

# THE DECOUPLING LIMIT IN THE GEORGI-MACHACEK MODEL

KUNAL KUMAR

CARLETON UNIVERSITY

SUSY 2014 - JULY 22, 2014

(arXiv : 1404.2640 + ongoing work with H. E. Logan and K. Hartling)

# Motivation

---

- SM-like Higgs and no new particles discovered so far could mean we are observing the decoupling limit of a model
- The Georgi-Machacek model adds scalar triplets in way to preserve  $\rho \equiv M_W/M_Z \cos \theta_W = 1$ 

H. Georgi, M. Machacek [NPB 262,463];  
Chanowitz, Golden, Phys.Lett. B 165, 105
- Uncommon features : doubly-charged scalar, enhancement of  $hVV$  couplings close to the decoupling limit
- Has been incorporated into little Higgs and SUSY models

Cort, Garcia,  
Quiros[PRD 88, 075010]  
  
Chang, Wacker [PRD 69 035002];  
S. Chang [JHEP 0312 057]
- The GM model is thus a valuable benchmark model to study Higgs properties

# The Model

---

- Proposed in 1985 as a possible scenario for EWSB
- SM doublet + real triplet ( $Y=0$ ) + complex triplet ( $Y=2$ )
- Arranged in terms of  $\Phi$  and  $X$  make global  $SU(2)_L \times SU(2)_R$  apparent

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} \quad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

- where  $S^{Q*} = (-1)^Q S^{-Q}$
- The scalar vevs preserve custodial  $SU(2)$

$$\langle \Phi \rangle = \frac{v_\phi}{\sqrt{2}} \mathbb{1}_{2 \times 2} \quad \langle X \rangle = v_\chi \mathbb{1}_{3 \times 3}$$

- Constrained by  $W$  and  $Z$  masses

$$v_\phi^2 + 8v_\chi^2 \equiv v^2 = \frac{4M_W^2}{g^2} \approx (246 \text{ GeV})^2 \quad \sin \theta_H = \frac{2\sqrt{2}v_\chi}{v}$$

# Scalar Sector

---

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} \quad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

- Expanding around the minima : 3 Goldstones, 10 physical scalars
- 10 scalars arranged as custodial multiplets : 1 five-plet, 1 triplet, 2 singlets
- Masses :  $m_5, m_3, m_h, m_H$  respectively
- $\alpha$  controls mixing between custodial singlets  $H_1^0$  and  $H_1^{0'}$

$$h = \cos \alpha H_1^0 - \sin \alpha H_1^{0'}, \\ H = \sin \alpha H_1^0 + \cos \alpha H_1^{0'}.$$

- $v_\chi/v$  controls contribution of states in  $X$  to Goldstones, custodial triplets

# Scalar Potential

- Most general gauge-invariant potential that preserves  $SU(2)_C$  :

$$\begin{aligned}
 V(\Phi, X) = & \frac{\mu_2^2}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{\mu_3^2}{2} \text{Tr}(X^\dagger X) + \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 + \lambda_2 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X) \\
 & + \lambda_3 \text{Tr}(X^\dagger X X^\dagger X) + \lambda_4 [\text{Tr}(X^\dagger X)]^2 - \lambda_5 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(X^\dagger t^a X t^b) \\
 & - M_1 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) (UXU^\dagger)_{ab} - M_2 \text{Tr}(X^\dagger t^a X t^b) (UXU^\dagger)_{ab}.
 \end{aligned}$$

Hartling, KK, Logan[arXiv: 1404.2640]; Aoki, Kanemura [PRD 77,095009]; Chiang, Yagyu [JHEP 1301, 026]

- $\mu_2$  and  $\lambda_1$  can be traded for  $v$  and  $m_h$  respectively and hence are not free parameters.
- Free parameters :  $\mu_3, \lambda_2, \lambda_3, \lambda_4, \lambda_5, M_1, M_2$ .
- Most literature on GM model impose  $Z_2$  symmetry for simplicity
  - e.g. Englert, Re, Spannowsky [PRD 87, 095014]
    - No  $M_1, M_2$  terms in this case and all  $m_i = \lambda_i v^2$
    - $\lambda_i$  bounded to be  $\mathcal{O}(1)$  by unitarity constraints  $\implies m_i < 700$  GeV
    - The  $Z_2$  symmetric version does not possess a decoupling limit

# Scalar Potential

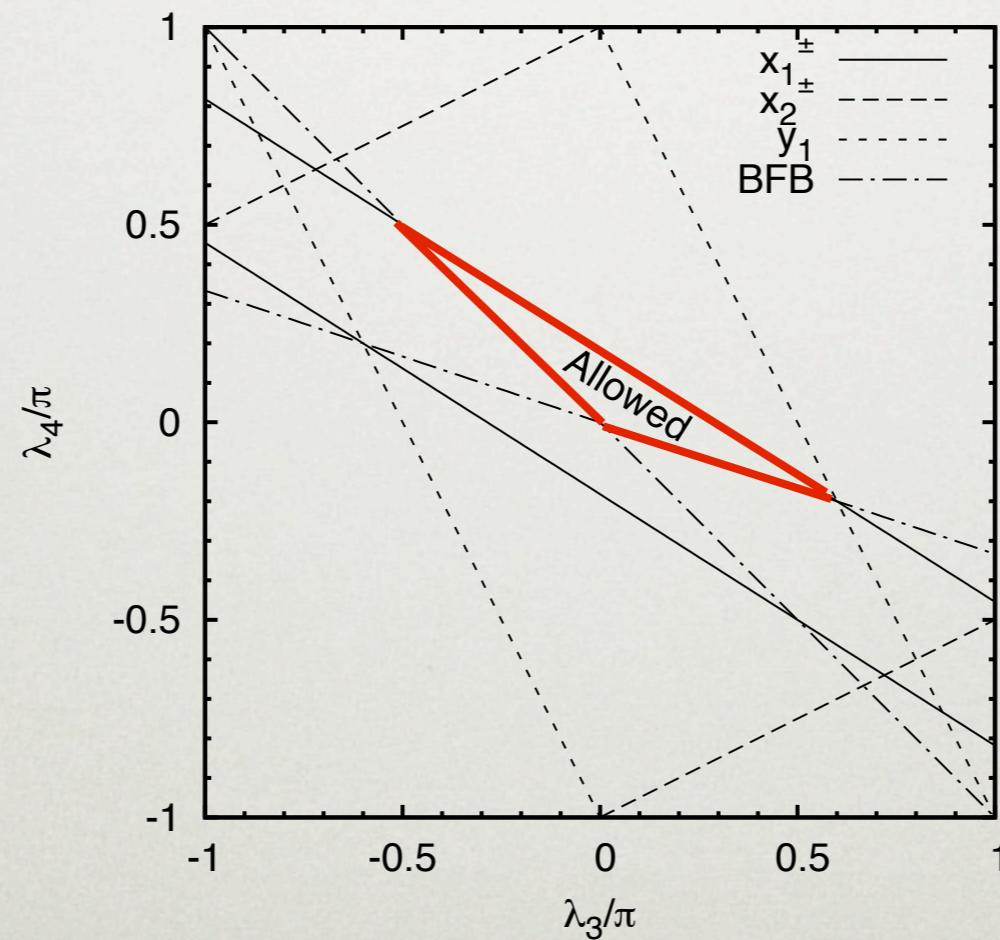
---

$$\begin{aligned} V(\Phi, X) = & \frac{\mu_2^2}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{\mu_3^2}{2} \text{Tr}(X^\dagger X) + \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 + \lambda_2 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X) \\ & + \lambda_3 \text{Tr}(X^\dagger X X^\dagger X) + \lambda_4 [\text{Tr}(X^\dagger X)]^2 - \lambda_5 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(X^\dagger t^a X t^b) \\ & - M_1 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) (U X U^\dagger)_{ab} - M_2 \text{Tr}(X^\dagger t^a X t^b) (U X U^\dagger)_{ab}. \end{aligned}$$

- No  $Z_2$  symmetry allows us to write  $M_1$  and  $M_2$  terms
- In this scenario the GM model does have a decoupling limit!
- $\mu_3$  defines the mass scale for new particles

# Theoretical Constraints

- $\lambda_i$  are constrained by **unitarity** limits on  $2 \rightarrow 2$  scalar scattering
- We also require the scalar potential to be **bounded from below** for all possible field values



- We ensure that our desired vacuum is the global minimum by imposing checks to **avoid alternative minima**

# Decoupling Behaviour

---

- Decoupling occurs when combinations of the three dimensional parameters :  $\mu_3$ ,  $M_1$  and  $M_2$  is taken large compared to  $v$ .

$$\lambda_1 \approx \frac{m_h^2}{8v^2} + \frac{3}{32} \frac{M_1^2}{\mu_3^2}$$

- $M_1$  can increase at most linearly with  $\mu_3$  because  $\lambda_1$  is bounded by unitarity.

$$|M_1|/\sqrt{\mu_3^2} \lesssim 3.3$$

- $M_2$  can increase at most linearly with  $\mu_3$  because increasing  $M_2$  increases  $v_\chi$  which is constrained by  $8v_\chi^2 < v^2$

$$|M_2|/\sqrt{\mu_3^2} \lesssim 1.2$$

# Decoupling Behaviour

---

- Case A :  $\mu_3$  is taken large but  $M_1$  and  $M_2$  are fixed.
- Case B :  $\mu_3$  is taken large and  $M_1$ ,  $M_2$  increase linearly with  $\mu_3$ .

Case	$\mu_3 \equiv \sqrt{ \mu_3^2 }$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	$\lambda_5$	$M_1$	$M_2$
A	300–1000 GeV derived	0.1	0.1	0.1	0.1	0.1	100 GeV	100 GeV
B	300–1000 GeV derived	0.1	0.1	0.1	0.1	$\mu_3/3$	$\mu_3/3$	

- We shall derive expressions for masses, higgs couplings, vevs and custodial-singlet mixing angle ( $\alpha$ ) up to leading order in  $\mu_3^{-1}$  (or equivalently the dimensionless quantity  $v/\mu_3$ )

# Decoupling Behaviour

---

- Case A :  $M_1$  and  $M_2$  fixed; Case B :  $M_1 = M_2 = \mu_3/3$
- We don't consider cases where only  $M_1$  or  $M_2$  is fixed
- In the expansion formulae  $M_2$  always appears with  $M_1$

$$v_\chi \simeq \frac{M_1 v^2}{4\mu_3^2} \left[ 1 - (2\lambda_2 - \lambda_5) \frac{v^2}{\mu_3^2} + \frac{M_1(3M_2 - M_1)v^2}{2\mu_3^4} \right]$$

- $M_1$  fixed,  $M_2 \propto \mu_3 \equiv$  Case A
- $M_1 \propto \mu_3$ ,  $M_2$  fixed  $\equiv$  Case B

# Decoupling Behaviour

---

- Case A :  $M_1$  and  $M_2$  fixed; Case B :  $M_1 = M_2 = \mu_3/3$

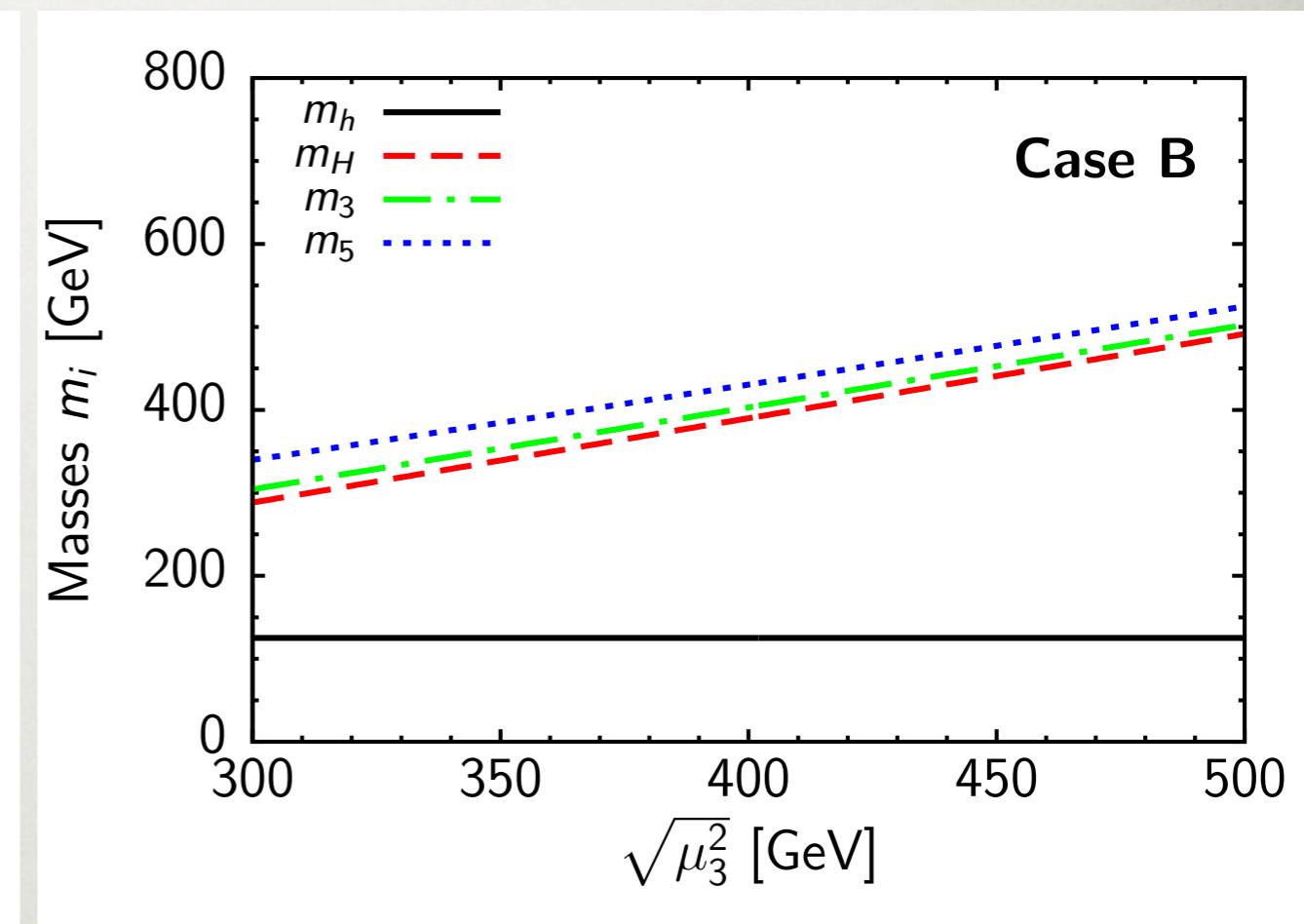
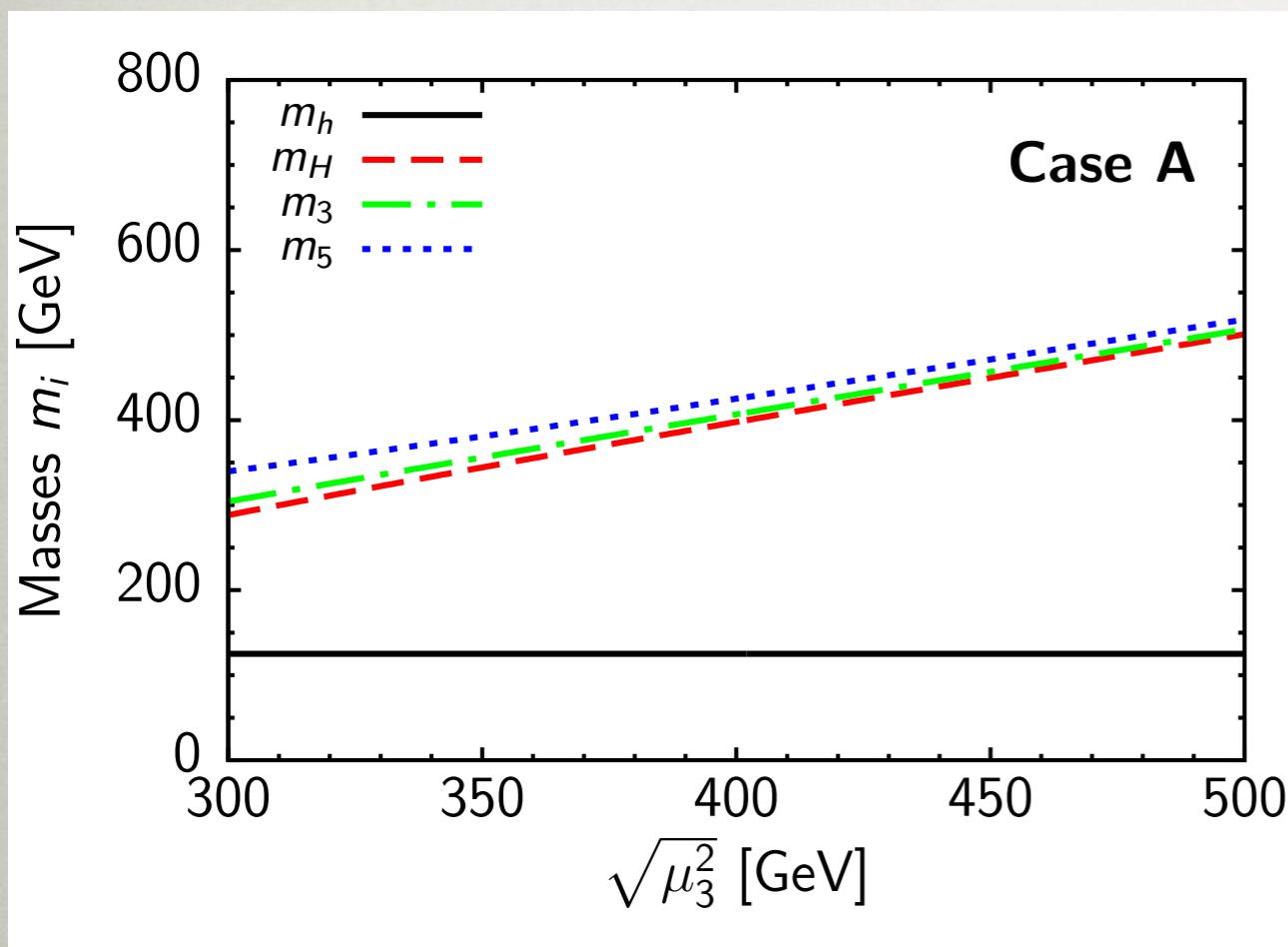
$$v_\chi \simeq \frac{M_1 v^2}{4\mu_3^2} \left[ 1 - (2\lambda_2 - \lambda_5) \frac{v^2}{\mu_3^2} + \frac{M_1(3M_2 - M_1)v^2}{2\mu_3^4} \right]$$

Quantity	Case A	Case B
$v_\chi$	$\mu_3^{-2}$	$\mu_3^{-1}$

- In general convergence to SM is more rapid in Case A.

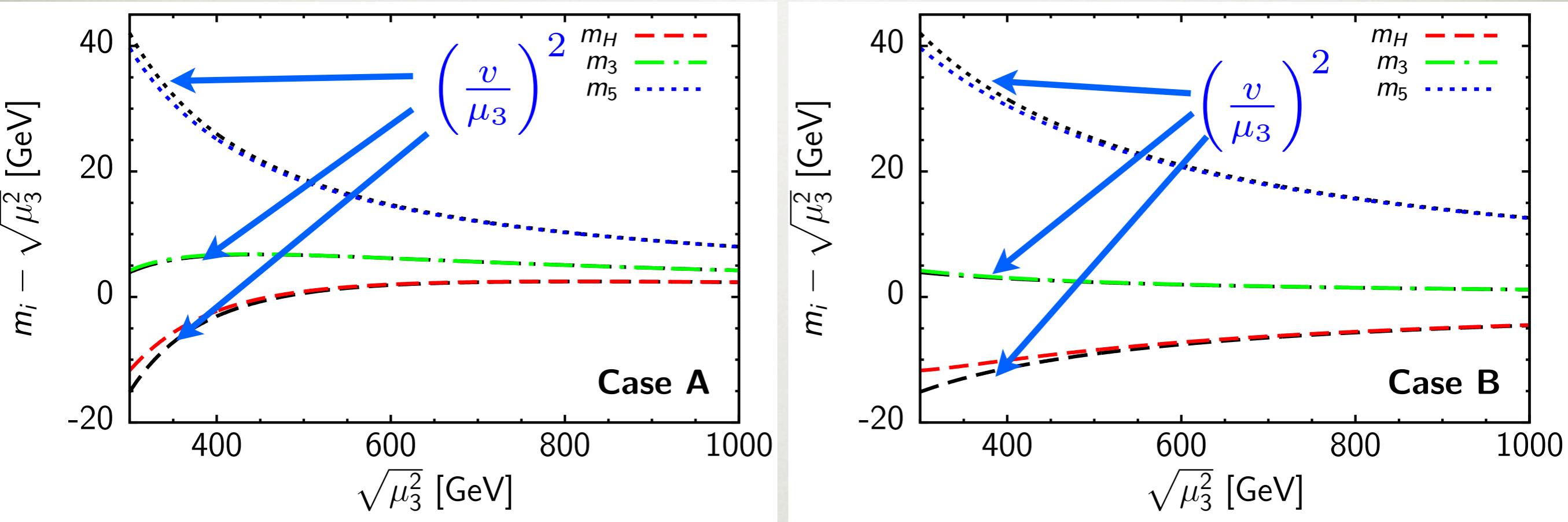
# Decoupling Behaviour: Masses

$$m_3 \simeq \mu_3 \left[ 1 + \left( 2\lambda_2 - \frac{\lambda_5}{2} \right) \frac{v^2}{2\mu_3^2} + \frac{M_1(M_1 - 3M_2)v^2}{4\mu_3^4} \right]$$



# Decoupling Behaviour: Masses

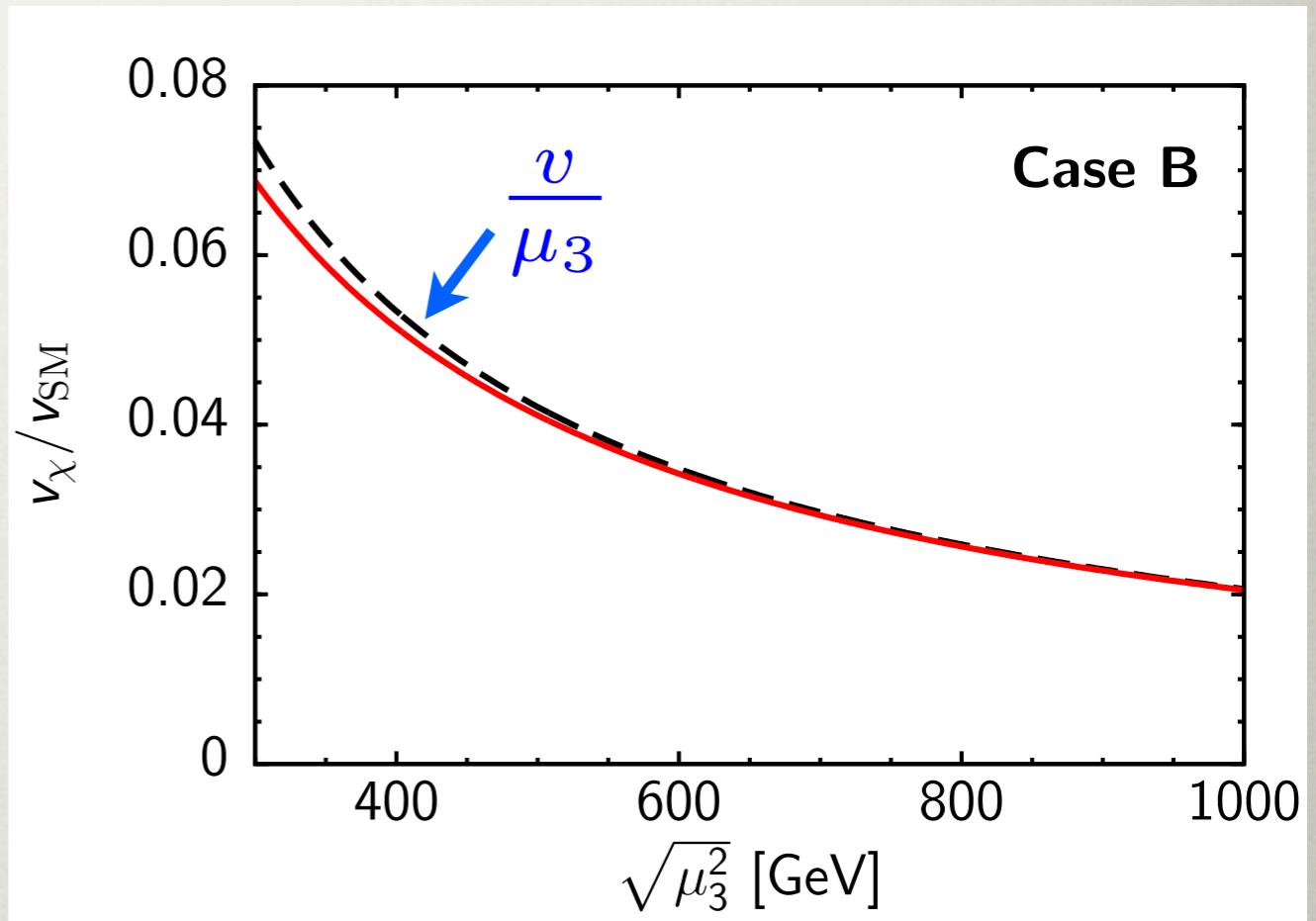
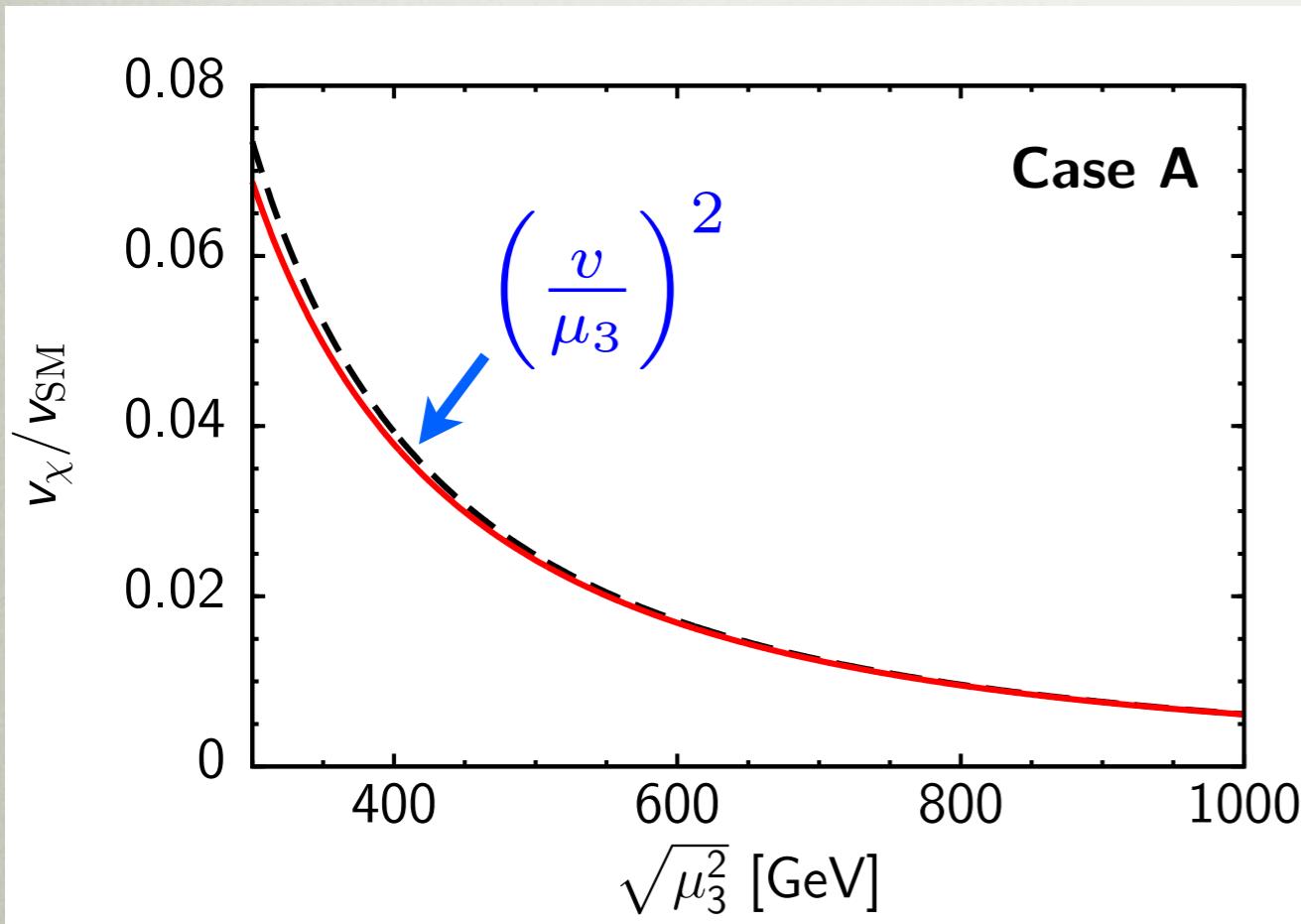
$$m_3 \simeq \mu_3 \left[ 1 + \left( 2\lambda_2 - \frac{\lambda_5}{2} \right) \frac{v^2}{2\mu_3^2} + \frac{M_1(M_1 - 3M_2)v^2}{4\mu_3^4} \right]$$



- Black curves correspond to expansions in  $\mu_3^{-1}$  while colored curves are exact.

# Decoupling Behaviour: vev

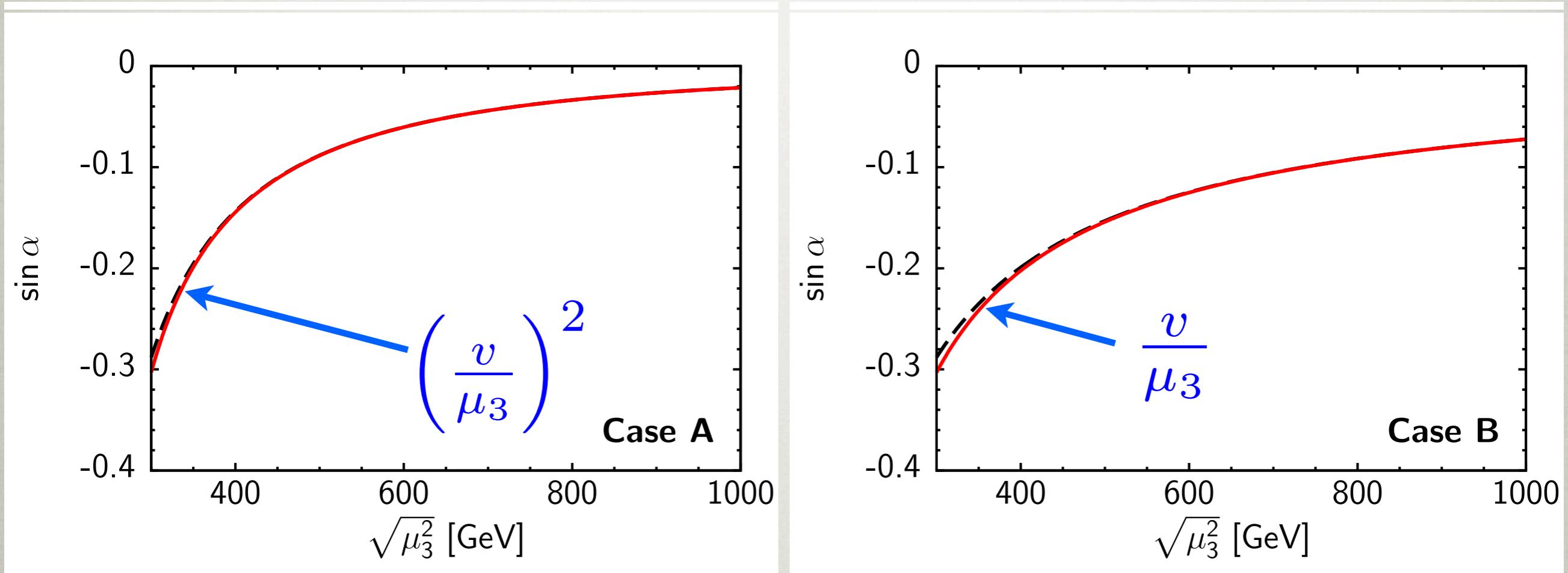
- triplet vev  $v_\chi \simeq \frac{M_1 v^2}{4\mu_3^2} \left[ 1 - (2\lambda_2 - \lambda_5) \frac{v^2}{\mu_3^2} + \frac{M_1(3M_2 - M_1)v^2}{2\mu_3^4} \right]$
- $v_\chi \rightarrow 0$  as  $\mu_3 \rightarrow \infty$



# Decoupling Behaviour: mixing angle

- custodial-singlet mixing angle

$$\sin \alpha \simeq -\frac{\sqrt{3}M_1 v}{2\mu_3^2} \left[ 1 - 2(2\lambda_2 - \lambda_5) \frac{v^2}{\mu_3^2} + \frac{m_h^2}{\mu_3^2} + \frac{M_1(24M_2 - 5M_1)v^2}{8\mu_3^4} \right]$$



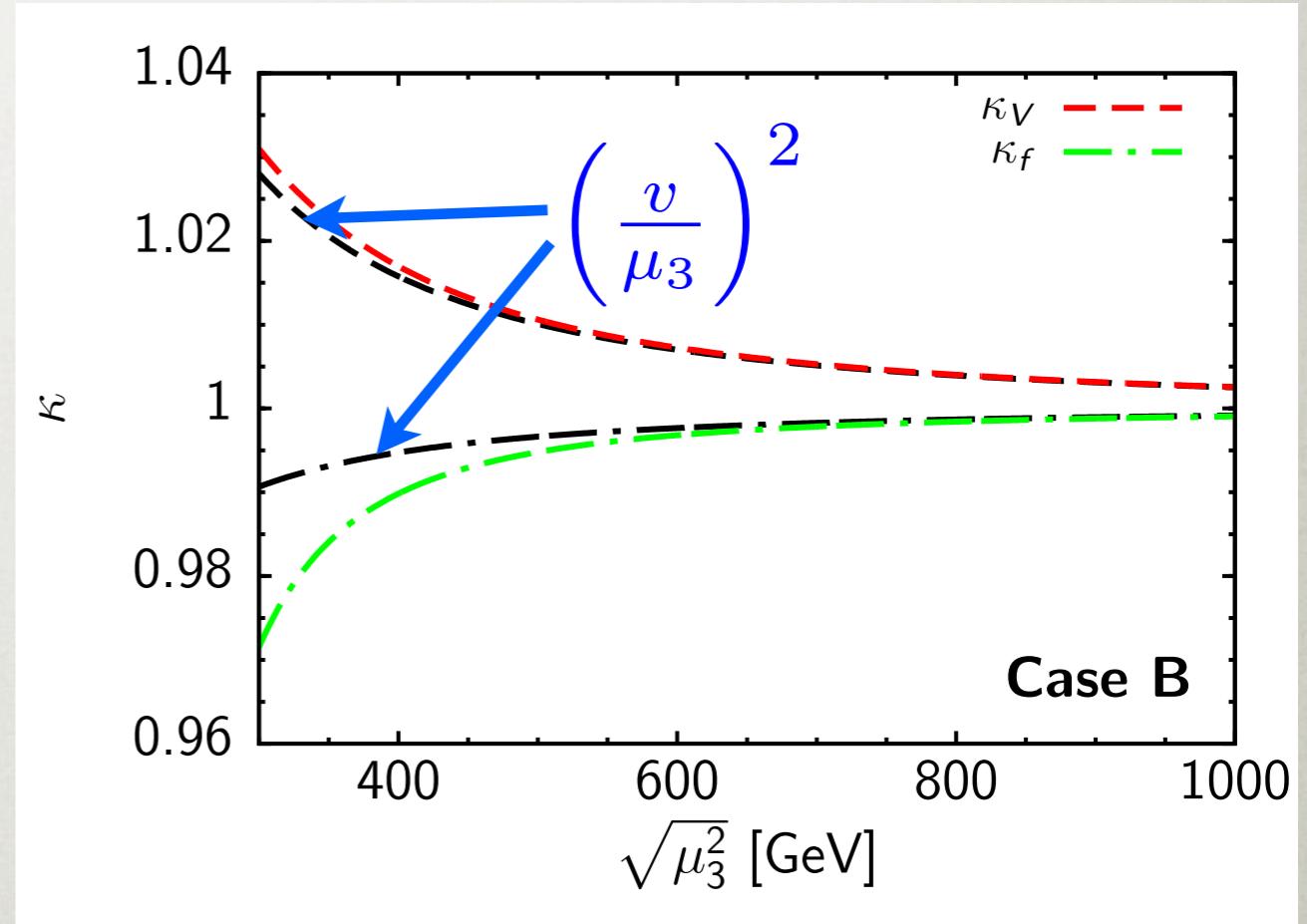
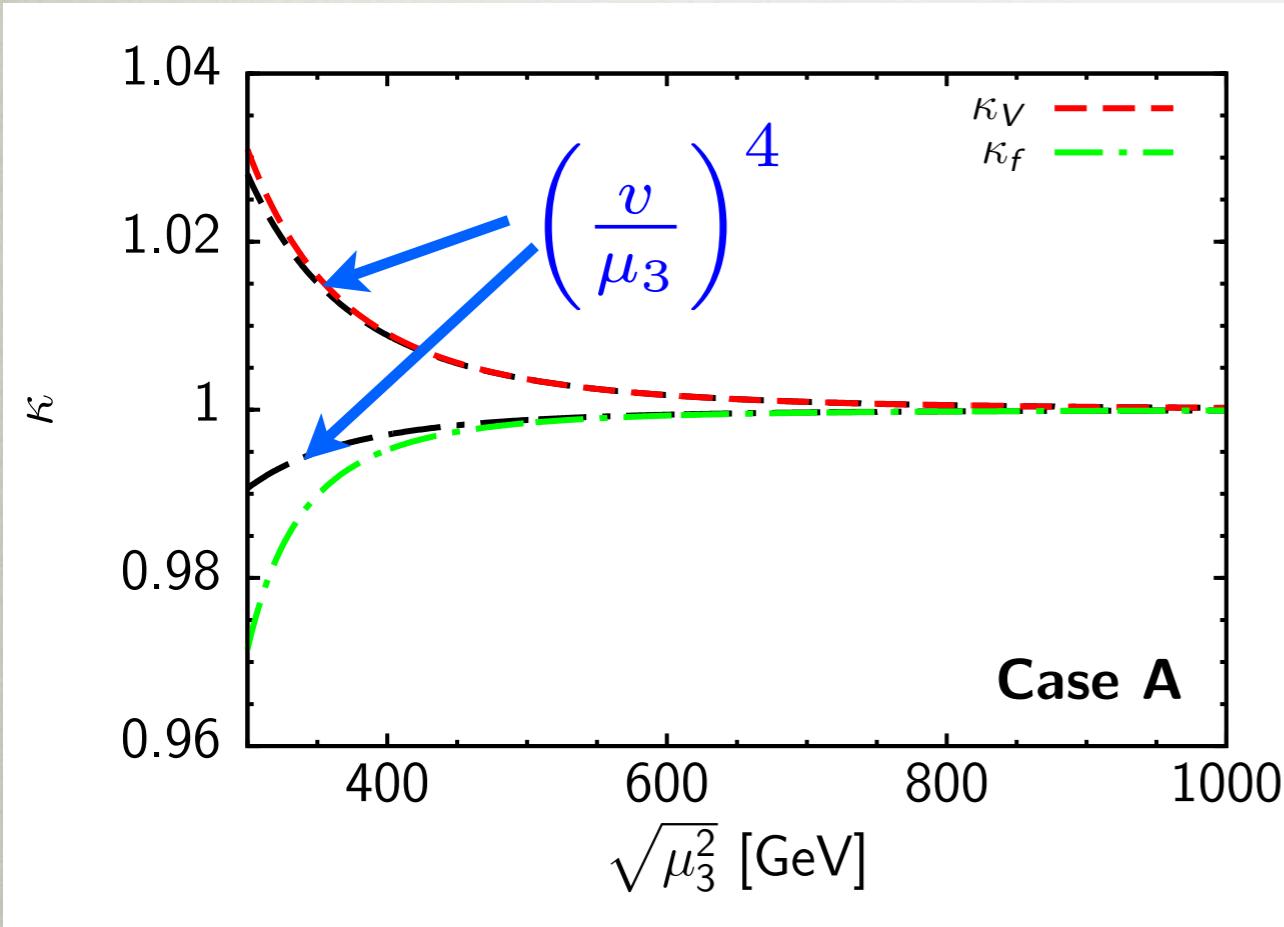
- $\sin \alpha \rightarrow 0$  as  $\mu_3 \rightarrow \infty$

# Decoupling Behaviour: hVV, hff

$\kappa$  : ratio of a coupling to its SM value

$$\kappa_V = \cos \alpha \frac{v_\phi}{v} - \frac{8}{\sqrt{3}} \sin \alpha \frac{v_\chi}{v} \simeq 1 + \frac{3}{8} \frac{M_1^2 v^2}{\mu_3^4},$$

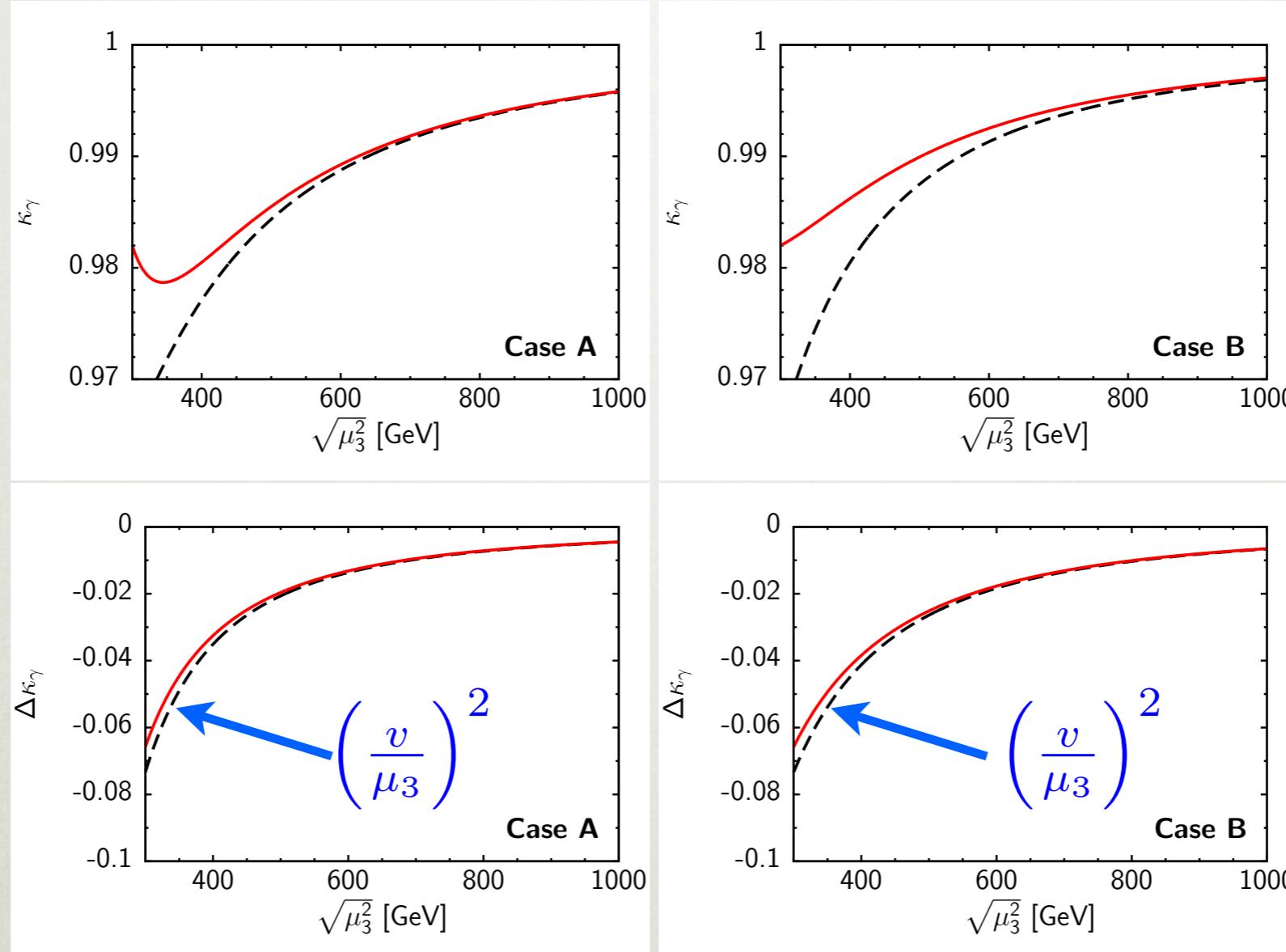
$$\kappa_f = \cos \alpha \frac{v}{v_\phi} \simeq 1 - \frac{1}{8} \frac{M_1^2 v^2}{\mu_3^4},$$



- Expansion formulae are not a very good approximation in the case of  $\kappa_f$  for  $\mu_3 \lesssim 400$

# Decoupling Behaviour: $h\gamma\gamma$

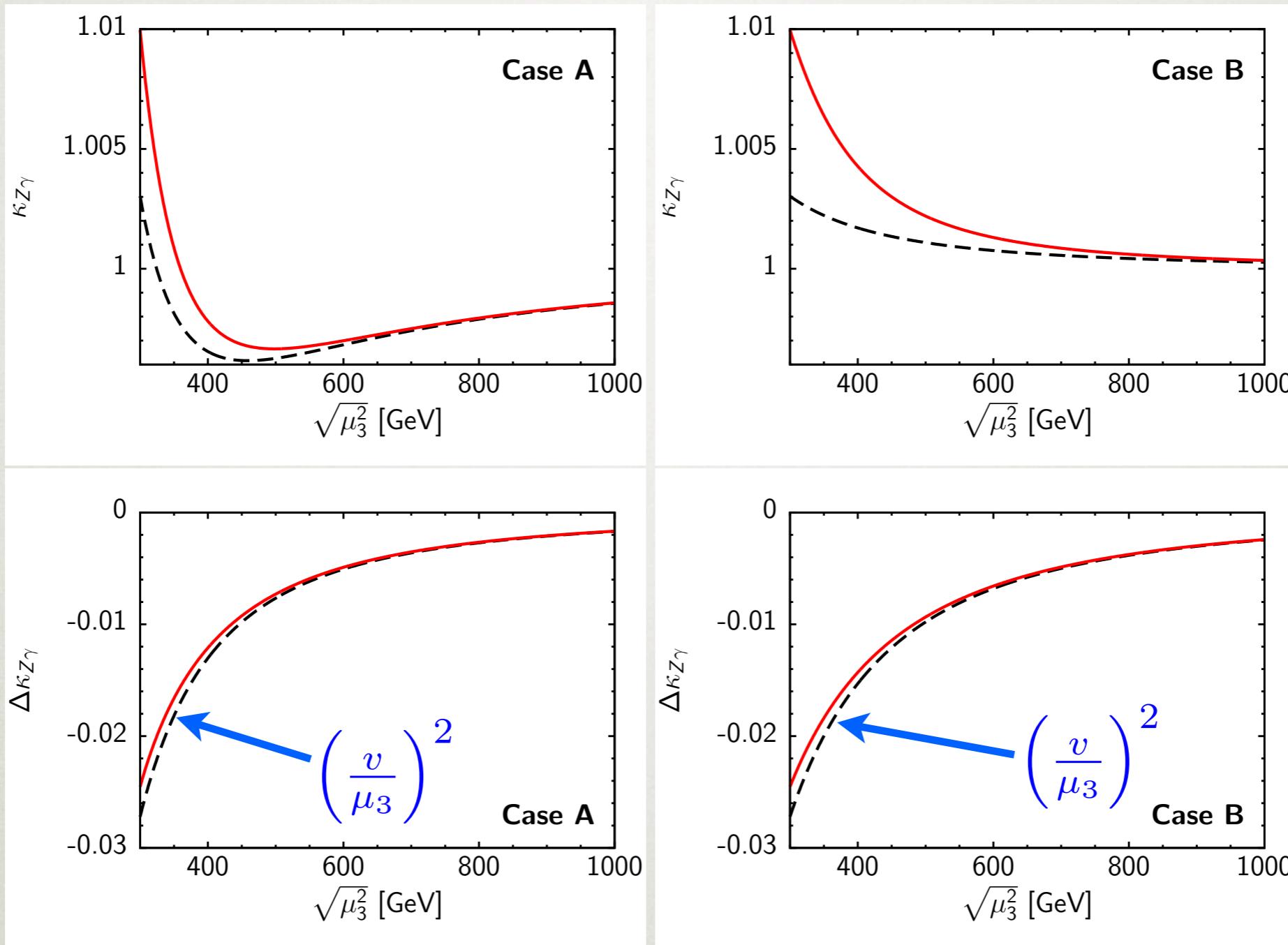
- Loop induced couplings: affected by changes to  $hVV$  and  $hff$ , new charged scalars in the loop



$$\Delta\kappa_\gamma \simeq -\frac{1}{F_1(M_W) + \frac{4}{3}F_{1/2}(m_t)} \frac{2v^2}{3\mu_3^2} \left[ 6\lambda_2 + \lambda_5 + \frac{M_1^2 + 12M_1M_2}{4\mu_3^2} \right]$$

- $\Delta\kappa$ : Ratio of contribution from non-SM particles in loop to SM coupling

# Decoupling Behaviour: $hZ\gamma$



$$\Delta\kappa_{Z\gamma} \simeq \frac{1}{2(A_W + A_f)} \frac{1 - 2s_W^2}{s_W c_W} \frac{2v^2}{3\mu_3^2} \left[ 6\lambda_2 + \lambda_5 + \frac{M_1^2 + 12M_1 M_2}{4\mu_3^2} \right]$$

- $\Delta\kappa$  : Ratio of contribution from non-SM particles in loop to SM coupling

# Comparison with Type-II 2HDM

## Type-II 2HDM

$$\kappa_V^{2\text{HDM}} \simeq 1 - \frac{\hat{\lambda}^2 v^4}{2 m_A^4},$$

$$\kappa_f^{2\text{HDM}} \simeq 1 + \frac{\hat{\lambda} v^2}{m_A^2} \times \begin{cases} \cot \beta & \text{for up type fermions} \\ -\tan \beta & \text{for down type fermions,} \end{cases}$$

$$g_{hhh}^{2\text{HDM}} \simeq \frac{3m_h^2}{v} \left[ 1 - \frac{3\hat{\lambda}^2 v^2}{\lambda m_A^2} \right],$$

J.F. Gunion, H.E. Haber [PRD 67, 075019]

## Georgi-Machacek

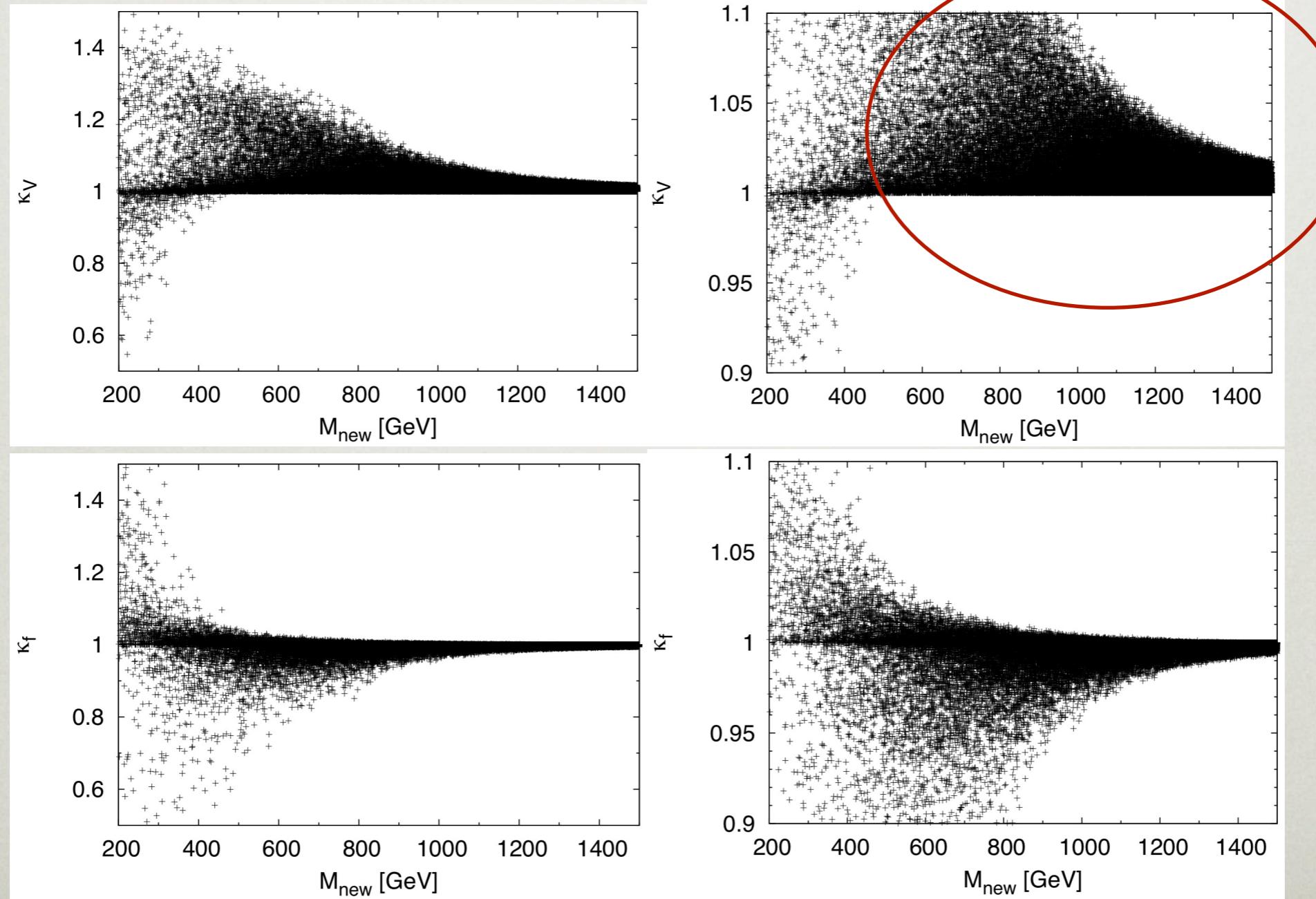
Quantity	Case A	Case B
$\kappa_V - 1$	$\mu_3^{-4}$	$\mu_3^{-2}$
$\kappa_f - 1$	$\mu_3^{-4}$	$\mu_3^{-2}$
$g_{hhh}/g_{hhh}^{\text{SM}} - 1$	$\mu_3^{-4}$	$\mu_3^{-2}$

- relatively large deviations in all couplings would favor GM Case B
- relatively large deviations fermion and trilinear couplings, but SM like vector couplings favors 2HDM
- Another distinguishing feature is the  $\kappa_V$  is enhanced in the decoupling limit of the GM model

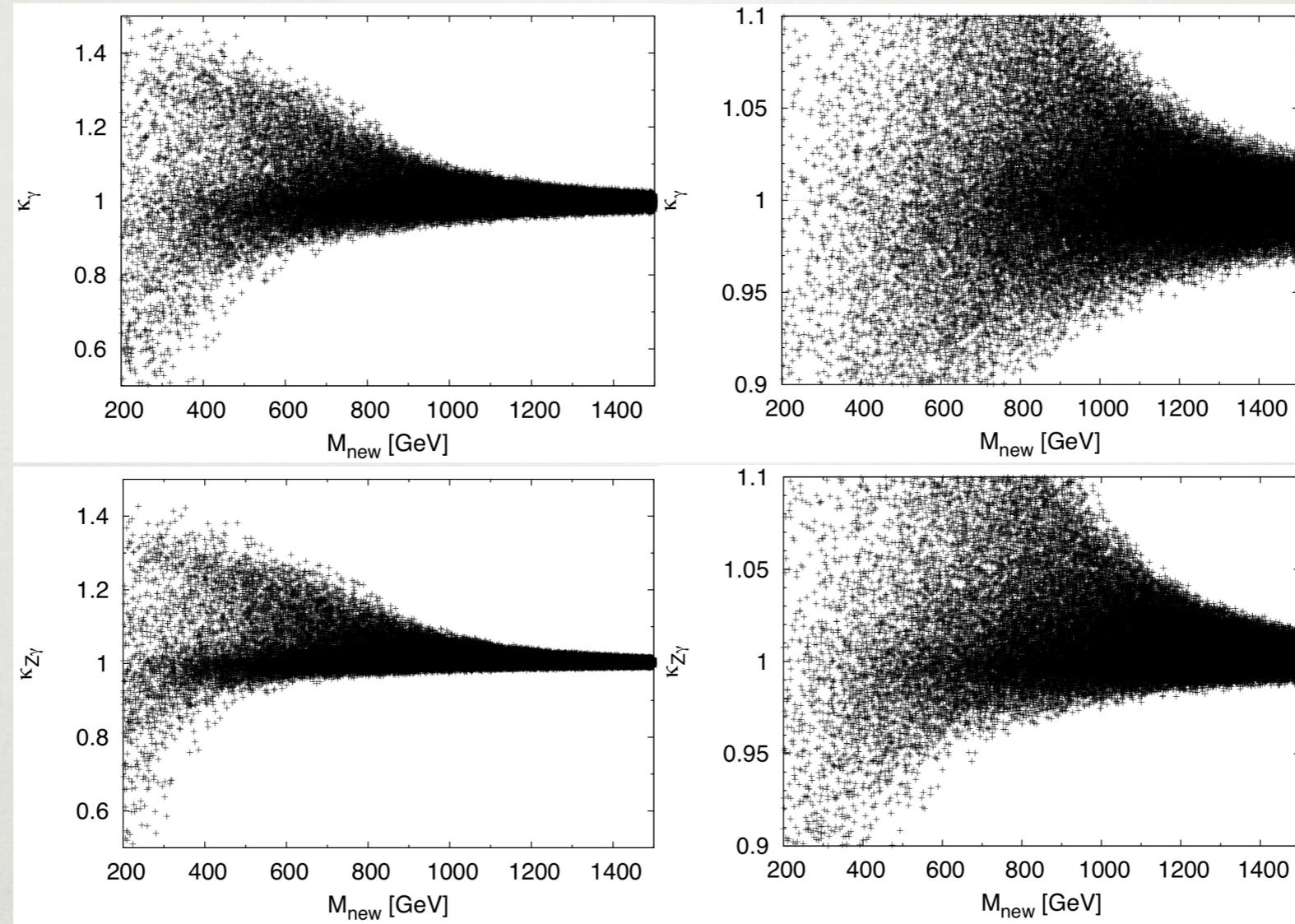
$$\kappa_V = \cos \alpha \frac{v_\phi}{v} - \frac{8}{\sqrt{3}} \sin \alpha \frac{v_\chi}{v} \simeq 1 + \frac{3}{8} \frac{M_1^2 v^2}{\mu_3^4},$$

# Numerical Scans

- Numerical scans to examine accessible range of couplings
- We allow all the free parameters to vary and impose theoretical constraints.



# Numerical Scans

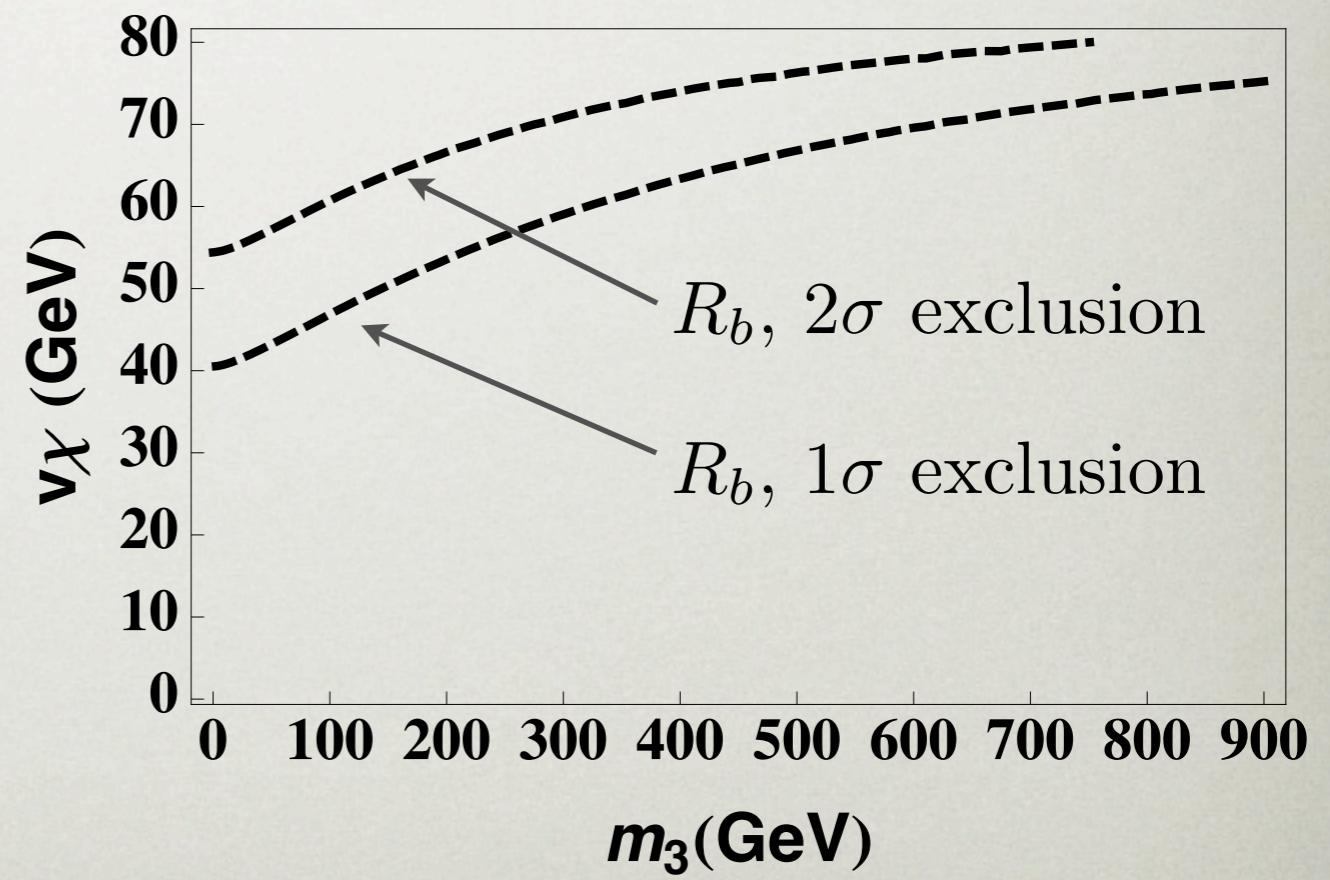
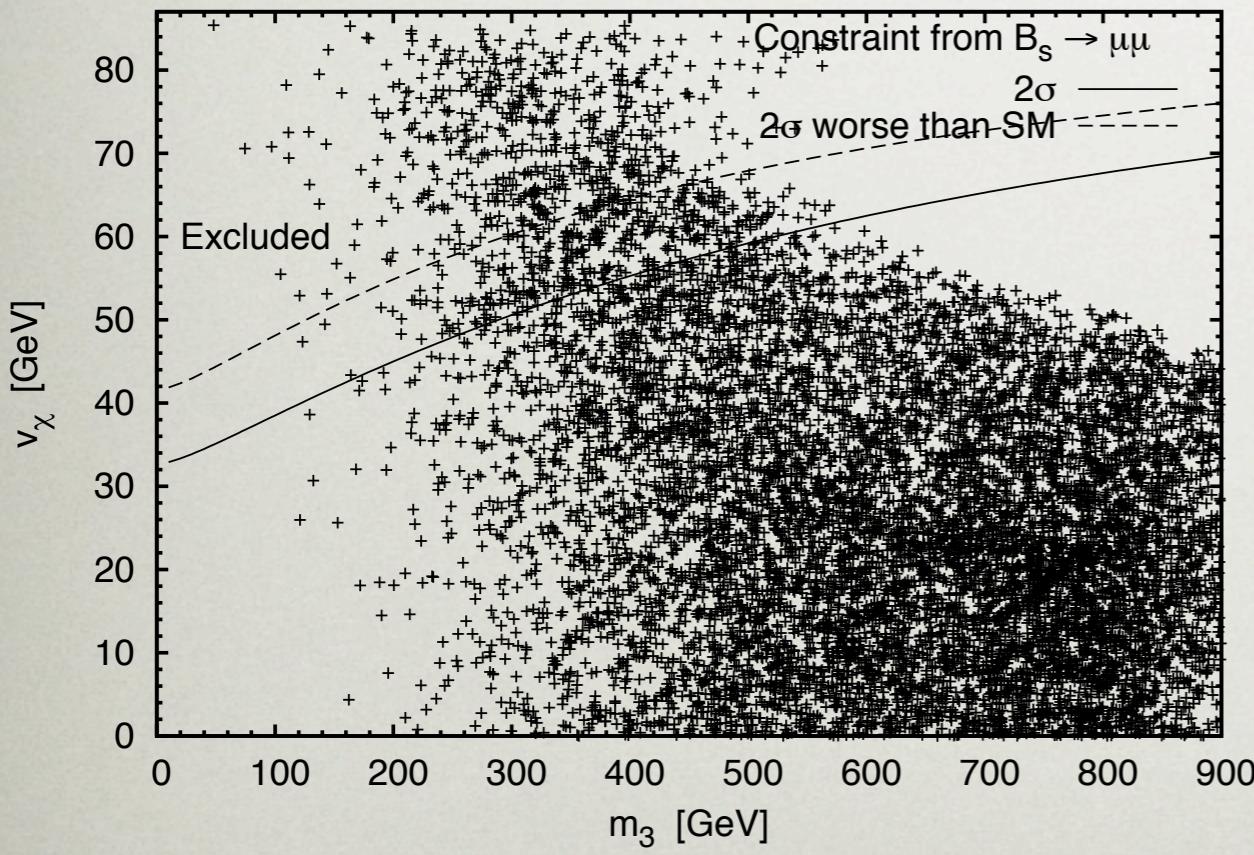


- All couplings can show 10% deviations even when the mass of the lightest scalar is around 800 GeV.
- The  $1\sigma$  allowed regions of higgs couplings measured at the LHC can be populated fully by the GM model with scalar masses below  $400 - 600$  GeV.

# Indirect Constraints

- Include constraints from : B meson mixing,  $B_s \rightarrow \mu\mu$ ,  $R_b$ ,  $b \rightarrow s\gamma$  ,Oblique parameters (ongoing)
- Most stringent constraint comes from  $B_s \rightarrow \mu\mu$

Preliminary Result



# Indirect Constraints

---

- At one-loop level  $\rho \neq 1$  and we require a counterterm to cancel the singularities in the  $T$  parameter Gunion, Vega, Wudka [PRD 43, 2322]
- We could approach the constraint from oblique parameters in the following ways:
  - Apply the constraint only from the  $S$  parameter
  - Perform the one-loop calculations with the required counterterm
  - Minimize  $\chi^2$  with respect to  $T$  and use  $T_{\min}$  to obtain the constraint

$$\chi^2 = \frac{1}{(1 - \rho_{ST}^2)} \left[ \frac{(S - S_{\text{exp}})^2}{(\Delta S_{\text{exp}})^2} + \frac{(T - T_{\text{exp}})^2}{(\Delta T_{\text{exp}})^2} - \frac{2(S - S_{\text{exp}})(T - T_{\text{exp}})}{\Delta S_{\text{exp}} \Delta T_{\text{exp}}} \right]$$

$$T_{\min} = T_{\text{exp}} + (S - S_{\text{exp}}) \frac{\Delta T_{\text{exp}}}{\Delta S_{\text{exp}}}$$

# Conclusions

---

- GM model with most general gauge-invariant and  $SU(2)_C$  preserving potential does possess a decoupling limit
- Approach to the SM is in general faster when  $M_1$  and  $M_2$  are fixed as compared to  $M_1 = M_2 = \mu_3/3$
- Numerical scans show that 10% coupling deviations are possible for new scalars even as heavy as 800 GeV
- GM model can fully populate the allowed  $1\sigma$  ranges of Higgs couplings when the new scalars are lighter than 400-600 GeV.
- Improved measurements of higgs couplings can help distinguish GM model from other extensions such as the 2HDM.

## Future Work

- Including exclusion due to oblique parameters.

# BACKUP SLIDES

# Theoretical Constraints : Unitarity

$Q$	$Y$	Basis states	Eigenvalues
0	0	$[\chi^{++*}\chi^{++}, \chi^{+*}\chi^+, \xi^{+*}\xi^+, \phi^{+*}\phi^+, \chi^{0*}\chi^0, \frac{\xi^0\xi^0}{\sqrt{2}}, \phi^{0*}\phi^0]$	$x_1^+, x_1^-, x_2^+, x_2^-, y_1, y_1, y_2$
0	1	$[\phi^+\xi^{+*}, \phi^0\xi^0, \chi^+\phi^{+*}, \chi^0\phi^{0*}]$	$y_3, y_4, y_4, y_5$
0	2	$[\frac{\phi^0\phi^0}{\sqrt{2}}, \chi^0\xi^0, \chi^+\xi^{+*}]$	$x_2^+, x_2^-, y_2$
0	3	$[\phi^0\chi^0]$	$y_3$
0	4	$[\frac{x^0x^0}{\sqrt{2}}]$	$y_2$
1	-2	$[\xi^+\chi^{0*}]$	$y_2$
1	-1	$[\phi^+\chi^{0*}, \xi^+\phi^{0*}]$	$y_3, y_4$
1	0	$[\xi^+\xi^0, \chi^{+*}\chi^{++}, \phi^+\phi^{0*}, \chi^{0*}\chi^+]$	$x_2^+, x_2^-, y_1, y_2$
1	1	$[\phi^0\xi^+, \phi^+\xi^0, \phi^{+*}\chi^{++}, \phi^{0*}\chi^+]$	$y_3, y_4, y_4, y_5$
1	2	$[\phi^+\phi^0, \chi^+\xi^0, \chi^{++}\xi^{+*}, \chi^0\xi^+]$	$x_2^+, x_2^-, y_1, y_2$
1	3	$[\phi^+\chi^0, \phi^0\chi^+]$	$y_3, y_4$
1	4	$[\chi^+\chi^0]$	$y_2$
2	0	$[\chi^{++}\chi^{0*}, \frac{\xi^+\xi^+}{\sqrt{2}}]$	$y_1, y_2$
2	1	$[\phi^+\xi^+, \chi^{++}\phi^{0*}]$	$y_3, y_4$
2	2	$[\frac{\phi^+\phi^+}{\sqrt{2}}, \chi^{++}\xi^0, \chi^+\xi^+]$	$x_2^+, x_2^-, y_2$
2	3	$[\phi^+\chi^+, \phi^0\chi^{++}]$	$y_3, y_4$
2	4	$[\chi^{++}\chi^0, \frac{x^+\chi^+}{\sqrt{2}}]$	$y_1, y_2$
3	2	$[\chi^{++}\xi^+]$	$y_2$
3	3	$[\chi^{++}\phi^+]$	$y_3$
3	4	$[\chi^{++}\chi^+]$	$y_2$
4	4	$[\frac{x^{++}x^{++}}{\sqrt{2}}]$	$y_2$

$$|x_i^\pm| < 8\pi \text{ and } |y_i| < 8\pi$$

# Theoretical Constraints : BFB

$$r \equiv \sqrt{\text{Tr}(\Phi^\dagger \Phi) + \text{Tr}(X^\dagger X)},$$

$$r^2 \cos^2 \gamma \equiv \text{Tr}(\Phi^\dagger \Phi),$$

$$r^2 \sin^2 \gamma \equiv \text{Tr}(X^\dagger X),$$

$$\zeta \equiv \frac{\text{Tr}(X^\dagger X X^\dagger X)}{[\text{Tr}(X^\dagger X)]^2},$$

$$\omega \equiv \frac{\text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(X^\dagger t^a X t^b)}{\text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X)}$$

$$[a + b\bar{y}^2 + c\bar{y}^4] \quad a > 0, \quad c > 0, \quad \text{and} \quad b + 2\sqrt{ac} > 0.$$

quartic terms in the potential

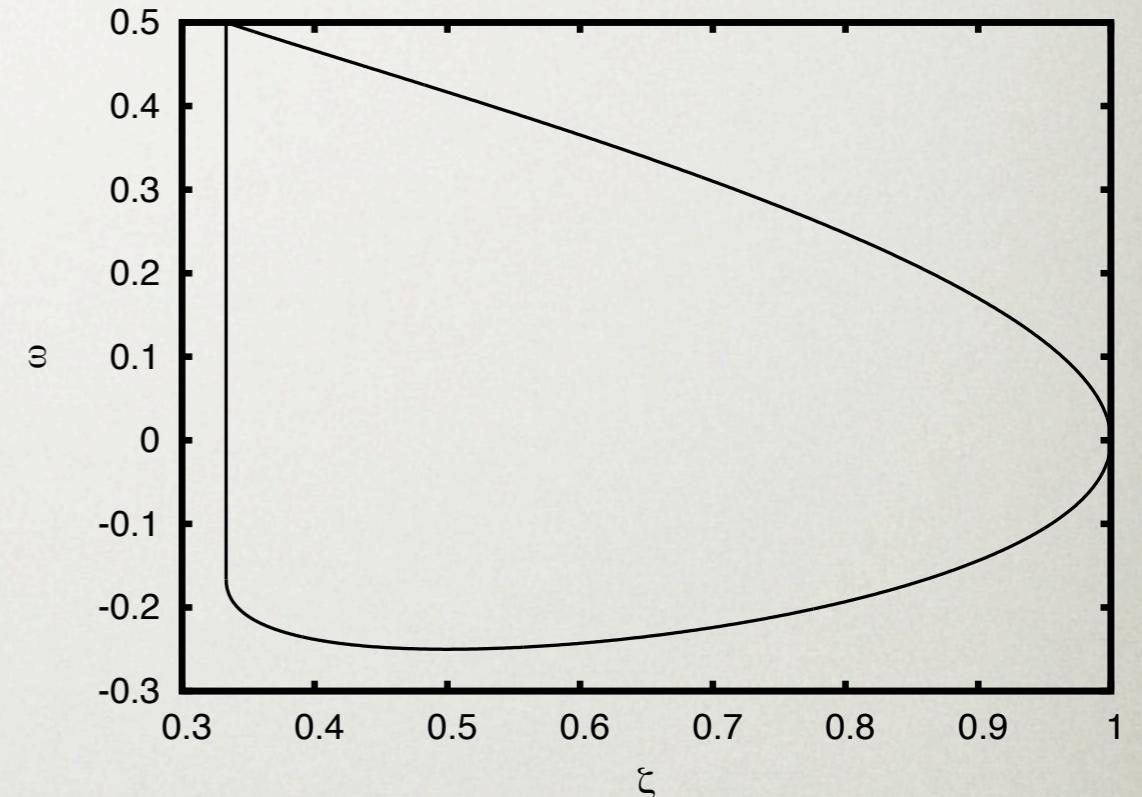
$$V^{(4)}(r, \tan \gamma, \zeta, \omega) = \frac{r^4}{(1 + \tan^2 \gamma)^2} [\lambda_1 + (\lambda_2 - \omega \lambda_5) \tan^2 \gamma + (\zeta \lambda_3 + \lambda_4) \tan^4 \gamma]$$

bounded-from-below conditions

$$\lambda_1 > 0, \quad \zeta \lambda_3 + \lambda_4 > 0, \quad \text{and} \quad \lambda_2 - \omega \lambda_5 + 2\sqrt{\lambda_1(\zeta \lambda_3 + \lambda_4)} > 0.$$

$$r \in [0, \infty), \quad \gamma \in \left[0, \frac{\pi}{2}\right]$$

$$\zeta \in \left[\frac{1}{3}, 1\right] \quad \text{and} \quad \omega \in \left[-\frac{1}{4}, \frac{1}{2}\right]$$



# Theoretical Constraints

---

- Maximum Ranges :

$$\lambda_1 \in \left(0, \frac{1}{3}\pi\right) \simeq (0, 1.05)$$

$$\lambda_4 \in \left(-\frac{1}{5}\pi, \frac{1}{2}\pi\right) \simeq (-0.628, 1.57)$$

$$\lambda_2 \in \left(-\frac{2}{3}\pi, \frac{2}{3}\pi\right) \simeq (-2.09, 2.09)$$

$$\lambda_5 \in \left(-\frac{8}{3}\pi, \frac{8}{3}\pi\right) \simeq (-8.38, 8.38)$$

$$\lambda_3 \in \left(-\frac{1}{2}\pi, \frac{3}{5}\pi\right) \simeq (-1.57, 1.88)$$

- Within these ranges the following conditions need to be satisfied :

$$\lambda_4 > \begin{cases} -\frac{1}{3}\lambda_3 & \text{for } \lambda_3 \geq 0, \\ -\lambda_3 & \text{for } \lambda_3 < 0, \end{cases}$$

$$\lambda_2 > \begin{cases} \frac{1}{2}\lambda_5 - 2\sqrt{\lambda_1 \left(\frac{1}{3}\lambda_3 + \lambda_4\right)} & \text{for } \lambda_5 \geq 0 \text{ and } \lambda_3 \geq 0, \\ \omega_+(\zeta)\lambda_5 - 2\sqrt{\lambda_1(\zeta\lambda_3 + \lambda_4)} & \text{for } \lambda_5 \geq 0 \text{ and } \lambda_3 < 0, \\ \omega_-(\zeta)\lambda_5 - 2\sqrt{\lambda_1(\zeta\lambda_3 + \lambda_4)} & \text{for } \lambda_5 < 0. \end{cases}$$

# Theoretical Constraints : Alternative Minima

$$\begin{aligned} V = & \frac{r^2}{(1 + \tan^2 \gamma)} \frac{1}{2} [\mu_2^2 + \mu_3^2 \tan^2 \gamma] \\ & + \frac{r^4}{(1 + \tan^2 \gamma)^2} [\lambda_1 + (\lambda_2 - \omega \lambda_5) \tan^2 \gamma + (\zeta \lambda_3 + \lambda_4) \tan^4 \gamma] \\ & + \frac{r^3}{(1 + \tan^2 \gamma)^{3/2}} \tan \gamma [-\sigma M_1 - \rho M_2 \tan^2 \gamma], \end{aligned}$$

$$\sigma \equiv \frac{\text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b)(UXU^\dagger)_{ab}}{\text{Tr}(\Phi^\dagger \Phi)[\text{Tr}(X^\dagger X)]^{1/2}},$$

$$\rho \equiv \frac{\text{Tr}(X^\dagger t^a X t^b)(UXU^\dagger)_{ab}}{[\text{Tr}(X^\dagger X)]^{3/2}}.$$

# Decoupling Behaviour

Case	$\mu_3 \equiv \sqrt{ \mu_3^2 }$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	$\lambda_5$	$M_1$	$M_2$
A	300–1000 GeV	derived	0.1	0.1	0.1	0.1	100 GeV	100 GeV
B	300–1000 GeV	derived	0.1	0.1	0.1	0.1	$\mu_3/3$	$\mu_3/3$

Quantity	Case A	Case B
$\frac{m_{H,3,5}}{\mu_3} - 1$	$\mu_3^{-2}$	$\mu_3^{-2}$
$v_\chi$	$\mu_3^{-2}$	$\mu_3^{-1}$
$\sin \alpha$	$\mu_3^{-2}$	$\mu_3^{-1}$
$\kappa_V - 1$	$\mu_3^{-4}$	$\mu_3^{-2}$
$\kappa_f - 1$	$\mu_3^{-4}$	$\mu_3^{-2}$
$g_{hhVV}/g_{hhVV}^{\text{SM}} - 1$	$\mu_3^{-4}$	$\mu_3^{-2}$
$g_{hhh}/g_{hhh}^{\text{SM}} - 1$	$\mu_3^{-4}$	$\mu_3^{-2}$
$\Delta\kappa_\gamma$	$\mu_3^{-2}$	$\mu_3^{-2}$
$\Delta\kappa_{Z\gamma}$	$\mu_3^{-2}$	$\mu_3^{-2}$