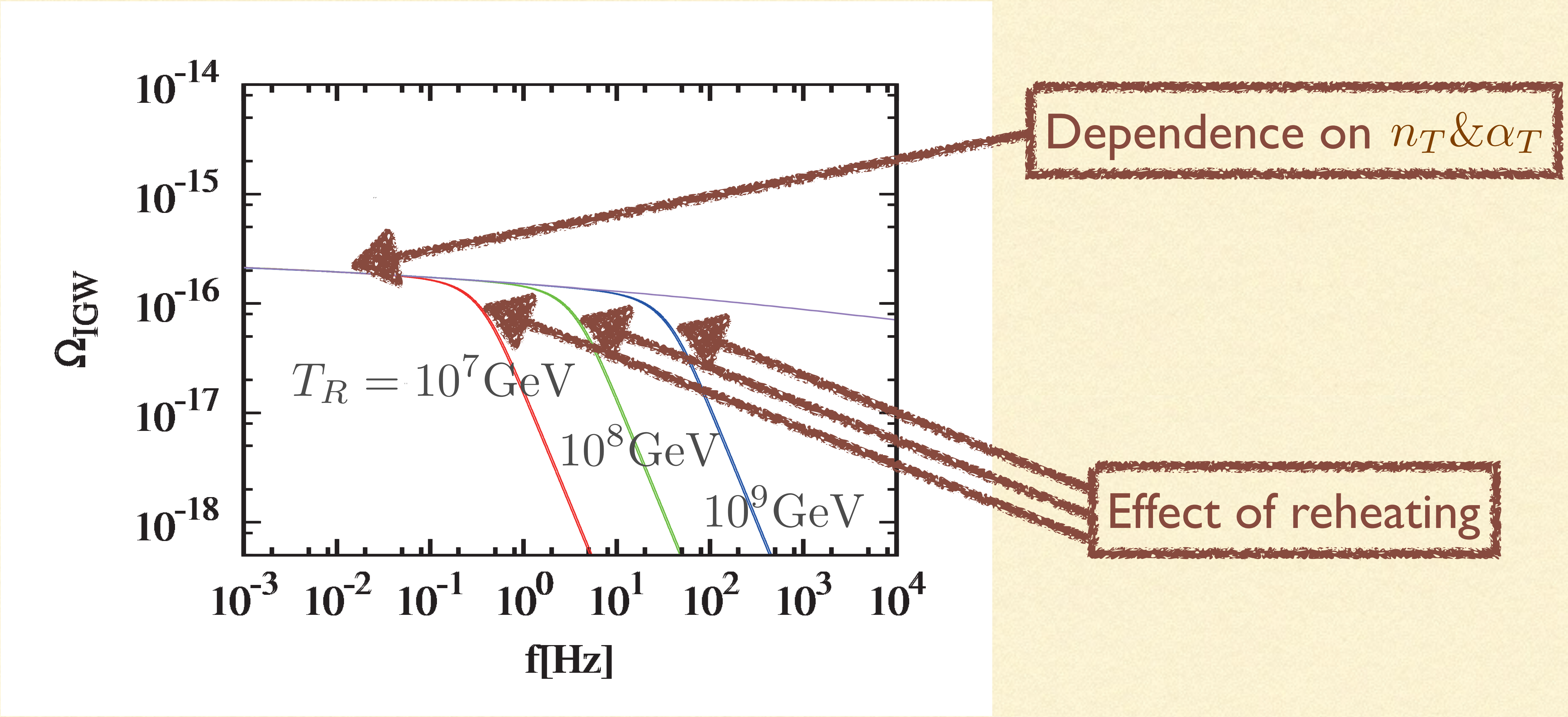
$$\mathcal{P}_{T,\mathrm{prim}}(k) = 64\pi G \left(\frac{H_{\mathrm{inf}}}{2\pi}\right)^2 = \underline{\mathcal{P}_{T,\mathrm{prim}}(k_*)} \left(\frac{k}{k_*}\right)^{\underline{n_T + \alpha_T \ln(k/k_*)/2}}$$

$$\left(\langle h_{ij}(x)^2\rangle = \int d\ln k\, \mathcal{P}_{T,\mathrm{prim}}(k)\right)$$

$$\begin{cases} n_T = -2\epsilon \\ \alpha_T = -4\epsilon(2\epsilon - \eta) \end{cases}$$

PROPERTIES OF IGWS

- Numerically-calculated spectrum



RESULT

- Sensitivities to $n_T \& \alpha_T$ $(1\sigma,\ T_{\rm obs} = 10{\rm yrs})$

Fiducial values : $\left\{ \begin{array}{l} r \simeq 0.15 \text{ (at CMB scale)} \\ n_T = -6.4 \times 10^{-2} \\ \alpha_T = -4.1 \times 10^{-3} \end{array} \right.$ $\left(\begin{array}{c} \text{predictions of} \\ \phi^2 \text{ inflation} \end{array} \right)$

$n_T \& \alpha_T$ can be determined with $O(10^{-2})$ error

	BBO-std	BBO-grand
$n_{\rm T}$ (w/ $\ln \bar{\Omega}_{\rm IGW}, \alpha_{\rm T}$)	9.6×10^{-2}	1.2×10^{-2}
$\alpha_{\rm T}$ (w/ $\ln \bar{\Omega}_{\rm IGW}, n_{\rm T}$)	0.28	3.5×10^{-2}

SUMMARY

- If tensor-to-scalar ratio is sizable ($r \sim 0.1$),

  IGWs may be observed by future experiments

- When T_R is relatively low, determination of its value is expected :

  Both upper & lower bounds are obtained for

  $T_R = 10^{6.3-7.2}\mathrm{GeV}$ (BBO standard), $T_R = 10^{6-7.4}\mathrm{GeV}$ (BBO grand)

- If r is sizable, space-interferometers are strongly suggested

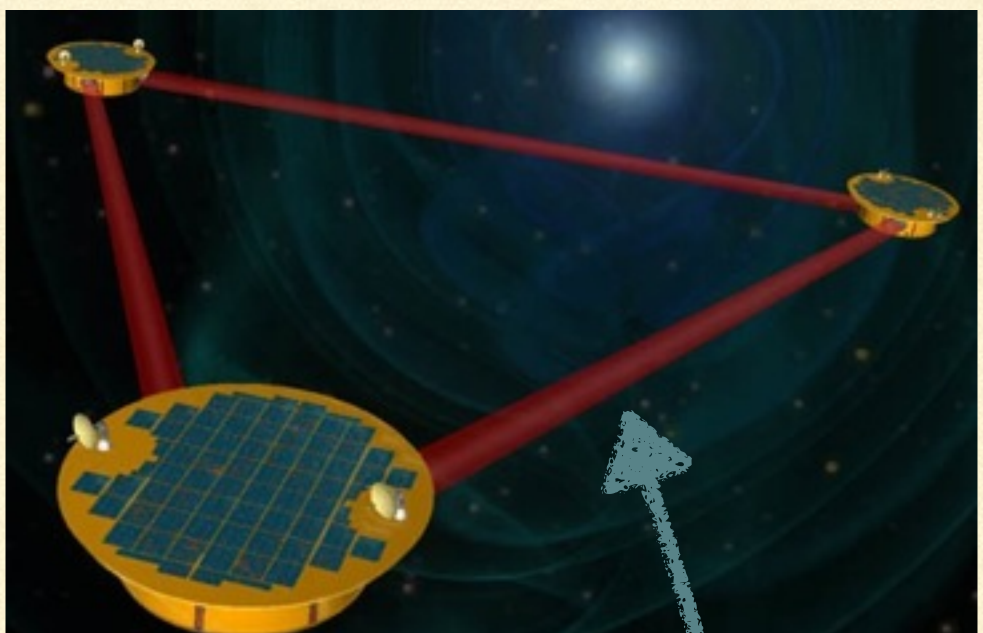
BACKUP

SPACE INTERFEROMETERS

- Configuration of LISA & BBO/DECIGO

～5×10^7m

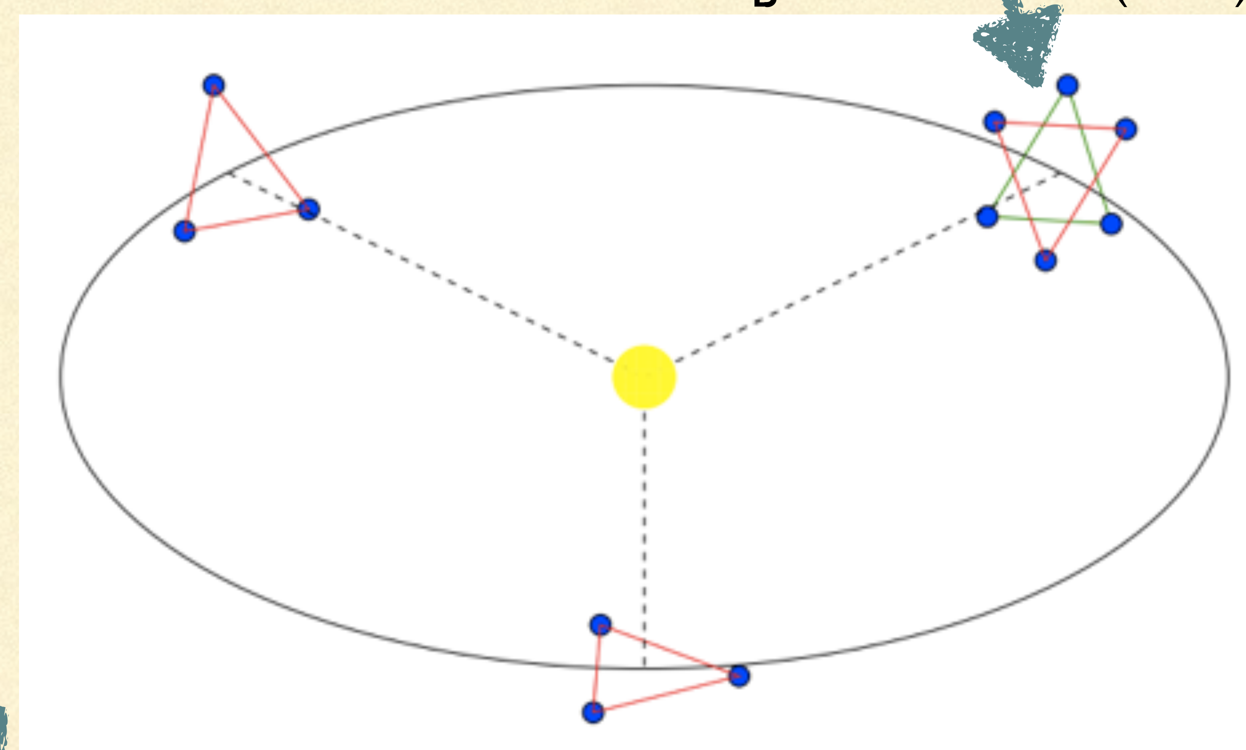
LISA



～5×10^6m

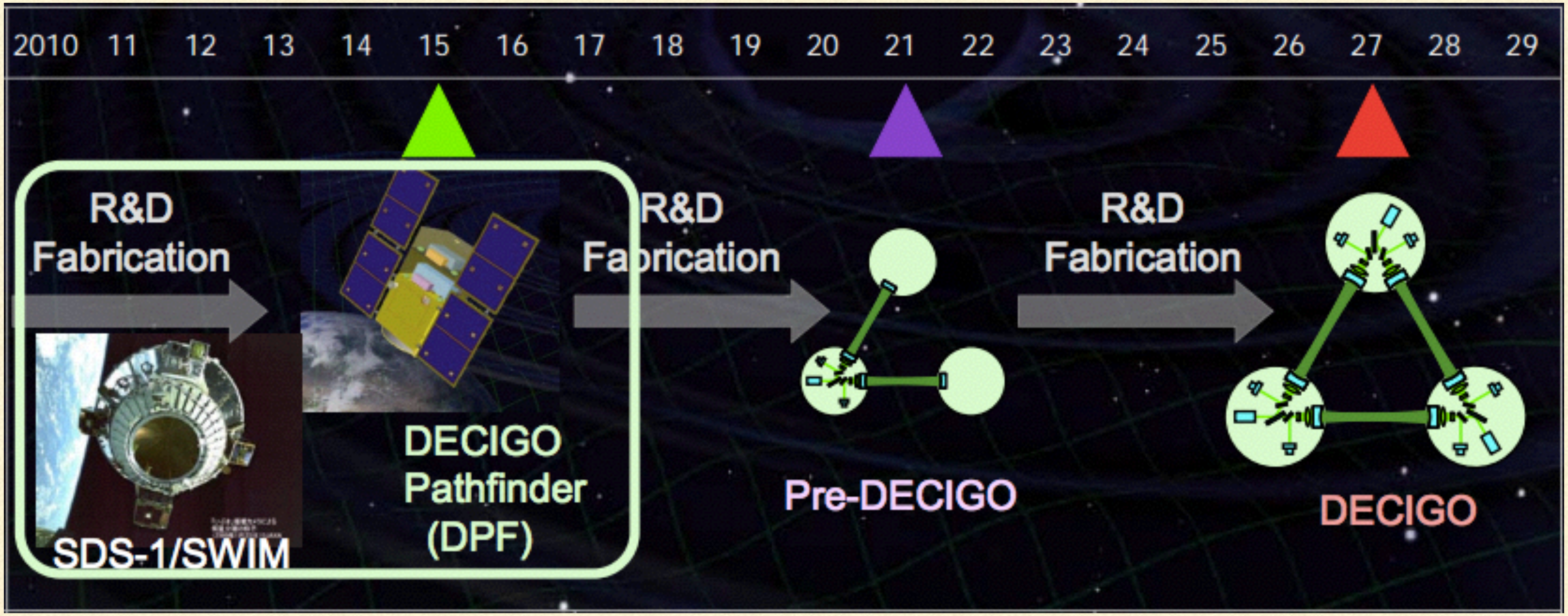
BBO/DECIGO

[J.Crowder et al.(2005)]



SPACE INTERFEROMETERS

- DECIGO roadmap



[M.Ando, DEIGO workshop(2010)]

χ^2 ANALYSIS

- $$\chi^2 = \sum_{\text{independent channels}} \sum_{\text{frequency bins}} \left(\begin{array}{c}\text{\# of independent} \\ \text{data} \\ \text{in each bin}\end{array} \right) \times \left(\begin{array}{c}\text{Detector} \\ \text{geometry} \\ \text{factor}\end{array} \right) \times \left(\frac{\text{Fiducial signal - Postulated signal}}{\text{Noise}} \right)^2$$

$$= \frac{2}{25} \sum_{I,I'} \int_{f\min} \frac{df}{1/T_{\text{obs}}} \gamma_{II'}^2(f) \left(\frac{S_{h,\text{postulated}}(f) - S_{h,\text{fiducial}}(f)}{N_{II'}} \right)^2$$

[H.Kudoh et al.(2006)]

- $S_h(f) = \frac{3H_0^2}{4\pi^2} f^{-3} \Omega_{\text{GW}}(f)$: Fundamental parameters $\Omega_{\text{GW}}, n_T, \alpha_T, T_R$ inside

χ^2 ANALYSIS

- Expression for chi2

$$\delta\chi^2(\{p\};\{\hat{p}\}) = -2\ln\mathcal{L}(\{p\};\{\hat{p}\}) = \frac{2}{25}T_{\rm obs}\sum_{(I,I')}\int_{f_{\rm min}}^{\infty} df\,\frac{\gamma_{II'}^2(f)}{\sigma_{II'}^2(f)}\left[S_h(f;\{p\}) - S_h(f;\{\hat{p}\})\right]^2 .$$

$$\sigma_{II'}^2(f) = \left[\frac{1}{2}S_I(f) + \frac{1}{5}\gamma_{II}(f)S_h(f)\right]\left[\frac{1}{2}S_{I'}(f) + \frac{1}{5}\gamma_{I'I'}(f)S_h(f)\right] + \frac{1}{25}\gamma_{II'}^2(f)S_h^2(f).$$

[H.Kudoh et al.(2006)]

$$S_h(f) = \frac{3H_0^2}{4\pi^2}f^{-3}\Omega_{\rm IGW}(f),$$

χ^2ANALYSIS

- Noise function for each channel

[CHECK]

$$S_A(f) = 8\sin^2(f/2f_L)\left[(2+\cos(f/f_L))\,S_{\rm shot} + 2\,(3+2\cos(f/f_L)+\cos(2f/f_L))\,S_{\rm accel}\right], \tag{3.9}$$

$$S_E(f) = S_A(f), \tag{3.10}$$

$$S_T(f) = 2\,(1+2\cos(f/f_L))^2\left[S_{\rm shot} + 4\sin^2(f/2f_L)S_{\rm accel}\right], \tag{3.11}$$

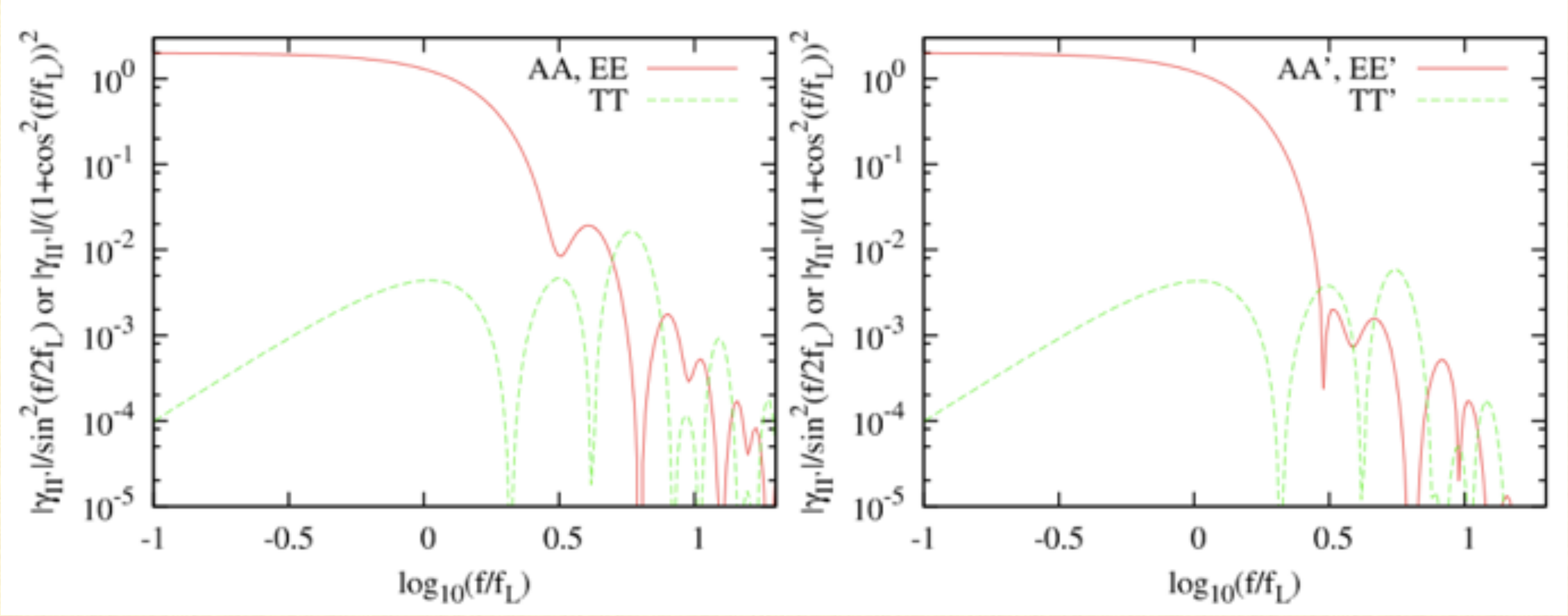
Experiments	L[m]	$S_{\rm shot}[(L/{\rm m})^{-2}{\rm Hz}^{-1}]$	$S_{\rm accel}[(2\pi f/{\rm Hz})^{-4}(L/{\rm m})^{-2}{\rm Hz}^{-1}]$
BBO-std	5×10^7	7.3×10^{-34}	9.9×10^{-33}
BBO-grand	2×10^7	8.9×10^{-35}	9.9×10^{-35}
ult-DECIGO	5×10^7	1.1×10^{-35}	0

Shot noise : noise in the laser power

Acceleration noise : noise in the mirror position

χ^2 ANALYSIS

- Overlap reduction function for each channel



[E.E.Flanagan(1993)]
[B.Allen et al.(1999)]
[N.J.Cornish et al.(2001)]
[N.Seto(2006)]
[V. Corbin et al.(2006)]

χ^2 ANALYSIS

- Standard quantum limit

$$\begin{cases} \text{Shot noise : noise in the laser power} \quad \propto N^{-1/2} \\ \text{Acceleration noise : noise in the mirror position} \quad \propto N^{1/2} \end{cases}$$

→We cannot improve both at the same time