# NNLL soft and Coulomb resummation for squark and gluino production at the LHC


Christian Schwinn

— Univ. Freiburg —

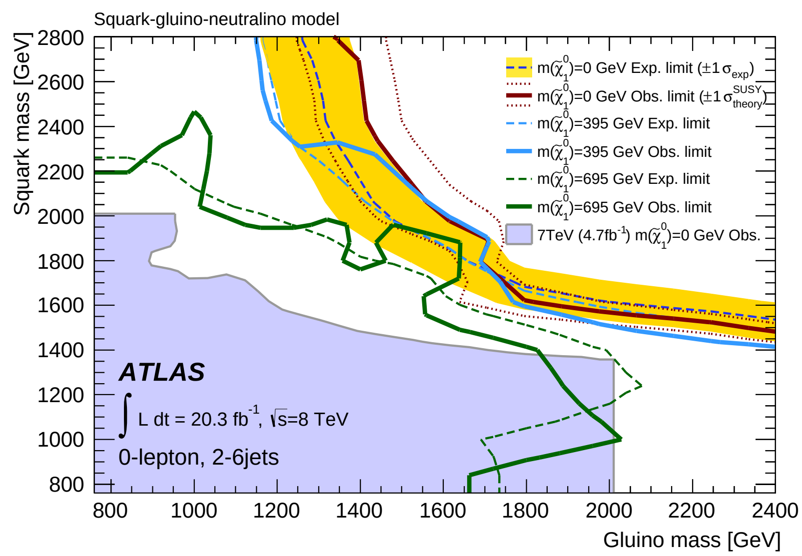

**22.7.2014**

P.Falgari, CS, C.Wever, arXiv:1202.2260 [hep-ph]
+ M.Beneke, J.Piclum, arXiv:1312.0837 [hep-ph] and in progress

## Coloured sparticle production

main search channels at LHC

$$pp \to \{\tilde{q}\bar{\tilde{q}}\,, \quad \tilde{q}\tilde{q}\,, \quad \tilde{q}\tilde{g}\,, \quad \tilde{g}\tilde{g}\,, \quad \tilde{t}_i\bar{\tilde{t}}_i\}$$




**No sign** of SUSY

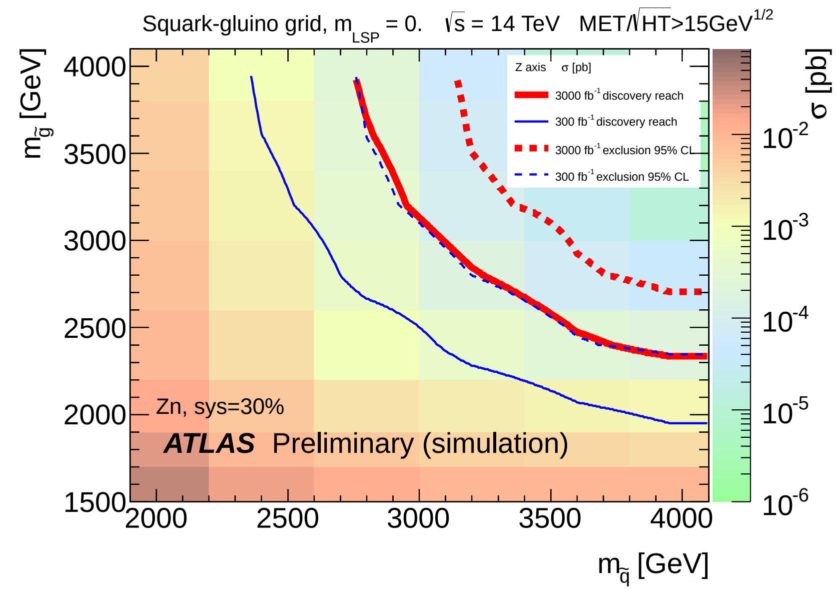
after LHC at $8\text{TeV}$

Limits: (ATLAS 14)

$$m_{\tilde{g}} \sim m_{\tilde{q}} \gtrsim 1.7\text{TeV}$$

$$m_{\tilde{g}} \gtrsim 1.3\text{TeV}$$

$$m_{\tilde{q}} \gtrsim 850\text{GeV}$$



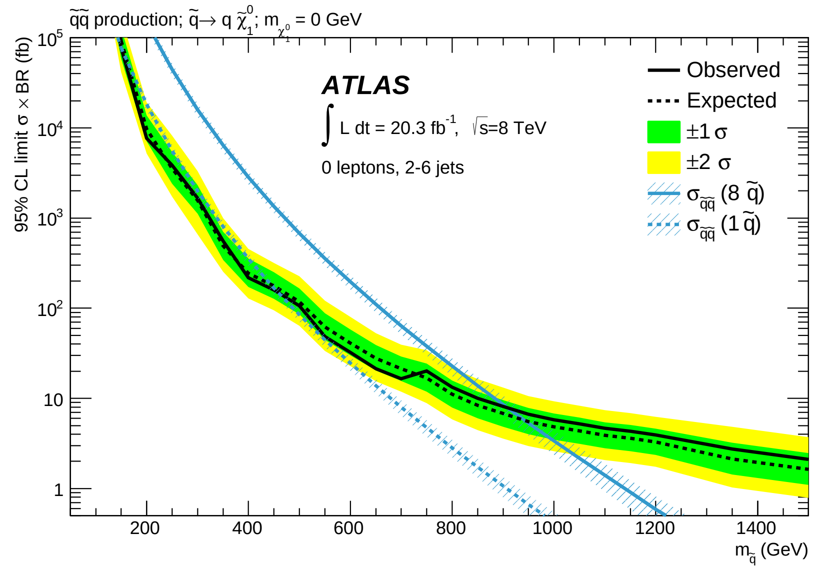

. . . search will continue

at $13 - 14\text{TeV}$

$\Rightarrow$ reach up to $4\text{TeV}$

**Precise knowledge of cross sections:**

- can help to distinguish models (if new particles observed)
- improve exclusion bounds (if no new particles observed)

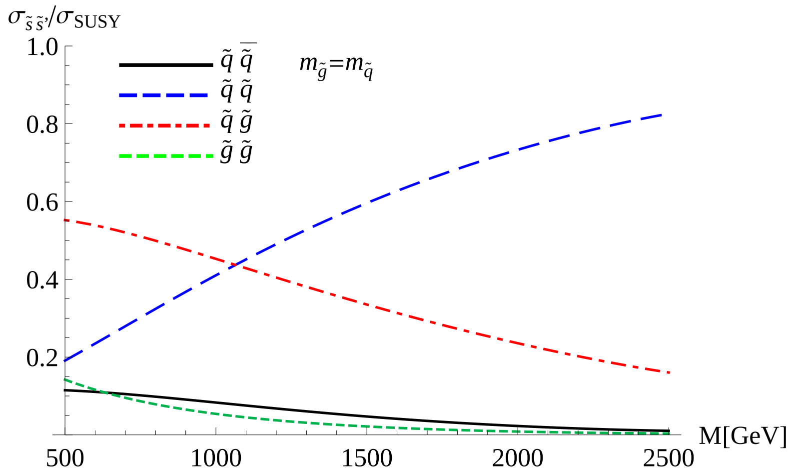


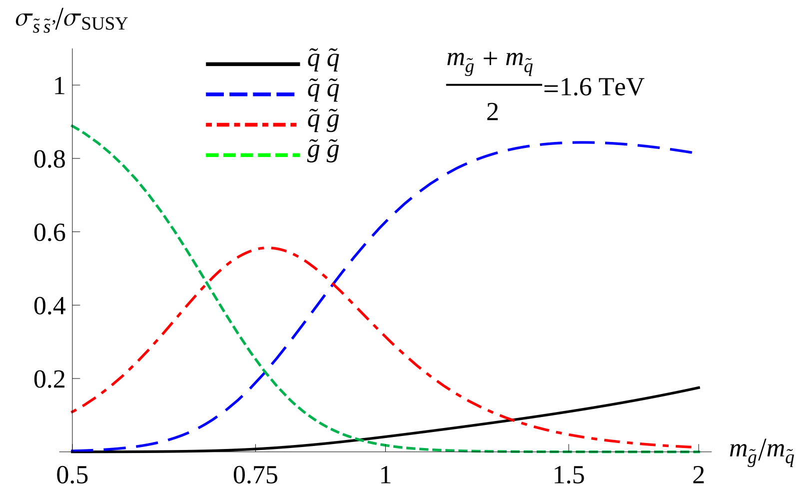

**Theory status:**

- NLO SUSY-QCD (Beenakker et al. 97, PROSPINO; Goncalves-Netto et al. 12, MADGOLEM, Parton-Shower matching: Gavin et al. 13)
- (N)NLL soft resummation (Beenakker et al. 09-13, Broggio et al. 13)
  NNLO$_{\text{approx}}$ for $\tilde{q}\bar{\tilde{q}}$, $\tilde{g}\tilde{g}$, $\tilde{t}\bar{\tilde{t}}$ (Langenfeld et al. 09–12, Broggio et al. 13)
- Bound state effects (Hagiwara/Yokoya 09, Kauth et al. 11; Kim et al. 14)
- NLO corrections to $\tilde{q}\tilde{q}$ production and decay (Hollik et al. 12)
- EW corrections (Bornhauser et al. 07; Germer/Hollik/Mirabella/Trenkel 08-11)

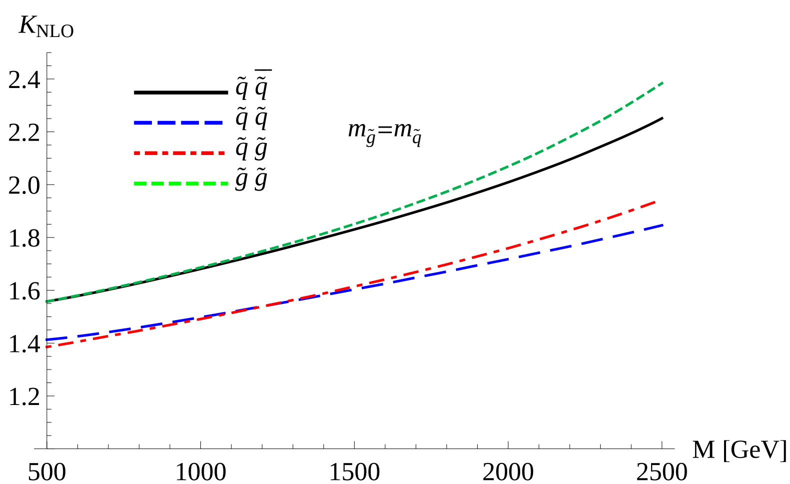
# Squark and gluino production

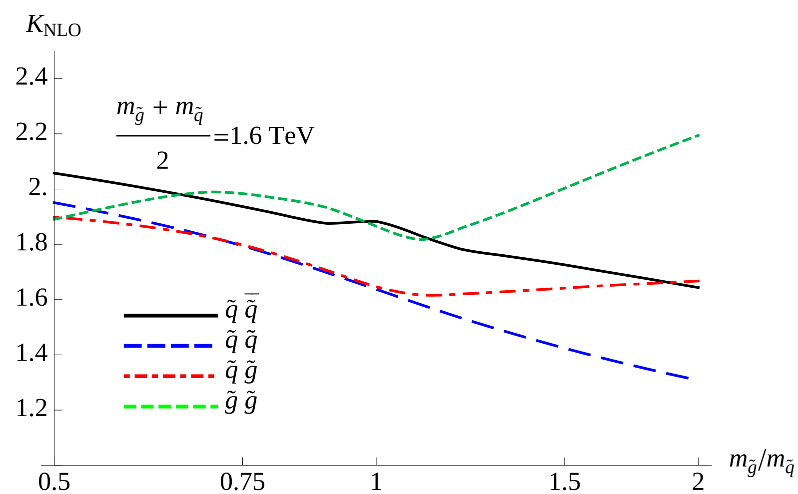
**Production processes** $pp' \to \tilde{s}\tilde{s}'$, $p, p' \in \{q, \bar{q}, g\}$ , $\tilde{s}, \tilde{s}' \in \{\tilde{q}, \bar{\tilde{q}}, \tilde{g}\}$




**NLO corrections** up to $> 100\%$ , scale uncertainty $\pm 20$–$30\%$



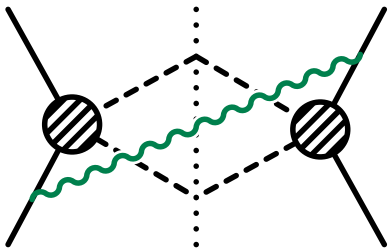



# Threshold resummation

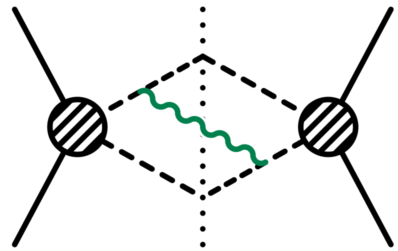


**Soft corrections:** Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et.al. 98, ...

$\beta = \sqrt{1 - \frac{4M^2}{\hat{s}}}$

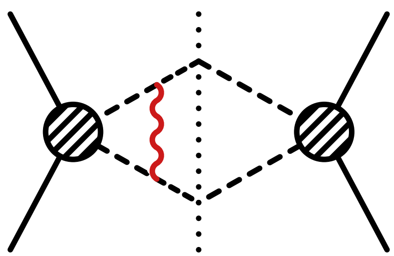


$\Rightarrow \alpha_s \; \log^2(8\beta^2)$



$\Rightarrow \alpha_s \; \log(8\beta^2)$

**Coulomb gluon corrections** (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,...)



$\Rightarrow \alpha_s \; \frac{1}{\beta}$

**Reorganization** of perturbative series:

$$\hat{\sigma}_{pp'} \quad \propto \quad \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \dots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \left\{1\,\text{(LL,NLL)}; \alpha_s, \beta\,\text{(NNLL)}; \dots\right\} :$$

**Combination of Coulomb- and soft effects?**

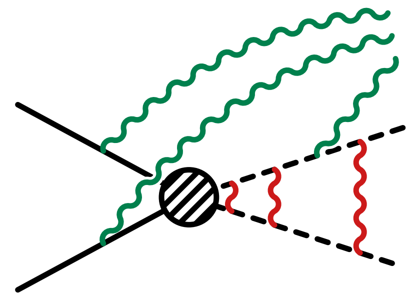
Heavy particles nonrelativistic near threshold:

$$E \sim M\beta^2\,, \qquad |\vec{p}| \sim M\beta$$

soft gluon momenta of same order:

$$q_s \sim M\beta^2 \sim E$$

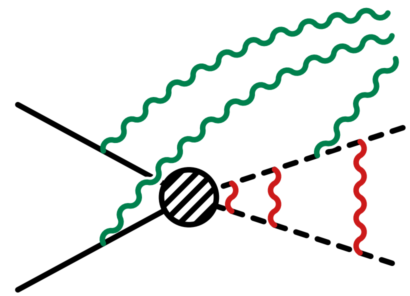
$\Rightarrow$ heavy particles “feel” soft radiation



**Combination of Coulomb- and soft effects?**

Heavy particles nonrelativistic near threshold:

$$E \sim M\beta^2 \,, \qquad |\vec{p}| \sim M\beta$$



soft gluon momenta of same order:

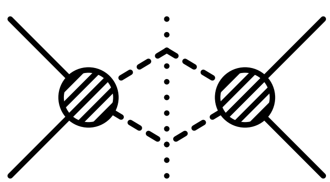
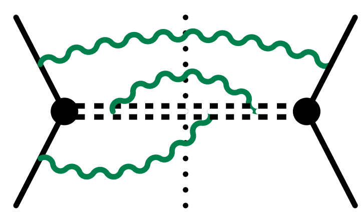
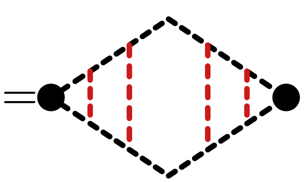
$$q_s \sim M\beta^2 \sim E$$

⇒ heavy particles “feel” soft radiation

**Factorization** of cross section (Beneke, Falgari, CS 09/10)

$$\Rightarrow \hat{\sigma}_{pp' \to \tilde{s}\tilde{s}'}|_{\hat{s} \to 4M^2} = \sum_{i,i'} H_{ii'}(M,\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(M\beta^2 - \frac{\omega}{2}) W_{ii'}^{R_\alpha}(\omega,\mu)$$

Hard, soft and Coulomb functions:

$H_i =$  , $W_i{}^R =$  , $J^R =$ 

Factorization scale dependence of $H$, $W$ cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu} \Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH_i}{d\mu} \Rightarrow$ IR singularities (2-loop: Becher, Neubert; Ferroglia et.al. 09)

$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

Factorization scale dependence of $H$, $W$ cancels against PDFs:

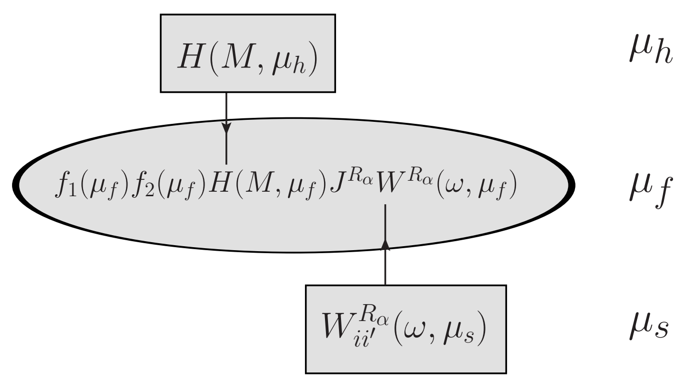
$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu} \Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH_i}{d\mu} \Rightarrow$ IR singularities (2-loop: Becher, Neubert; Ferroglia et.al. 09)

$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

**Recipe for Resummation:**

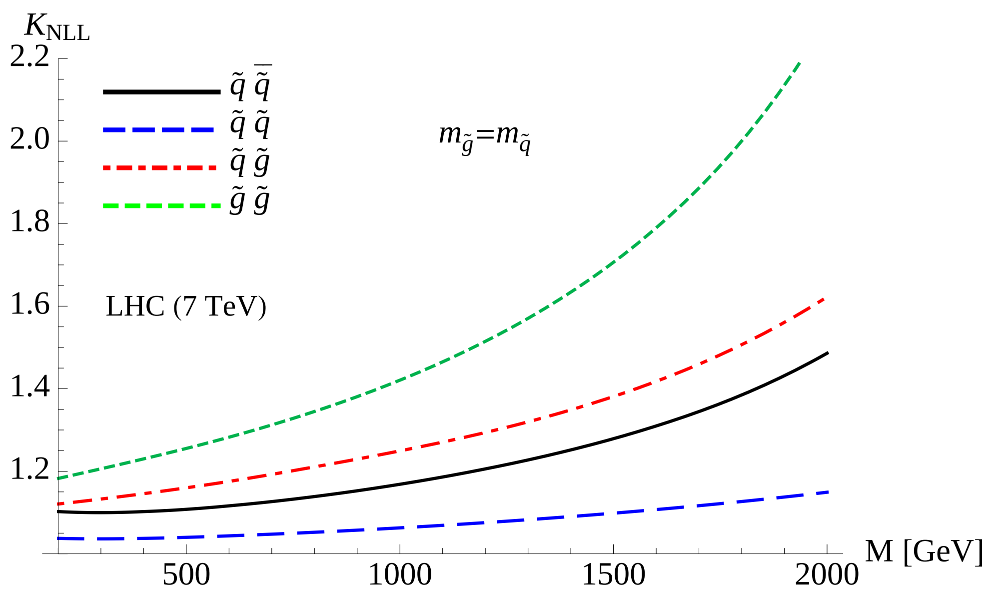
- Momentum-space solution to RGE
  ( Becher, Neubert, Pecjak 07)




- evolve hard function from $\mu_h \sim 2M$ to $\mu_f$
- evolve soft function from $\mu_s \sim M\beta^2$ to $\mu_f$

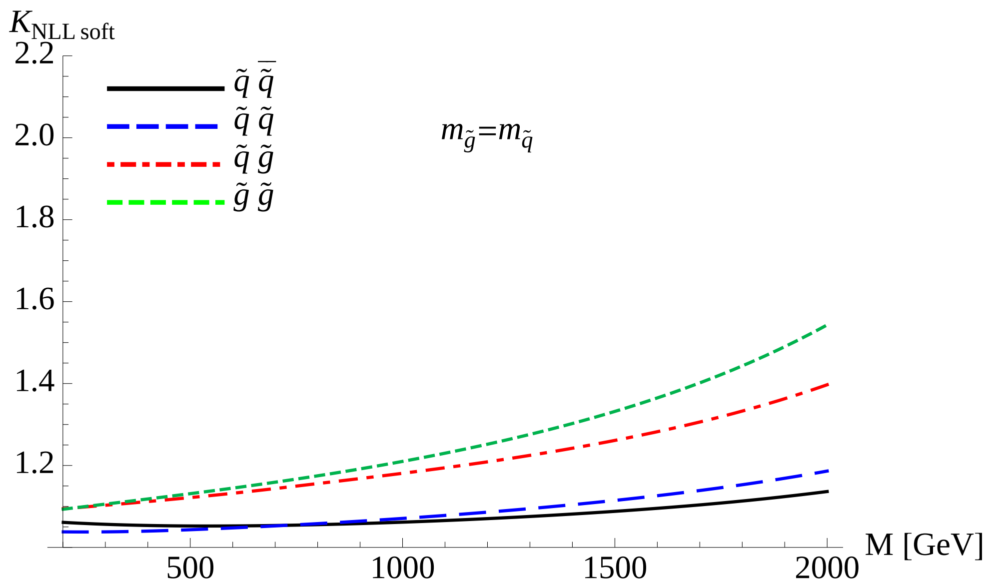
**NLL soft/Coulomb resummation** (Falgari/CS/Wever 12)

- Large corrections depending on process:
  $\Delta K_{\text{NLL}} = \frac{\Delta\sigma_{\text{NLL}}}{\sigma_{\text{NLO}}} \sim 10\text{–}120\%$
- Coulomb effects can be large
- interpolations provided for LHC7/8 included with `arXiv:1202.2260`



**NLL soft/Coulomb resummation** (Falgari/CS/Wever 12)

- Large corrections depending on process:
  $\Delta K_{\text{NLL}} = \frac{\Delta\sigma_{\text{NLL}}}{\sigma_{\text{NLO}}} \sim 10\text{–}120\%$
- Coulomb effects can be large
- interpolations provided for LHC7/8 included with `arXiv:1202.2260`



**Ingredients at NNLL**

- One-loop hard function $H$

  (Beenakker et al. 11/13; Kauth et al. 11; Langenfeld et al. 12),

- one-loop soft function $W$ (Beneke/Falgari/CS 09)

- Coulomb Green function $J$ with higher-order potentials

  (extension of $t\bar{t}$ results from Beneke/Signer/Smirnov 98)

  (In progress: additional $\alpha_s^2 \log\beta$ for $\tilde{g}\tilde{g}$ and $\tilde{g}\tilde{q}$ $\Rightarrow$ $t\bar{t}$: Bärnreuther/Czakon/Fiedler 13)

## Ingredients at NNLL

- One-loop hard function $H$

(Beenakker et al. 11/13; Kauth et al. 11; Langenfeld et al. 12),

- one-loop soft function $W$ (Beneke/Falgari/CS 09)
- Coulomb Green function $J$ with higher-order potentials

(extension of $t\bar{t}$ results from Beneke/Signer/Smirnov 98)

(In progress: additional $\alpha_s^2 \log\beta$ for $\tilde{g}\tilde{g}$ and $\tilde{g}\tilde{q}$ $\Rightarrow$ $t\bar{t}$: Bärnreuther/Czakon/Fiedler 13)

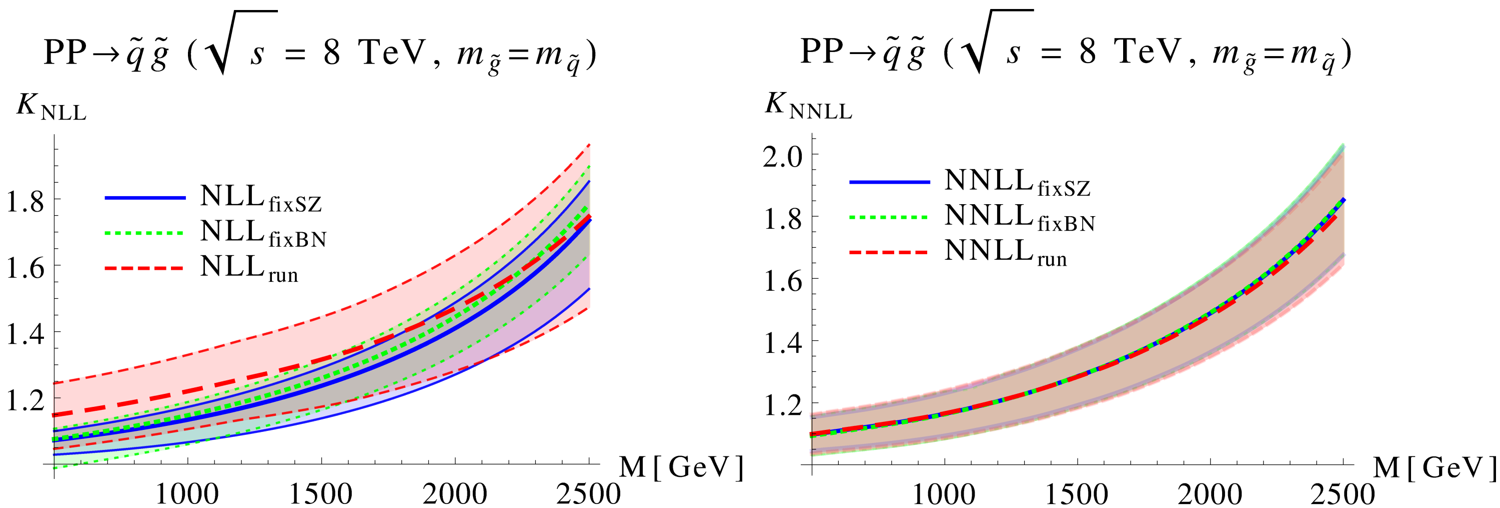
## Matching to NLO+NNLO$_{\text{approx}}$

$$\hat{\sigma}^{\text{NNLL}}_{pp'\,\text{matched}}(\hat{s}) = \left[\hat{\sigma}^{\text{NNLL}}_{pp'}(\hat{s}) - \hat{\sigma}^{\text{NNLL}(2)}_{pp'}(\hat{s})\right] + \hat{\sigma}^{\text{NLO}}_{pp'}(\hat{s}) + \hat{\sigma}^{\text{NNLO}}_{\text{app},pp'}(\hat{s})\,.$$

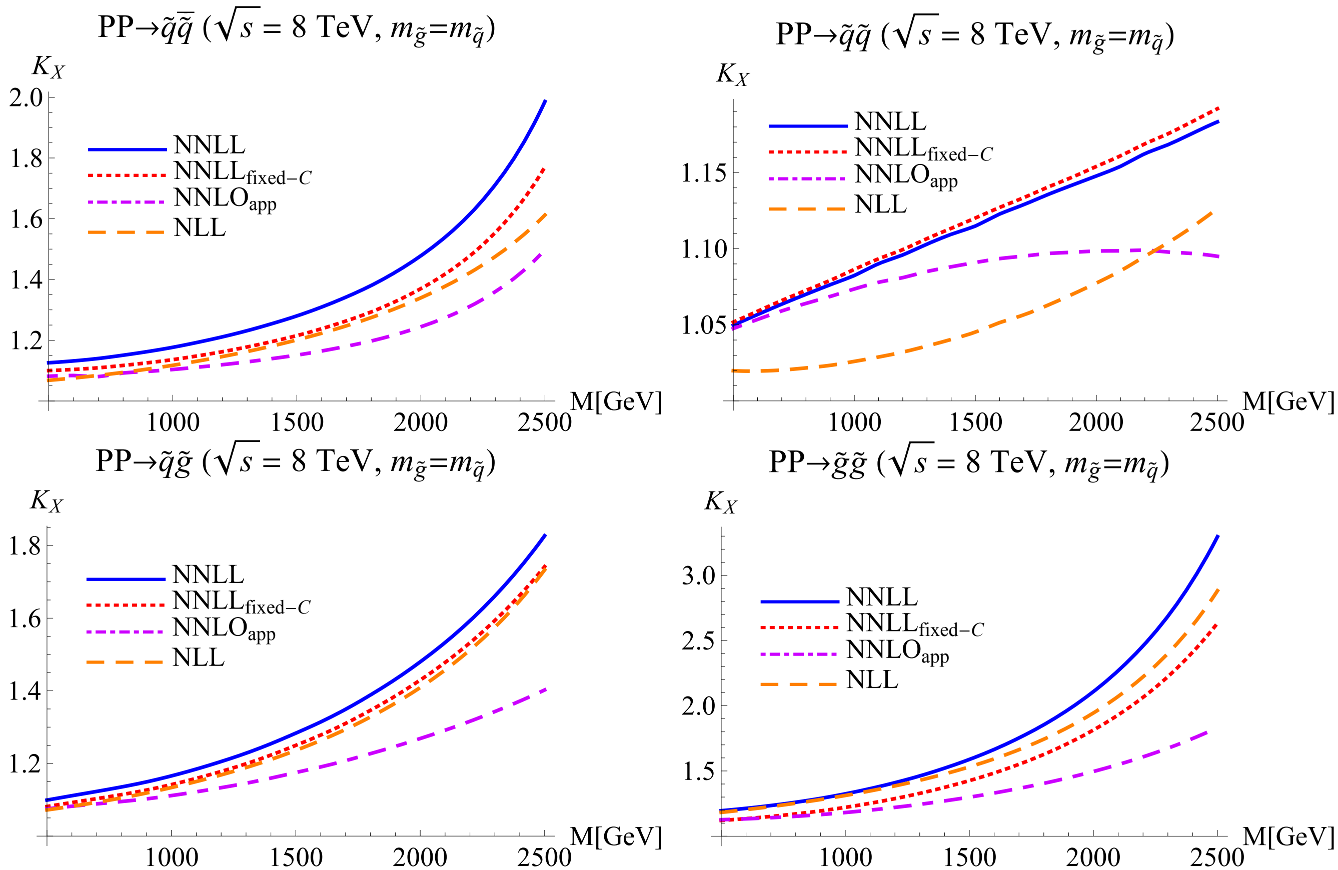
- $\hat{\sigma}^{\text{NNLL}(2)}$: NNLO expansion of NNLL,
- $\hat{\sigma}^{\text{NLO}}$: NLO cross section from Prospino,
- $\hat{\sigma}^{\text{NNLO}}_{\text{app}}$: Threshold enhanced NNLO terms (Beneke/Czakon/Falgari/Mitov/CS 09)

**Choice of soft scale**

- Scale choice $\mu_s \sim M\beta^2$ yields all $\log\beta$ terms; not viable for $\beta \to 0$
- **Fixed** scale $\mu_s$:
  - RGE approach: minimize $\Delta\sigma^{\mathrm{NLO}}_{\mathrm{soft}}$ (Becher, Neubert, Xu 07)
  - $\mu_s$ from logarithmic derivative of parton luminosity
    $\Rightarrow$ **default choice** (relation to Mellin-space resummation: Sterman/Zeng 13)
- **Running scale** frozen at $\beta_{\text{cut}}$ (Beneke/Falgari/Klein/CS 11)
- Small dependence on prescription at NNLL



## NNLL soft/Coulomb resummation (Beneke/Falgari/Piclum/CS/Wever)




- NNLL corrections $10-30\%$ relative to NLL
- Coulomb resummation effects significant
- non-negligible corrections beyond NNLO

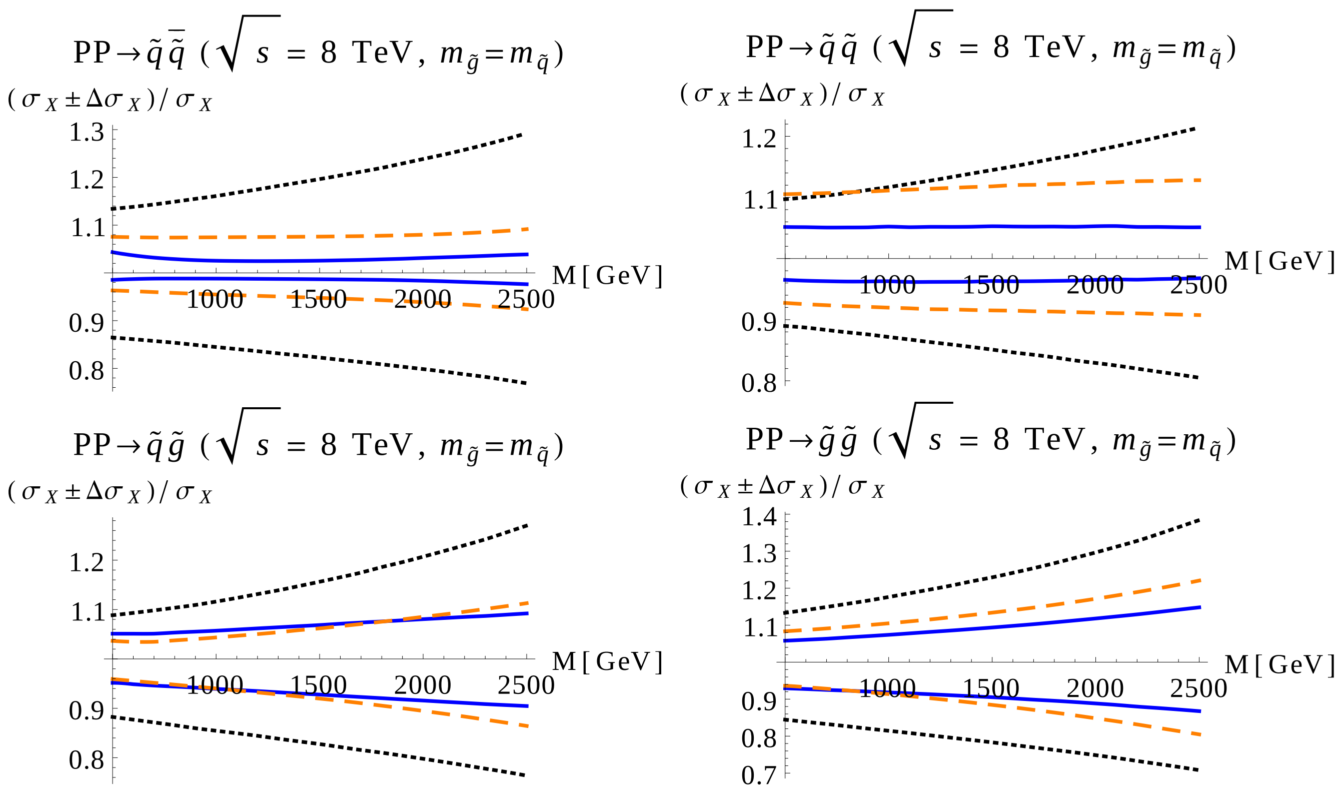
$K_X = \frac{\sigma_X}{\sigma_{\text{NLO}}}$

NLO/NLL: MSTW2008NLO;

NNLO/NNLL: MSTW2008NNLO

## NNLL soft/Coulomb resummation

(Beneke/Falgari/Piclum/CS/Wever)




- Reduced scale dependence:

  $\pm 20$–$30\%$ (NLO)

  $\Rightarrow \pm 10$–$20\%$ (NLL)

  $\Rightarrow \pm 5$–$15\%$ (NNLL)

$$K_X = \frac{\sigma_X}{\sigma_{\text{NLO}}}$$

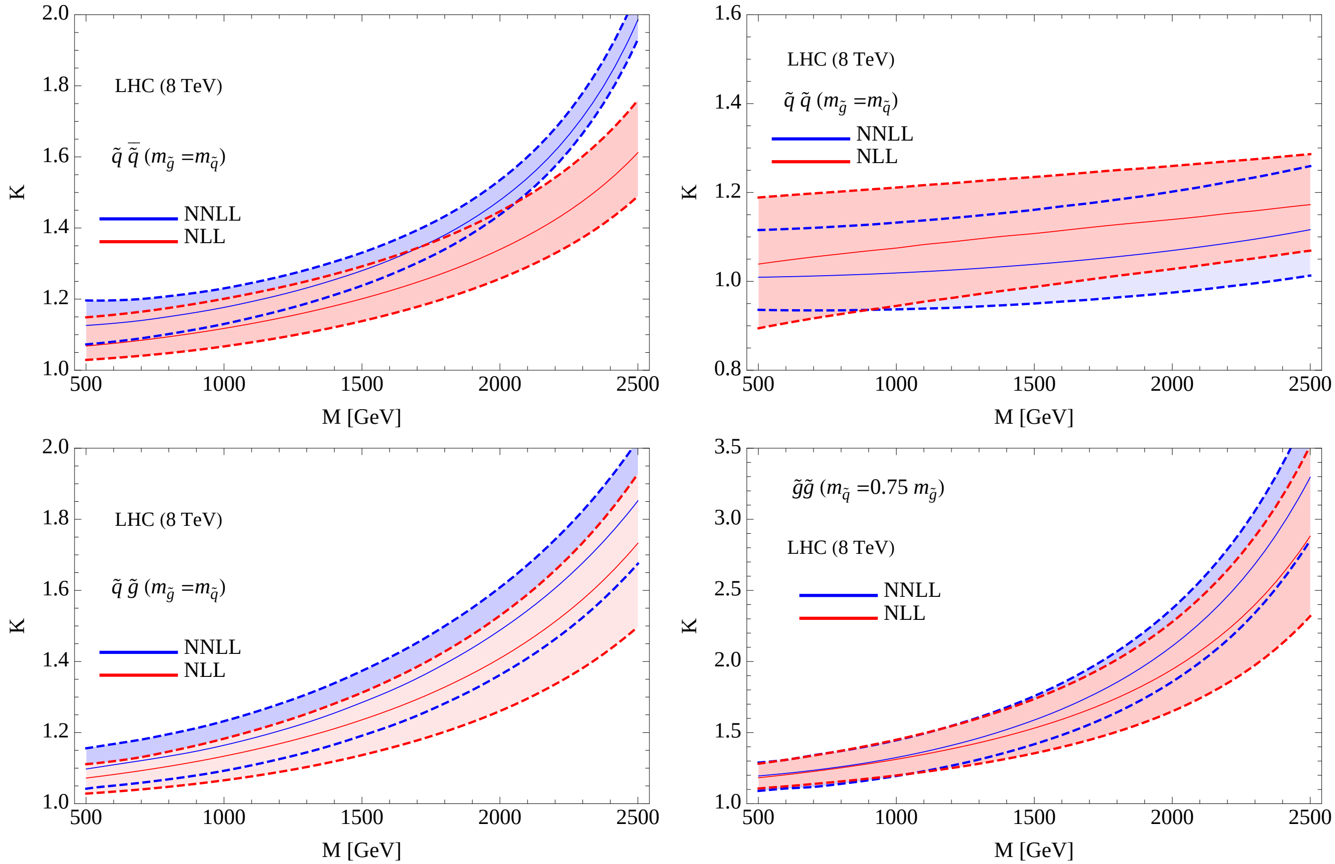
NLO/NLL: MSTW2008NLO;

NNLO/NNLL: MSTW2008NNLO

# Squark and gluino production at NNLL

## NNLL soft/Coulomb resummation

(Beneke/Falgari/Piclum/CS/Wever)




- Combined soft/Coulomb resummation improves concergence

$$K_X = \frac{\sigma_X}{\sigma_{\mathrm{NLO}}}$$

NLO/NLL: MSTW2008NLO;

NNLO/NNLL: MSTW2008NNLO

**Threshold corrections** $\sim \log^n \beta$, $\frac{1}{\beta^n}$

- Factorization of soft and Coulomb corrections
- $\log \beta$ resummation from momentum space solution to RGEs
- combined Soft and Coulomb resummation possible

**NNLL resummation** for squark and gluino production

- Corrections from $10 - 30\%$ ($\tilde{q}\tilde{q}$) to $30 - 200\%$ ($\tilde{g}\tilde{g}$)
- Coulomb corrections can be sizable
- Uncertainties reduced to $\pm 5$–$15\%$
- (not discussed: stop pair production at NLL, finite width effects.)

**Outlook:**

- Results for $13/14\mathrm{TeV}$; public program in preparation
- Future plans:
  stop production, non-degenerate squark masses

# Bonus slides

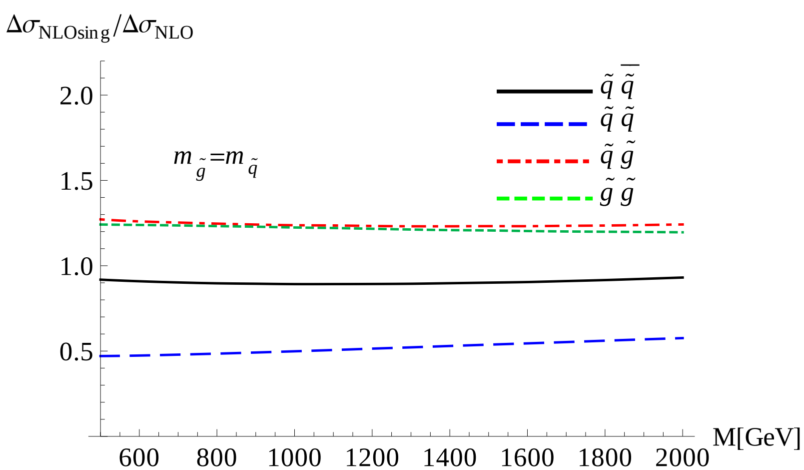
**Universal limit** $\beta = \sqrt{1 - \frac{4M^2}{\hat{s}}} \to 0$ (Beenakker et al. 97 , Beneke, Falgari, CS 09)

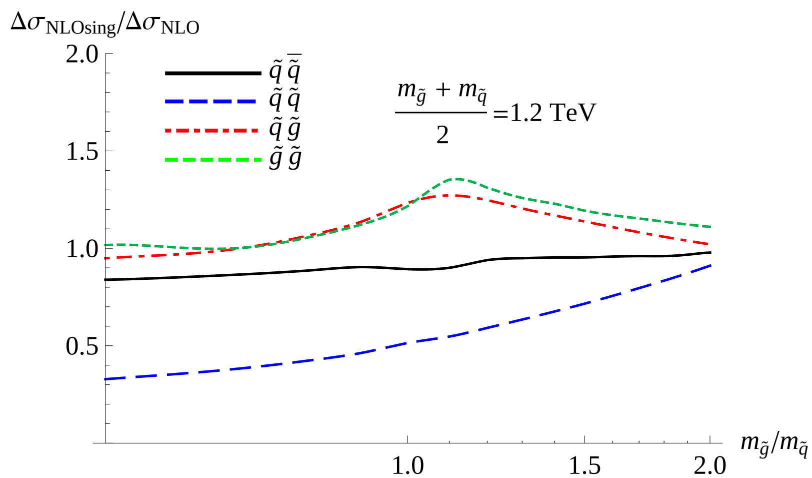
$$\sigma_{\text{NLO,app}} = \sigma^{(0)} \frac{\alpha_s}{(4\pi)} \left\{ -\frac{2\pi^2 D_{R_\alpha}}{\beta} \sqrt{\frac{2m_r}{M}} + 4(C_r + C_{r'}) \ln^2 \left( \frac{8M\beta^2}{\mu_f} \right) \right.$$
$$\left. -4(C_{R_\alpha} + 4(C_r + C_{r'})) \ln \left( \frac{8M\beta^2}{\mu_f} \right) \right\}$$

(Average and reduced mass: $M = (m_s + m_{s'})/2$, $m_r = m_s m_{s'}/(m_s + m_{s'})$

$C_r$: quadratic $SU(3)$ Casimir for rep. $r$ Coulomb correction: $D_{R_\alpha} = \frac{1}{2}(C_{R_\alpha} - C_R - C_{R'})$ )

**Accuracy of threshold approximation**: (NLO:Prospino, Plehn et al.)



**Potential function** related to

Coulomb Green function: (Fadin, Khoze 87; Peskin, Strassler 90,...)

$$J_R(E) = 2\mathrm{Im}\, G_C^R(0,0;E) = \begin{cases} \frac{M^2 \pi D_R \alpha_s}{2\pi} \left( e^{\pi D_R \alpha_s \sqrt{\frac{M}{E}}} - 1 \right)^{-1} & E > 0 \\ \sum_{n=1}^{\infty} \delta(E - E_n) 2R_n & E < 0 \end{cases}$$

Bound-state poles at

$$E_n = -\frac{\alpha_s^2 D_R^2 M}{4n^2}$$

Smeared out by finite decay with

$$E \to E + \mathrm{i}\Gamma$$

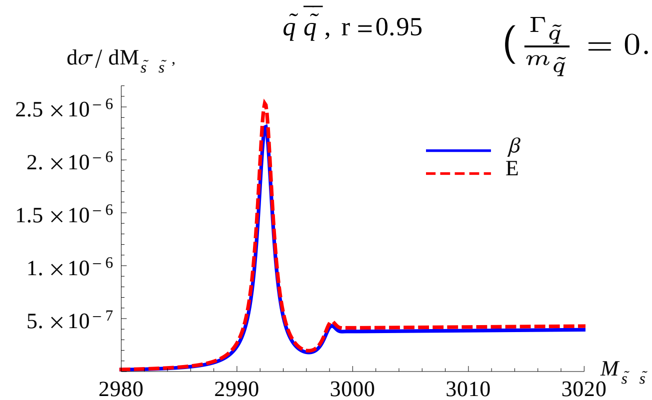
Default:

include bound-states with $\Gamma = 0$

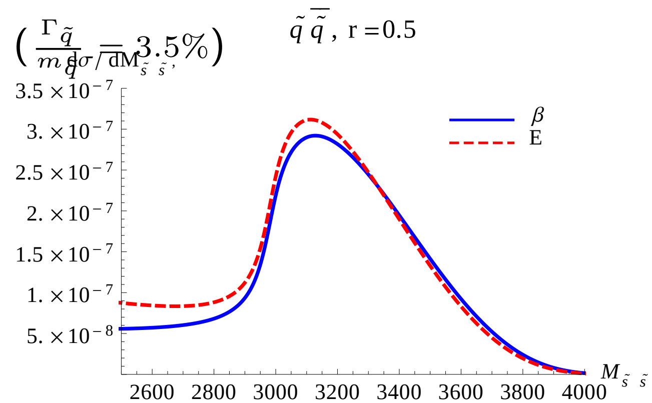
**Finite-width effects** negligible

for $\Gamma/M \lesssim 5\%$ (Falgari, CS, Wever 12)

$\left(\frac{\Gamma_{\tilde{q}}}{m_{\tilde{q}}} = 0.05\%\right)$




$\left(\frac{\Gamma_{\tilde{q}}}{m_{\tilde{q}}} = 3.5\%\right)$



# Input for NNLL resummation

**One-loop hard functions** $h_i^{(1)}(\mu_f)$ e.g. from threshold expansion
(Beenakker et al. 11/13; Kauth et al. 11; Langenfeld et al. 12)

$$\sigma_{\text{NLO,app}} = \sigma^{(0)} \frac{\alpha_s}{(4\pi)} \left\{ -\frac{2\pi^2 D_{R\alpha}}{\beta} \sqrt{\frac{2m_r}{M}} + 4(C_r + C_{r'}) \left[ \ln^2 \left( \frac{8M\beta^2}{\mu_f} \right) + 8 - \frac{11\pi^2}{24} \right] \right.$$
$$\left. - 4\left(C_{R_\alpha} + 4\left(C_r + C_{r'}\right)\right) \ln \left( \frac{8M\beta^2}{\mu_f} \right) + 12 C_{R_\alpha} + h_i^{(1)}(\mu_f) \right\}$$

**One-loop soft function** $(s = 1/(e^{\gamma_E} \mu e^{\rho/2}))$ (Beneke/Falgari/CS 09)

$$\tilde{s}_i^R(\rho, \mu) = \int_{0_-}^{\infty} d\omega e^{-s\omega}\, \overline{W}_i^{R_\alpha}(\omega, \mu) = 1 + \frac{\alpha_s}{4\pi} \left[ (C_r + C_{r'}) \left( \rho^2 + \frac{\pi^2}{6} \right) - 2C_R\,(\rho - 2) \right] + \mathcal{O}(\alpha_s^2)$$

**Coulomb Green function** (Beneke/Signer/Smirnov 98)

with insertion of NLO potentials

$$\delta\tilde{V}(\boldsymbol{p}, \boldsymbol{q}) = \frac{4\pi D_R \alpha_s(\mu_C)}{\boldsymbol{q}^2} \left[ \frac{\alpha_s(\mu_C)}{4\pi} \left( a_1 - \beta_0 \ln \frac{\boldsymbol{q}^2}{\mu_C^2} + \frac{\pi^2 |\boldsymbol{q}|}{m_r} \left( \frac{D_R}{2} \frac{2m_r}{M} + C_A \right) \right) + \frac{\boldsymbol{p}^2}{m_1 m_2} + \frac{\boldsymbol{q}^2}{4m_r^2} v_{\text{spin}} \right],$$

$$v_{\text{spin}} = -\frac{2m_r}{4M}\,\text{(s-s)}, \quad \frac{2m_r - M}{2M}\,\text{(f-f-triplet)}, \quad -\frac{2m_r + 3M}{6M}\,\text{(f-f-singlet)}, \quad -\frac{2m_r}{4M} - \frac{4m_r^2}{8m_f^2}\,\text{(s-f)}$$

**Resummed cross section** in momentum space

$$\hat{\sigma}^{\text{res}}_{pp'}(\hat{s},\mu_f) = \sum_i H_i(\mu_h)\, U_i \left(\frac{2M}{\mu_s}\right)^{-2\eta} \tilde{s}_i^{R_\alpha}(\partial_\eta,\mu_s)\,\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)} \int_0^\infty d\omega\, \frac{J^S_{R_\alpha}(E-\frac{\omega}{2})}{\omega}\left(\frac{\omega}{\mu_s}\right)^{2\eta}$$

- Resummation functions depend on scales $\mu_s, \mu_f, \mu_h$:

$$U_i = e^{-\frac{\alpha_s \Gamma_{\text{cusp}}}{2\pi}\left(\log^2(\frac{\mu_s}{\mu_f}) - \log^2(\frac{\mu_h}{\mu_f}) - \log(\frac{4M^2}{\mu_f^2})\log(\frac{\mu_s}{\mu_h})\right)+\dots}, \eta = \frac{\alpha_s\Gamma_{\text{cusp}}}{2\pi}\log(\frac{\mu_s}{\mu_f}) + \dots$$

- fixed-order soft function generates $\log(E/\mu_s)$-terms

$$\tilde{s}_i^{R_\alpha}(\partial_\eta,\mu_s)\left(\frac{\omega}{\mu_s}\right)^{2\eta} = \left(\frac{\omega}{\mu_s}\right)^{2\eta}\tilde{s}_i^{R_\alpha}\left(2\ln\left(\frac{\omega}{\mu_s}\right)+\partial_\eta,\mu_s\right)$$

- Expansion in $\alpha_s$ generates all logs in $\hat{\sigma}$ for $\mu_s \sim M\beta^2$
- **RGE approach:** fixed $\mu_s$ that minimizes soft corrections to hadronic $\sigma$ (Becher, Neubert, Xu 07)
- Running scale frozen at $\beta_{\text{cut}}$ (Beneke/Falgari/Klein/CS 11)

$$\mu_s = 2M\max\{\beta^2, \beta^2_{\text{cut}}\}$$

Hadronic cross section:

$$\sigma_{N_1 N_2 \to \tilde{s}\tilde{s}' X}(s) = \int_{\tau_0 = 4M^2/s}^{1} d\tau \sum_{p,p'=q,\bar{q},g} L_{pp'}(\tau, \mu_f)\hat{\sigma}_{pp'}(\tau s, \mu_f)\,,$$

with parton luminosity:

$$L_{pp'}(\tau, \mu) = \int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau) f_{p/N_1}(x_1, \mu) f_{p'/N_2}(x_2, \mu)\,.$$

Single-power approximation

$$L_{pp'}(\tau, \mu) = L_{pp'}(\tau_0, \mu) \left(\frac{\tau_0}{\tau}\right)^{s_{1,pp'}(\tau,\mu)}$$

with $\quad s^{(1)}_{pp'}(\tau_0, \mu) = -\dfrac{d \ln L_{pp'}(\tau, \mu)}{d \ln \tau}\Big|_{\tau=\tau_0}.$

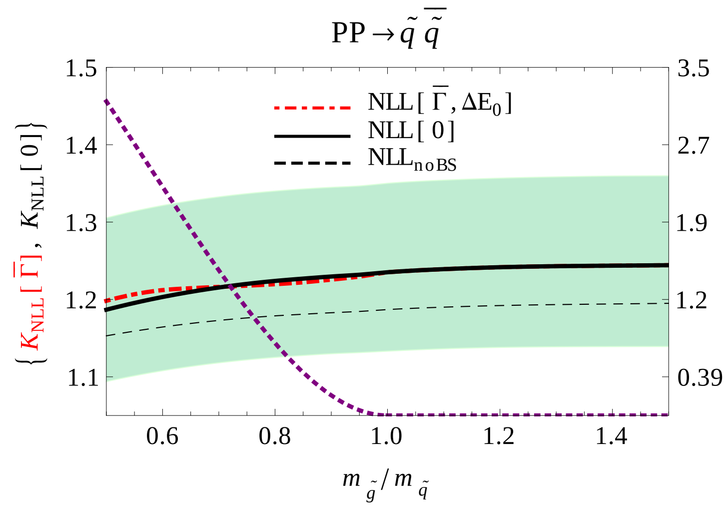
For single-power approximation, soft scale choice (Sterman/Zeng 2013)

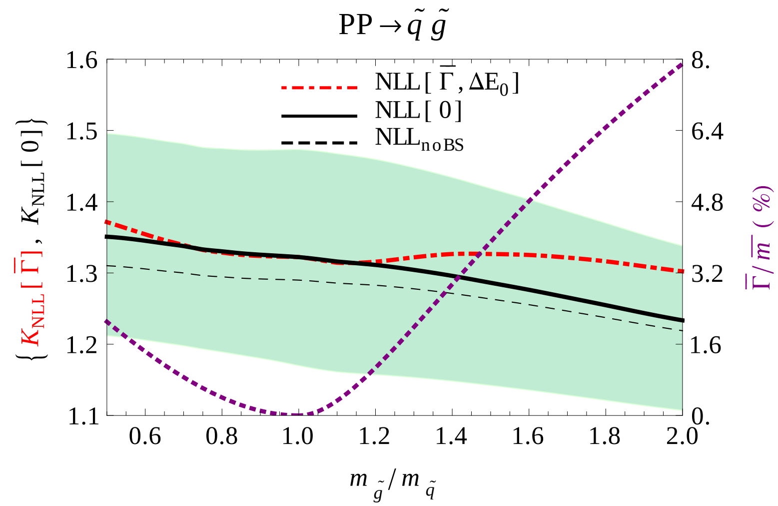
$$\mu_s = \frac{2Me^{-\gamma_E}}{s^{(1)}_{pp'}}$$

is equivalent to Mellin-space formalism.

**Total cross sections**, LO-SQCD decays as example:

$$\Gamma_{\tilde{q}\to q\tilde{g}} = \frac{\alpha_s C_F m_{\tilde{q}}}{2}(1-(\frac{m_{\tilde{g}}}{m_{\tilde{q}}})^2)^2\,, \qquad m_{\tilde{q}}>m_{\tilde{g}}, \qquad\qquad \Gamma_{\tilde{g}\to q\bar{\tilde{q}}} = \frac{\alpha_s n_f m_{\tilde{g}}}{2}(1-(\frac{m_{\tilde{g}}}{m_{\tilde{q}}})^{-2})^2\,, \qquad m_{\tilde{q}}<m_{\tilde{g}}.$$

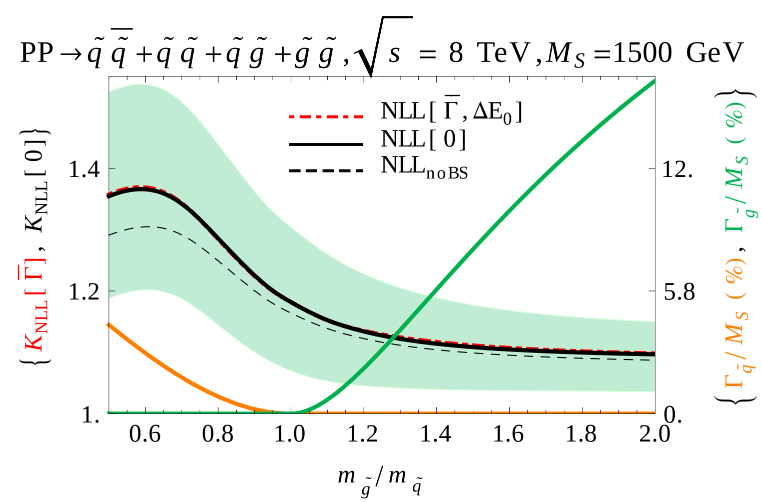



**Negligible effect**

on total SUSY production rate

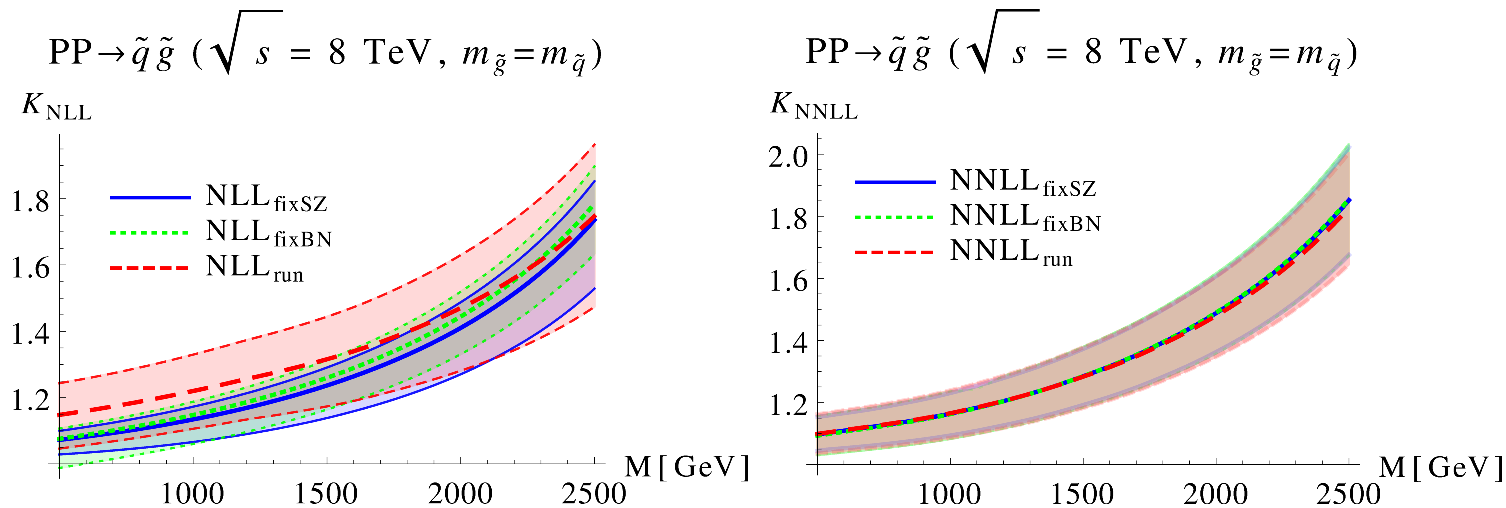
but relevant bound-state corrections



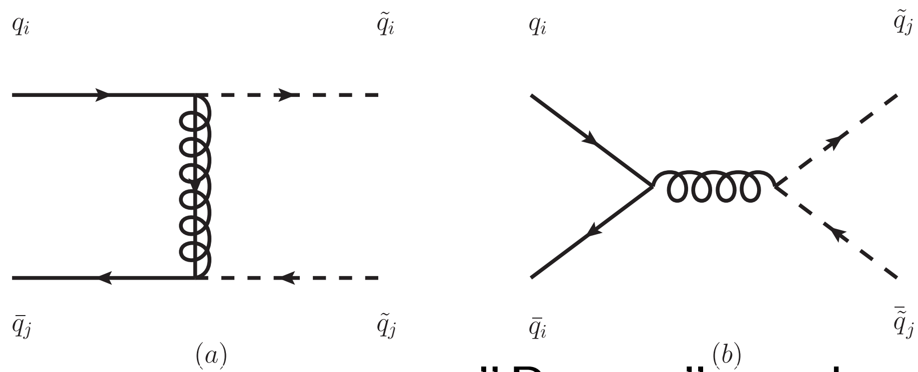
**Ambiguities in resummation**

- variation of $\mu_f, \mu_h, \mu_C$ by $0.5, \ldots 2$,
  ambiguity $E = M\beta^2 \Leftrightarrow \sqrt{\hat{s}} - 2M$,
  estimate NNLO constant.
- Running soft scale: variation of $\beta_{\text{cut}}$, envelope of several approximations (matching to NLO/NNLO, approximate NNLO/N$^3$LO)
- Fixed soft scale: variation of $\mu_s$

Resummation ambiguities reduced at NNLL



Difference to light-flavour squarks for $q\bar{q}$ initial state:

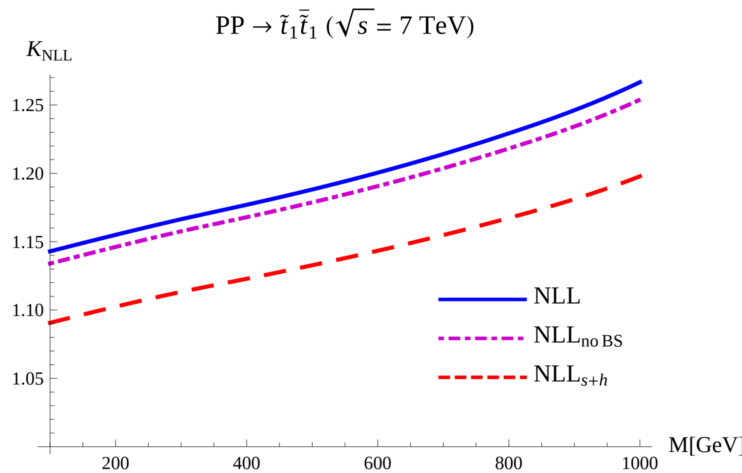



$\Rightarrow$ "P-wave" production, $\sim \beta^3$

absent for stops

Resummation formalism works also for $q\bar{q}$ channel (Falgari, CS, Wever 12)

use Coulomb Green function for $P$-waves (Bigi/Fadin/Khoze 92)



**All threshold enhanced $\mathcal{O}(\alpha_s^2)$ terms** (Beneke, Czakon, Falgari, Mitov, CS 09)

**Pure soft** corrections: (also Moch/Uwer+Langenfeld (08/09))

$$\Delta\sigma_{\mathrm{s}}^{(2)} \sim \alpha_s^2 (c_{\mathrm{LL}}^{(2)} \ln^4\beta + c_{\mathrm{NLL}}^{(2)} \ln^3\beta + c_{\mathrm{NNLL,2}}^{(2)} \ln^2\beta + \underbrace{c_{\mathrm{NNLL,1}}^{(2)} \ln\beta}_{\text{2-loop } \gamma_{H,s}})$$

**Potential** corrections: 2nd Coulomb, NLO potentials

$$\Delta\sigma_{\mathrm{p}}^{(2)} \sim \alpha_s^2 \left(\frac{c_{C^2}}{\beta^2} + \frac{1}{\beta}(c_{\mathrm{C,0}}^{(2)} + c_{\mathrm{C,1}}^{(2)} \log\beta) + \underbrace{c_{\text{n-C}}^{(2)} \ln\beta}_{\text{spin-dependent}}\right)$$

(using Beneke, Signer, Smirnov 99, Czarnecki/Melnikov 97/01)

**mixed Coulomb/soft/hard** corrections:

$$\Delta\sigma_{\mathrm{p}\otimes\,\mathrm{sh}}^{(2)} \sim \frac{\alpha_s}{\beta}\alpha_s (c_{\mathrm{LL}}^{(1)} \ln\beta^2 + c_{\mathrm{NLL}}^{(1)} \ln\beta + c + H^{(1)})$$

$$\Delta\sigma_{\mathrm{s}\otimes\mathrm{h}}^{(2)} \sim \alpha_s^2 H^{(1)} (c_{\mathrm{LL}}^{(1)} \ln\beta^2 + c_{\mathrm{NLL}}^{(1)} \ln\beta)$$

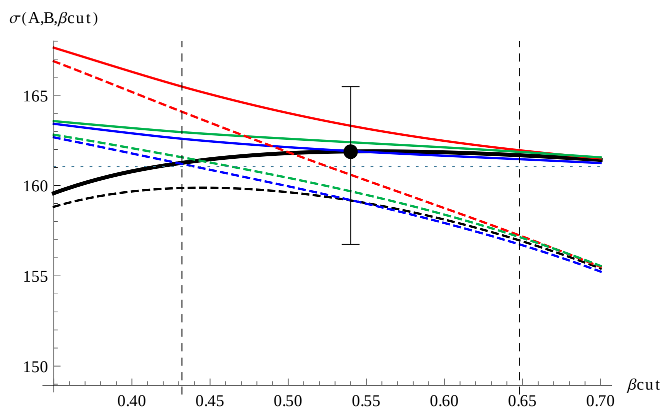
($H_1$: process and colour-channel dependent, $t\bar{t}$: Czakon/Mitov 09 )

**Determination of $\beta_{\text{cut}}$**

- allow for different implementations

  $\beta < \beta_{\text{cut}}$ : NNLL ($\mu_s = k_s M \beta^2_{\text{cut}}$) with/without constant at $\mathcal{O}(\alpha_s^2)$

  $\beta > \beta_{\text{cut}}$ : NNLL ($\mu_s = k_s M \beta^2$); NNLO$_{\text{approx}}$; NNNL$_3$(A/B)



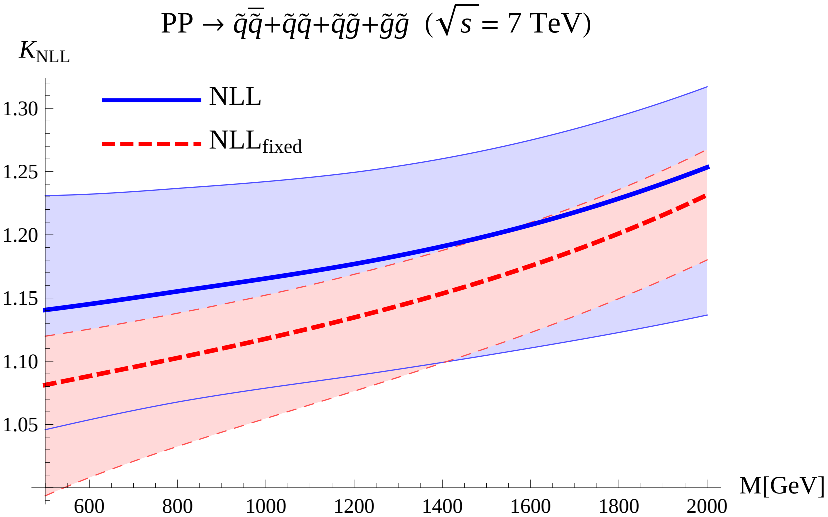

- Choose $\beta_{\text{cut}}$ so that not too sensitive to
  - ambiguities for $\beta \to 1$
  - breakdown of perturbation theory for $\beta \to 0$

(E.g. LHC7: $\mu_s = 2M\beta^2$, $\beta_{\text{cut}} = 0.54 \Rightarrow \mu_s > 100$ GeV)

## Ambiguities in resummation

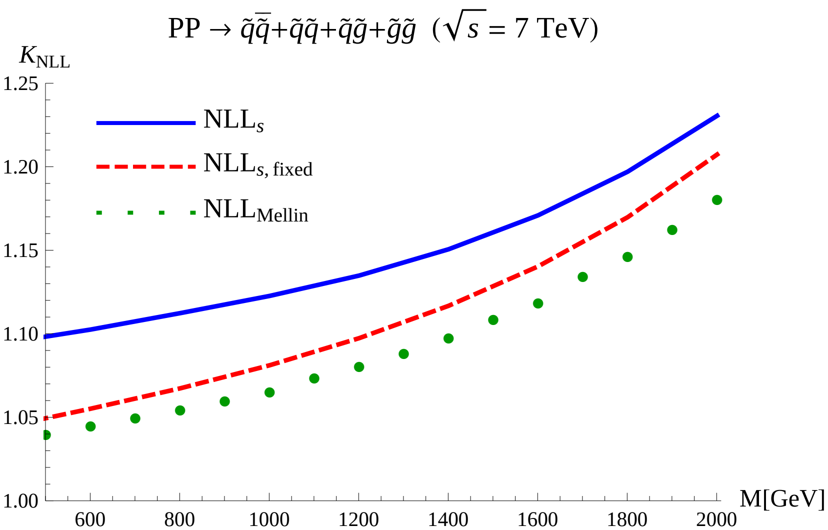
Running-scale and fixed-scale implementations agree within resummation uncertainties

($\beta_{\text{cut}}$, $\mu_s$ variation...)




For soft resummation reasonable consistency with Mellin-space resummation

(Beenakker et al. 09)




(LHC7, $m_{\tilde{q}} = m_{\tilde{g}}$)