keV sterile neutrino model building

Viviana Niro

Universitat de Barcelona

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in collaboration with *M. Lindner, A. Merle, D. Schmidt*

Outline

1. Introduction
2. keV-sterile neutrinos as Dark Matter
   - $L_e - L_\mu - L_\tau$ flavour symmetry
   - Froggatt-Nielsen model
   - keV-sterile from a frozen-in FIMP
3. Conclusions

The missing mass problem

26.8% of the total mass-energy present in the Universe is in the form of Dark Matter:

- many independent observations consistent with a missing matter component: galaxy rotation curves, weak lensing reconstruction in interacting clusters of galaxies, angular spectrum of anisotropies in the CMB

  $\Omega_{\rm CDM} h^2 = 0.1199 \pm 0.0027$ *Planck Collaboration, 1303.5076 [astro-ph.CO]*

- can active neutrinos be the Dark Matter? NO $\rightarrow$ bounds on m_ν imply that neutrino density is too low; neutrinos are "hot DM" (particles relativistic when they decouple from the primordial plasma) $\rightarrow$ inconsistent with structure formation: destroys small scale structures in cosmological evolution

- the Universe has Dark Matter (and neutrinos have mass) $\rightarrow$ physics beyond the Standard Model required

The missing mass problem

- Different Dark Matter candidates present in the literature:
  - CDM: particles non-relativistic when they decouple from the primordial plasma, WIMP particles
    $\rightarrow$ but so far no detection
  - WDM: hardly relativistic, e.g. keV neutrinos
    $\rightarrow$ may provide a solution to some problems of the DM simulations, e.g. the number of Dwarf satellite galaxies, predicts smoother profiles

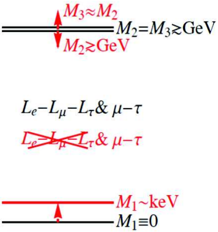
    A. Boyarsky, O. Ruchayskiy, D. Iakubovskyi, arXiv:0808.3902 [hep-ph], D. Gorbunov, A. Khmelnitsky, V. Rubakov, arXiv:0808.3910 [hep-ph]

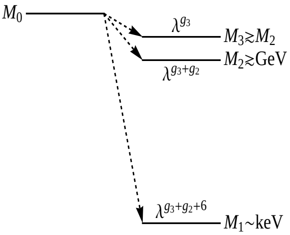
    Note: Right-handed neutrinos exist probably anyway
    $\rightarrow$ need to build a model that involve one sterile neutrino with a keV-scale mass and two heavy sterile neutrinos

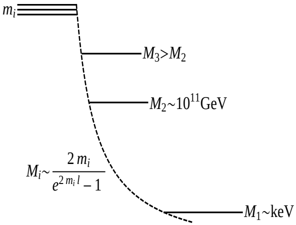
Some models on keV Neutrinos as WDM

- Models based on broken flavour symmetries
  - Soft $L_e - L_\mu - L_\tau$ flavour symmetry breaking

    M. Lindner, A. Merle, VN; arXiv:1011.4950 [hep-ph]
  - see also *J. Barry, W. Rodejohann, H. Zhang, arXiv:1110.6382 [hep-ph]; H. Zhang, arXiv:1110.6838 [hep-ph]*
- Models based on suppression mechanism
  - Froggatt-Nielsen mechanism *A. Merle, VN 1105.5136[hep-ph]*
  - Split Seesaw $\rightarrow$ idea: use the splitting between SM brane and hidden brane *A. Kusenko, F. Takahashi, T. Yanagida: arXiv:1006.1731 [hep-ph]*







keV Neutrinos from $L_e - L_\mu - L_\tau$ symmetry

L. Lavoura, W. Grimus, hep-ph/0008020: $L_e - L_\mu - L_\tau$ for 3 light & 2 heavy neutrinos
$\Rightarrow$ application of the same symmetry to the heavy sector

	L_{eL}	$L_{\mu L}$	$L_{\tau L}$	e_R	μ_R	τ_R	N_{1R}	N_{2R}	N_{3R}	ϕ	Δ
$\mathcal{F}$	1	-1	-1	1	-1	-1	1	-1	-1	0	0

Only certain contributions to the mass terms are allowed:

$$\mathcal{L}_{\text{mass}} = -M_R^{12}\,\overline{(N_{1R})^C}\,N_{2R} - M_R^{13}\,\overline{(N_{1R})^C}\,N_{3R} + h.c.$$

$$\begin{aligned}\mathcal{L}_{\text{mass}} \quad = \quad & -Y_D^{e1}\,\overline{L_{eL}}\,\tilde{\phi}\,N_{1R} - Y_D^{\mu 2}\,\overline{L_{\mu L}}\,\tilde{\phi}\,N_{2R} - Y_D^{\mu 3}\,\overline{L_{\mu L}}\,\tilde{\phi}\,N_{3R} - \\ & -Y_D^{\tau 2}\,\overline{L_{\tau L}}\,\tilde{\phi}\,N_{2R} - Y_D^{\tau 3}\,\overline{L_{\tau L}}\,\tilde{\phi}\,N_{3R} + h.c.,\end{aligned}$$

with $\tilde{\phi} = i\sigma_2\phi^*$. Using the triplet scalar, we can also have a Majorana mass term for the left-handed neutrinos:

$$\mathcal{L}_{\text{mass}} = -Y_L^{e\mu}\,\overline{(L_{eL})^C}\,(i\sigma_2\Delta)\,L_{\mu L} - Y_L^{e\tau}\,\overline{(L_{eL})^C}\,(i\sigma_2\Delta)\,L_{\tau L} + h.c.$$

keV Neutrinos from $L_e - L_\mu - L_\tau$ symmetry

Light neutrino spectrum: $(0, m, m)$; heavy neutrino spectrum: $(0, M, M) \rightarrow N_1$ massless!
$\Rightarrow$ flavour symmetry must be broken with "soft-breaking" terms that must be smaller than the symmetry-preserving terms (similar idea: *M. Shaposhnikov, hep-ph/0605047*)
$\Rightarrow$ these terms give N_1 a small mass, and lift the degeneracy between N_2 and N_3

$$\mathcal{L}_{\rm mass} = -\frac{1}{2}\overline{\Psi^C}\mathcal{M}_\nu\Psi + h.c.\,,$$

with $\Psi \equiv \left((\nu_{eL})^C, (\nu_{\mu L})^C, (\nu_{\tau L})^C, N_{1R}, N_{2R}, N_{3R}\right)^T$ and

$$\mathcal{M}_\nu = \left(\begin{array}{ccc|ccc} s_L^{ee} & m_L^{e\mu} & m_L^{e\tau} & m_D^{e1} & 0 & 0 \\ m_L^{e\mu} & s_L^{\mu\mu} & 0 & 0 & m_D^{\mu 2} & m_D^{\mu 3} \\ m_L^{e\tau} & 0 & s_L^{\tau\tau} & 0 & m_D^{\tau 2} & m_D^{\tau 3} \\ \hline m_D^{e1} & 0 & 0 & S_R^{11} & M_R^{12} & M_R^{13} \\ 0 & m_D^{\mu 2} & m_D^{\tau 2} & M_R^{12} & S_R^{22} & 0 \\ 0 & m_D^{\mu 3} & m_D^{\tau 3} & M_R^{13} & 0 & S_R^{33} \end{array}\right)$$

simplifications: $s_L^{\alpha\alpha} \simeq s$, $S_R^{ii} \simeq S$, $M_R^{ij} \simeq M_R$ ($S_{2L} \times S_{2R}$ symmetry), and $m_L^{\alpha\beta} \ll m_D^{\alpha i} \ll M_R^{ij}$
$\Rightarrow \Lambda_s = S$, $\Lambda'_\pm = S \pm \sqrt{2}M_R$, $\lambda_s = s$, $\lambda'_\pm = s \pm \sqrt{2}\left[m_L - \frac{m_D^2}{M_R}\right] + \frac{5m_D^2 S}{4M_R^2}$

keV Neutrinos from $L_e - L_\mu - L_\tau$ symmetry

conditions: $s \ll m_L - m_D^2/M_R$ and $S \ll M_R$ possible to have $M_1 \simeq$keV and $M_{2,3} \geq$GeV
masses: neutrino masses are given by $m_1 = s + b$, $m_2 = s - b$, and $m_3 = s$
$\rightarrow |m_1| = 0.0486$ eV, $|m_2| = 0.0494$ eV, and $|m_3| = 0.0004$ eV
problem: we predict bimaximal mixing

$$\mathcal{U}_\nu = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

way out: this mixing can come from the charged lepton sector
$\Rightarrow$ we must assume small $s_D^{e\tau}$ and $m_D^{\mu\tau}$

$$\mathcal{M}_l \mathcal{M}_l^\dagger \simeq \begin{pmatrix} (m_D^{ee})^2 + (s_D^{e\mu})^2 & s_D^{e\mu}\left(m_D^{ee} + m_D^{\mu\mu}\right) & 0 \\ s_D^{e\mu}\left(m_D^{ee} + m_D^{\mu\mu}\right) & (m_D^{\mu\mu})^2 + (s_D^{e\mu})^2 & 0 \\ 0 & 0 & (m_D^{\tau\tau})^2 \end{pmatrix}$$

If we identify $s_D^{e\mu} = m_D^{\mu\mu}\lambda$, $m_D^{ee} = m_e$, $m_D^{\mu\mu} = m_\mu$, and $m_D^{\tau\tau} = m_\tau$
$\Rightarrow \mathcal{U}_L$ such that $\mathcal{U}_{PMNS} = \mathcal{U}_L^\dagger \mathcal{U}_\nu$ with $\theta_{12} \simeq 33.4^\circ$, $\theta_{13} \simeq 8^\circ$, $\theta_{23} \simeq 45^\circ$

keV Neutrinos from Froggatt-Nielsen models

Froggatt-Nielsen: Nucl. Phys. B147, 277 (1979)

- the different generations are differently charged under a new $U(1)_{FN}$ symmetry
- there is a high energy sector of fermions and scalars. The scalars develop vevs to break the symmetry
- this leads to multiple seesaw-like diagrams

$$\begin{aligned}\mathcal{L}_{\text{leptons}} \quad = & \quad -Y_e^{ij}\,\overline{e_{iR}}\,H\,L_{jL}\left(\frac{\Theta}{\Lambda}\right)^{k_i+f_j} - Y_D^{ij}\,\overline{N_{iR}}\,\tilde{H}\,L_{jL}\left(\frac{\Theta}{\Lambda}\right)^{g_i+f_j} \\ - & \quad \frac{1}{2}\,\overline{N_{iR}}\,\tilde{M}_R^{ij}\,(N_{jR})^C\left(\frac{\Theta}{\Lambda}\right)^{g_i+g_j} - \frac{1}{2}\,Y_L^{ij}\,\overline{(L_{iL})^C}\,(i\sigma_2\Delta)\,L_{jL}\left(\frac{\Theta}{\Lambda}\right)^{f_i+f_j} + h.c.\,,\end{aligned}$$

integrating out the heavy fermions: $M_R^{ij} \simeq \tilde{M}_R^{ij}\lambda^{g_i+g_j}$, where $\lambda = \langle\theta\rangle/\Lambda \rightarrow$ suppression

- we can use the Froggatt-Nielsen mechanism to suppress a higher mass scale
- leads naturally to a split mass spectrum
- the FN-charges of the RH-neutrinos drop out in the seesaw formula, like any global $U(1)$ charge

keV Neutrinos from Froggatt-Nielsen models

Find the minimal assignments needed to explain the pattern in the sterile neutrino sector

- key point: for the charges (g_1, g_2, g_3), we need at least $g_1 \geq g_2 + 3$ and $g_2 \geq g_3$ in order to create the required hierarchy
- two example scenarios: A=(g_1, g_2, g_3)=(3,0,0) & B=(g_1, g_2, g_3)=(4,1,0)

A Froggatt-Nielsen charge assignment is not as arbitrary as it may seem, in the context of keV neutrinos:

- needs two FN-fields to combine predictivity and CP violation (auxiliar Z_2 symmetry is also necessary)
- incompatible with left-right symmetry
- excludes bimaximal mixing from neutrino sector
- GUTs: favors SU(5), disfavours SO(10)
- disfavours democratic Yukawa couplings
- nice feature: RGE-effects negligible (due to tiny y_D)

keV Neutrinos from Froggatt-Nielsen models

Choosing the vev $\langle \Theta_1 \rangle$ to be real:

$$\lambda = \frac{\langle \Theta_1 \rangle}{\Lambda}, \quad R = \frac{\langle \Theta_1 \rangle}{\langle \Theta_2 \rangle} = R_0 e^{i\alpha_0}, \quad B_{2n} = 1 + R^2 + \ldots + R^{2n}$$

we adopt the following FN charges and Z_2 assignments:

inspired by *T. Asaka hep-ph/0304124*, *S. Kanemura, 0704.0697*

$$\begin{array}{ll} \Theta_{1,2} : (-1,-1;+,-), & L_{1,2,3} : (a+1, a, a; +,+,-), \quad \text{with } a{=}0,1 \\ \overline{e_{1,2,3}} : (3,2,0;+,+,-), & \overline{N_{1,2,3}} : (g_1, g_2, g_3; +,+,-) \end{array}$$

$$M_R^{(\mathrm{A,B})} = \begin{pmatrix} \tilde{M}_R^{11} B_{6,8} \lambda^{6,8} & \tilde{M}_R^{12} B_{2,4} \lambda^{3,5} & \tilde{M}_R^{13} R B_2 \lambda^{3,4} \\ \bullet & \tilde{M}_R^{22} B_{0,2} \lambda^{0,2} & 0, \ \tilde{M}_R^{23} R \lambda \\ \bullet & \bullet & \tilde{M}_R^{33} \end{pmatrix}$$

$\rightarrow$ scenario A: $M_1 = M_0 \lambda^6 \mathcal{O}(1)$, $M_2 = M_0$, $M_3 = M_0[1 + \lambda^6 \mathcal{O}(1)]$
$\rightarrow$ scenario B: $M_1 = M_0 \lambda^8 \mathcal{O}(1)$, $M_2 = M_0 \lambda^2$, $M_3 = M_0[1 + \lambda^2 \mathcal{O}(1)]$

yield the required hierarchical spectra of the sterile neutrinos! With mild deviations of the Yukawa couplings from democratic assignments, one can find fully working models.

Production mechanisms for keV sterile neutrinos

- Dodelson-Widrow, thermal production of sterile neutrinos by mixing with the active neutrinos

  S. Dodelson, L.M. Widrow, hep-ph/9303287

- Shi-Fuller, non-thermal resonant active-sterile neutrino transformation driven by a lepton number asymmetry

  X. Shi and G.M. Fuller, astro-ph/9810076

- primordial abundance by scalar decays (inflaton)

  M. Shaposhnikov and I. Tkachev, hep-ph/0604236; A. Anisimov, Y. Bartocci, and F. L. Bezrukov, arXiv:0809.1097; T. Asaka, M. Shaposhnikov, and A. Kusenko, hep-ph/0602150; F. Bezrukov, D. Gorbunov, arXiv:0912.0390 [hep-ph]

- thermal overproduction + entropy dilution by out-of-equilibrium decay of the other right-handed neutrinos

  F. Bezrukov, H. Hettmansperger, M. Lindner, arXiv:0912.4415 [hep-ph]; M. Nemevsek, G. Senjanovic, Y. Zhang, arXiv:1205.0844 [hep-ph]

keV-sterile from a frozen-in FIMP

A. Merle, VN, Daniel Schmidt, arXiv:1306.3996 [hep-ph]

The particle content of the SM is extended by three right-handed sterile neutrinos N_a ($a = 1, 2, 3$) and one real scalar singlet S. The Lagrangian is

$$\mathcal{L} \quad = \quad \mathcal{L}_{\rm SM} + \left[i\overline{N_a}\partial\!\!\!/ N_a + \frac{1}{2}(\partial_\mu S)(\partial^\mu S) - \frac{y_a}{2} S \, \overline{N_a^c} N_a \ + h.c. \right] - V_{\rm scalar} + \mathcal{L}_\nu \,,$$

which consists of the SM, kinetic terms of the sterile neutrinos N_a, Yukawa interactions f_a of the singlet S with N_a, $\mathcal{L}_\nu$ is the part of the Lagrangian giving mass to the light neutrinos and a scalar potential $V_{\rm scalar}$:

$$V_{\rm scalar} = -\mu_H^2 H^\dagger H - \frac{1}{2}\mu_S^2 S^2 + \lambda_H (H^\dagger H)^2 + \frac{1}{4}\lambda_S S^4 + 2\lambda (H^\dagger H) S^2.$$

Assumption:

- feebly Higgs portal coupling $\lambda \ll 10^{-6}$
- sterile neutrino masses: $m_{N_{R1}} \ll m_\sigma \ll m_{N_{R2,3}}$
- a global $Z_4 = \{\pm 1, \pm i\}$ symmetry, such that $S \to -S$ and for all generation of leptons $L_\alpha \to i L_\alpha$ and $e_{R\alpha} \to i e_{R\alpha}$ ($N_a \to i N_a$)

keV-sterile from a frozen-in FIMP

The relic density of N_1 is produced by the decays of a frozen-in real scalar singlet particle $\sigma \Rightarrow$ solve a system of coupled equations describing simultaneously the annihilation and decay processes:

$$\frac{d}{dT} Y_\sigma = \frac{d}{dT} Y_\sigma^{\mathcal{A}} + \frac{d}{dT} Y_\sigma^{\mathcal{D}} , \quad \frac{d}{dT} Y_{N_1} = \frac{d}{dT} Y_{N_1}^{\mathcal{D}} ,$$

with

$$\tfrac{d}{dT} Y_\sigma^{\mathcal{A}} = -\sqrt{\tfrac{\pi}{45 G_N}} \sqrt{g_*} \langle \sigma_{\rm ann} v \rangle \, Y_{\sigma,\rm eq}^2 , \quad \tfrac{d}{dT} Y_\sigma^{\mathcal{D}} = -\tfrac{1}{2} \tfrac{d}{dT} Y_{N_1}^{\mathcal{D}} ,$$

$$\tfrac{d}{dT} Y_{N_1}^{\mathcal{D}} = -\sqrt{\tfrac{45}{\pi^3 G_N}} \tfrac{1}{T^3} \tfrac{1}{\sqrt{g_{\rm eff}}} \langle \Gamma(\sigma \to N_1 N_1) \rangle \, Y_\sigma \qquad ,$$

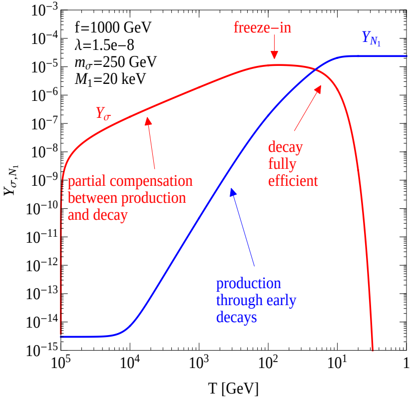
The equilibrium yield is given by

$$Y_{\sigma,\rm eq} = \frac{45 g_\sigma}{4\pi^4} \frac{x^2 K_2(x)}{h_{\rm eff}(T)} ,$$

$\langle \sigma_{\rm ann} v \rangle$ is the thermally averaged cross section (calculated with micrOMEGAs) and $\langle \Gamma(\sigma \to N_1 N_1) \rangle$ is the thermally averaged decay rate.

keV-sterile from a frozen-in FIMP

Example variation of yields Y_{N_1} and Y_σ as a function of the temperature T.
A significant abundance of σ gradually builds up due to freeze-in, before the decays $\sigma \to N_1 N_1$ set in and, at the same time, a significant amount of keV sterile neutrinos N_1 is produced.

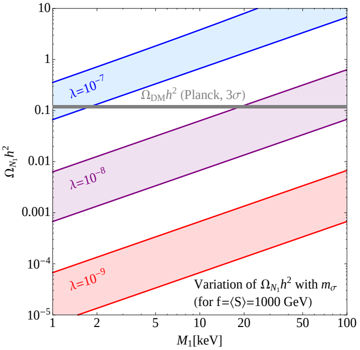


keV-sterile from a frozen-in FIMP

The relic density can be obtained using the following formula:

$$\Omega_{\rm DM} h^2 = 2.733 \times 10^8 \, \frac{m_{\rm DM}}{\rm GeV} \, Y_0 \,,$$

with $Y_0 = Y(T_0)$ being the yield at late times.



Relic density $\Omega_{N_1} h^2$ as a function of the sterile neutrino mass M_1, for Higgs portal coupling $\lambda = 10^{-7,8,9}$.

Constraints on the free streaming horizon

The *(co-moving) free-streaming horizon* $r_{\rm FS}$ can be interpreted as the mean distance which the DM particles would travel if they were not bound by gravitation at some point:

$$r_{\rm FS} = \int_{t_{\rm in}}^{t_0} \frac{\langle v(t)\rangle}{a(t)}\, dt,$$

where $t_{\rm in}$ is the initial time at which the integration starts, t_0 is the current time, $v(t)$ is the mean velocity of the DM particles, and $a(t)$ is the scale factor.

$$\begin{aligned} \text{Cold Dark Matter (CDM)} \quad &:\Longleftrightarrow \quad r_{\rm FS} < 0.01 \text{ Mpc},\\ \text{Warm Dark Matter (WDM)} \quad &:\Longleftrightarrow \quad 0.01 \text{ Mpc} < r_{\rm FS} < 0.1 \text{ Mpc},\\ \text{Hot Dark Matter (HDM)} \quad &:\Longleftrightarrow \quad 0.1 \text{ Mpc} < r_{\rm FS}. \end{aligned}$$

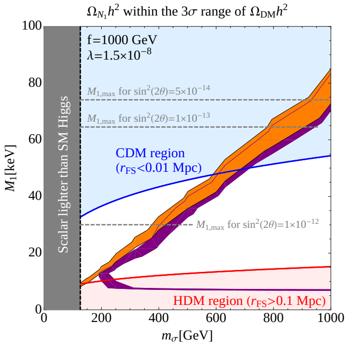
The production time of the DM particles can be approximated by $t_{\rm in} \equiv t_{\rm prod} + \tau$, where $t_{\rm prod}$ is the time of freeze-in and $\tau = 1/\Gamma$ is the lifetime of the scalar particle σ.
The scale factor $a(t) \propto t^{1/2}$ [$a(t) \propto t^{2/3}$] for radiation [matter] dominance.
The average velocity $\langle v(t)\rangle$: we assume an instantaneous transition between the highly relativistic and the fully non-relativistic regimes,

$$\langle v(t)\rangle \simeq \begin{cases} 1 & \text{if } t < t_{\rm nr}, \\ \frac{\langle p(t)\rangle}{M_1} & \text{if } t \geq t_{\rm nr}, \end{cases}$$

Part of the keV sterile neutrinos could be produced by the ordinary Dodelson-Widrow mechanism (thermal production by mixing).
This contribution depends on the active-sterile mixing angle θ_1 of the keV sterile neutrino N_1, and it can be estimated by the approximate formula: *A. Kusenko, arXiv:0906.2968*

$$\Omega_{N_1,\mathrm{DW}} h^2 \approx 0.2 \cdot \frac{\sin^2\theta_1}{3 \cdot 10^{-9}} \left(\frac{M_1}{3\ \mathrm{keV}}\right)^{1.8} .$$

We have considered the corresponding *maximal* (i.e., for the largest allowed value of $\sin\theta_1^2$) addition of particle production due to the DW mechanism.



Orange (purple) bands: relic abundance $\Omega_{N_1} h^2$ within the 3σ experimental value, through the decay of a freeze-in scalar (considering also the DW mechanism).

Other examples

Several other models are present in the literature $\Rightarrow$ some examples:

- composite Dirac neutrinos
  The neutrinos are naturally light due to compositeness (Dirac neutrinos are produced) *Y. Grossman, D.J. Robinson, arXiv:1009.2781 [hep-ph]*
  The right-handed active neutrino is a composite state, while elementary sterile neutrinos gain keV masses similarly to the quarks in extended Technicolor
  D.J. Robinson, Y. Tsai, arXiv:1404.7118 [hep-ph]; D.J Robinson, Y. Tsai, arXiv:1205.0569 [hep-ph]

- minimal extended seewsaw
  H. Zhang, arXiv:1110.6838 [hep-ph]; J. Heeck, H. Zhang, arXiv:1211.0538 [hep-ph]
  Minimal extension of the type-I seesaw by adding one extra singlet fermion
  $\mathcal{L} = -\overline{\nu_L} m_D N_R - \overline{(S_R)^c} M_S^T N_R - \frac{1}{2}\overline{N_R^c} M_R N_R + h.c.,$
  For $M_S > M_D \rightarrow m_s = M_S M_R^{-1} M_S^T \Rightarrow$ keV neutrinos

- and many other models...

Conclusions

keV sterile neutrinos are an interesting candidate for Warm Dark Matter
$\Rightarrow$ models where a mechanism "naturally" explains the light sterile neutrinos mass hierarchy ($M_{N_1} \ll M_{N_{2,3}}$)

- all models: deep connections between Dark Matter and light neutrino sector; sterile neutrino models have to fulfil light neutrino data
- models that generates a splitting in the N_i masses:
  - Randall-Sundrum model: exp-suppression
  - $L_e - L_\mu - L\tau$: soft breaking makes massless N_R massive
  - Froggatt-Nielsen: multiple seesaw-like diagrams
- ultraviolet completion of the theory is missing

Conclusions

keV-sterile from a frozen-in FIMP
$\Rightarrow$ new non-thermal production mechanism

- requires a feebly coupling constants
- the abundance Y increases with increasing coupling constants
- new region in the parameter space for obtaining the correct relic density

Many models exist to explain the pattern in the right-handed neutrino sector
$\Rightarrow$ keV sterile neutrinos are a well motivated candidate for DM, they could even cure some of the problems of the CDM scenario and give a connection between neutrinos and DM.

BACKUP SLIDES

Free-streaming horizon

With the distribution function $f(p,t)$ of the DM particle, given as

$$f(p,t) = \frac{\beta}{p/T_{\rm WDM}} \exp\left(-\frac{p^2}{T_{\rm WDM}^2}\right),$$

its average momentum $\langle p(t)\rangle$ equals

$$\langle p(t)\rangle = \frac{\int d^3p\ p\ f(p,t)}{\int d^3p\ f(p,t)} = \frac{\int_{p=0}^{\infty} dp\ p^2\ e^{-p^2/T_{\rm WDM}^2}}{\int_{p=0}^{\infty} dp\ p\ e^{-p^2/T_{\rm WDM}^2}},$$

which determines the average velocity $\langle v(t)\rangle = \langle p(t)\rangle/M_1$ of the DM particle with mass M_1. The free streaming horizon $r_{\rm FS}$ can then be calculated as

$$r_{\rm FS} = \int_{t_{\rm in}}^{t_0} \frac{\langle v(t)\rangle}{a(t)} dt,$$

where $t_{\rm in}$ is the initial time at which the integration starts, t_0 is the current time and $a(t)$ is the scale factor.

Free-streaming horizon

In the case of an early non-relativistic transition, i.e., the DM particle becomes non-relativistic at the time $t_{\rm nr} < t_{\rm eq}$ the time of matter-radiation equality, the integral can be splitted up into three regions resulting in

$$\begin{aligned} r_{\rm FS} &= \int_{t_{\rm in}}^{t_0} \frac{\langle v(t)\rangle}{a(t)} dt \\ &= \int_{t_{\rm in}}^{t_{\rm nr}} \frac{dt}{a(t)} + \int_{t_{\rm nr}}^{t_{\rm eq}} \frac{\langle v(t)\rangle}{a(t)} dt + \int_{t_{\rm eq}}^{t_0} \frac{\langle v(t)\rangle}{a(t)} dt \\ &\simeq \frac{2\sqrt{t_{\rm eq} t_{\rm nr}}}{a_{\rm eq}} + \frac{\sqrt{t_{\rm eq} t_{\rm nr}}}{a_{\rm eq}} \ln\left(\frac{t_{\rm eq}}{t_{\rm nr}}\right) + \frac{3\sqrt{t_{\rm eq} t_{\rm nr}}}{a_{\rm eq}} \\ &= \frac{\sqrt{t_{\rm eq} t_{\rm nr}}}{a_{\rm eq}} \left[5 + \ln\left(\frac{t_{\rm eq}}{t_{\rm nr}}\right)\right] \end{aligned}$$

Free-streaming horizon

In the case of a late transition, i.e., $t_{\rm nr} > t_{\rm eq}$, it follows instead:

$$
\begin{aligned}
r_{\rm FS} &= \int_{t_{\rm in}}^{t_0} \frac{\langle v(t)\rangle}{a(t)} dt \\
&= \int_{t_{\rm in}}^{t_{\rm eq}} \frac{dt}{a(t)} + \int_{t_{\rm eq}}^{t_{\rm nr}} \frac{\langle v(t)\rangle}{a(t)} dt + \int_{t_{\rm nr}}^{t_0} \frac{\langle v(t)\rangle}{a(t)} dt \\
&\simeq \frac{2t_{\rm eq}}{a_{\rm eq}} + \left(\frac{3t_{\rm eq}^{2/3} t_{\rm nr}^{1/3}}{a_{\rm eq}} - \frac{3t_{\rm eq}}{a_{\rm eq}} \right) + \frac{\sqrt{\pi}}{2} \frac{m_\sigma/2}{M_1} \sqrt{\frac{t_{\rm in}}{t_{\rm eq}}} \frac{3\ t_{\rm eq}^{4/3}}{a_{\rm eq}\, t_{\rm nr}^{1/3}} \\
&= \frac{3t_{\rm eq}^{2/3} t_{\rm nr}^{1/3}}{a_{\rm eq}} - \frac{t_{\rm eq}}{a_{\rm eq}} + \frac{\sqrt{\pi}}{2} \frac{m_\sigma/2}{M_1} \sqrt{\frac{t_{\rm in}}{t_{\rm eq}}} \frac{3\ t_{\rm eq}^{4/3}}{a_{\rm eq}\, t_{\rm nr}^{1/3}}
\end{aligned}
$$

Effective degrees of freedom

The energy density ρ_i and the entropy density s_i are defined as:

$$\begin{aligned}
\rho_i\left(T_i\right) &= g^i_{\mathrm{eff}}\left(T_i\right)\frac{\pi^2}{30}T_i^4\,,\\
s_i\left(T_i\right) &= h^i_{\mathrm{eff}}\left(T_i\right)\frac{2\pi^2}{45}T_i^3\,,
\end{aligned}$$

where T_i is the temperature of the particle species i. The effective degrees of freedom g^i_{eff} and h^i_{eff} for energy and entropy density, respectively, are defined as

$$\begin{aligned}
g^i_{\mathrm{eff}}\left(T_i\right) &= \frac{15g_i}{\pi^4}x_i^4\int_1^\infty dy\, y^2\sqrt{y^2-1}\,\frac{1}{e^{yx_i}+\eta_i}\,,\\
h^i_{\mathrm{eff}}\left(T_i\right) &= \frac{45g_i}{12\pi^4}x_i^4\int_1^\infty dy\,\left(4y^2-1\right)\sqrt{y^2-1}\,\frac{1}{e^{yx_i}+\eta_i}\,,
\end{aligned}$$

with $x_i \equiv m_i/T_i$, $\eta_i = 1$ for Fermi-Dirac, $\eta_i = -1$ for Bose-Einstein and $\eta_i = 0$ for Maxwell-Boltzmann. The number of internal degrees of freedom is denoted by g_i.

Modified Bessel functions

The modified Bessel functions $K_n(x)$ of the second kind obey the identity

$$K_n(x) = \frac{\sqrt{\pi}}{(n-\frac{1}{2})!}\left(\frac{1}{2}x\right)^n \int_1^\infty dy\, \frac{(y^2-1)^{n-\frac{1}{2}}}{e^{xy}}\,.$$

Assuming a Maxwell Boltzmann distribution with zero chemical potential, the equilibrium number density $n_{\rm eq}$ of a particle with g internal degrees of freedom and mass m can, therefore, be written as

$$\begin{aligned} n_{\rm eq} &= \frac{g}{(2\pi)^3}\int_0^\infty d^3p\; e^{-E/T} = \frac{g}{2\pi^2}\int_m^\infty dE\; E\sqrt{E^2-m^2}\; e^{-E/T} = \\ &= m^3\frac{g}{2\pi^2}\frac{1}{x}K_2(x)\,, \end{aligned}$$

where $x = m/T$. In terms of the abundance $Y = \frac{n}{s}$ with the entropy density $s = \frac{2\pi^2}{45}h_{\rm eff}\,T^3$, it follows:

$$Y_{\rm eq} = \frac{45g}{4\pi^4}\frac{x^2}{h_{\rm eff}}K_2(x)\,.$$

Annihilation and decay reactions

In the Friedman-Robertson-Walker metric, the Boltzmann equation for the number density n of a particle species can be written as

$$\frac{d}{dt}n + 3Hn = C[n]\,,$$

where C is the collision operator expressing the number of particles per phase space volume that are lost or gained per unit time due to interactions with other particles. For the standard annihilaton process $\sigma\,\sigma \rightleftharpoons \mathrm{SM}\ \ \mathrm{SM}$:

$$\frac{d}{dt}n_\sigma + 3Hn_\sigma = -\langle\sigma_{\rm ann}v\rangle(n_\sigma^2 - n_{\sigma,\rm eq}^2) \simeq \langle\sigma_{\rm ann}v\rangle n_{\sigma,\rm eq}^2\,,$$

where the last approximation is valid for the freeze-in case, for which the initial number density can be neglected.

$\langle\sigma_{\rm ann}v\rangle$ is the relativistic thermally averaged annihilation cross section and v is the Møller velocity:

$$\langle\sigma_{\rm ann}v\rangle = \frac{1}{8m_\sigma^4\,TK_2^2(m_\sigma/T)}\int_{4m_\sigma^2}^{\infty} ds\ \sigma_{\rm ann}(s-4m_\sigma^2)\sqrt{s}\ K_1\left(\frac{\sqrt{s}}{T}\right)\,.$$

We have generated the correct Feynman rules using LanHEP and we have used micrOMEGAs for the calculation of $\langle\sigma_{\rm ann}v\rangle$.

Annihilation and decay reactions

In the radiation dominated era, the Hubble expansion rate can be expressed as

$$H = \sqrt{\frac{4\pi^3 G_N g_{\rm eff}}{45}}\, T^2 \,,$$

where G_N is Newton's gravitational constant. Furthermore, in the radiation dominated era, the expansion age t of the Universe with $\Omega_{\rm tot} = 1$ equals:

$$t = \frac{1}{2H} \,.$$

In terms of the abundance $Y = \frac{n}{s}$ with the entropy density $s = \frac{2\pi^2}{45} h_{\rm eff}\, T^3$, it follows:

$$\frac{d}{dT} Y_\sigma^{\mathcal{A}} = -\sqrt{\frac{\pi}{45 G_N}} \sqrt{g_*} \langle \sigma_{\rm ann} v \rangle Y_{\sigma,{\rm eq}}^2 \,,$$

with the definition

$$\sqrt{g_*} \equiv \frac{h_{\rm eff}}{\sqrt{g_{\rm eff}}} \left(1 + \frac{1}{3} \frac{T}{h_{\rm eff}} \frac{dh_{\rm eff}}{T} \right) \,.$$

The superscript $\mathcal{A}$ serves as indication of the annihilation process.

Annihilation and decay reactions

The decay processes $\sigma \to N_1 N_1$ of a real scalar singlet σ into two sterile neutrinos N_1 is described by the following phase space integration:

$$\int \frac{d^3 p_\sigma}{(2\pi)^3 2E_\sigma} \frac{d^3 p_{N_1}}{(2\pi)^3 2E_{N_1}} \frac{d^3 p'_{N_1}}{(2\pi)^3 2E'_{N_1}} (2\pi)^4 \delta^{(4)}(p_{N_1} + p'_{N_1} - p_\sigma) |\mathcal{M}|^2_{\sigma \to N_1 N_1} f_\sigma (1 - f_{N_1})(1 - f'_{N_1}).$$

Neglecting, the Pauli blocking and enhancing factors, we can define

$$\int \frac{d^3 p_{N_1}}{(2\pi)^3 2E_{N_1}} \frac{d^3 p'_{N_1}}{(2\pi)^3 2E'_{N_1}} (2\pi)^4 \delta^{(4)}(p_{N_1} + p'_{N_1} - p_\sigma) |\mathcal{M}|^2_{\sigma \to N_1 N_1} \equiv 2E_\sigma \Gamma^*(\sigma \to NN)\,,$$

with $\Gamma^*(\sigma \to N_1 N_1)$ the decay width for the particle at energy E_σ. The above phase space integration yields:

$$\int dn_\sigma \, \Gamma^*(\sigma \to N_1 N_1) = n_\sigma \, \langle \Gamma(\sigma \to N_1 N_1) \rangle \,,$$

where

$$\langle \Gamma(\sigma \to N_1 N_1) \rangle = \frac{\int d^3 p_\sigma \Gamma^*(\sigma \to N_1 N_1) e^{-E_\sigma / T}}{\int d^3 p_\sigma e^{-E_\sigma / T}} = \frac{K_1(x)}{K_2(x)} \Gamma(\sigma \to N_1 N_1)\,,$$

with $\Gamma(\sigma \to N_1 N_1)$ the decay width in the rest frame of the decaying particle σ, i.e., $\Gamma(\sigma \to N_1 N_1) = \frac{E_\sigma}{m_\sigma} \Gamma^*(\sigma \to N_1 N_1)$.

Annihilation and decay reactions

For the decay process $\sigma \to N_1 N_1$ of a real scalar singlet σ into two sterile neutrinos N_1, the Boltzmann equation for the number density n_{N_1} reads as

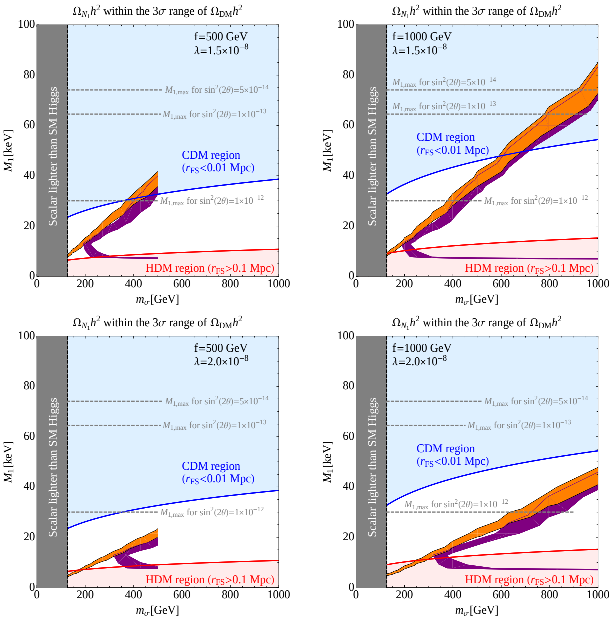
$$\frac{d}{dt} n_{N_1} + 3 H n_{N_1} = 2 \frac{K_1(x)}{K_2(x)} \Gamma(\sigma \to N_1 N_1)\, n_\sigma \,.$$

The factor 2 accounts for the fact that two sterile neutrinos N_1 are produced per decay. In terms of the abundance $Y = \frac{n}{s}$ with the entropy density $s = \frac{2\pi^2}{45} h_{\mathrm{eff}} T^3$, it follows:

$$\frac{d}{dT} Y_{N_1}^{\mathcal{D}} = -\sqrt{\frac{45}{\pi^3 G_N}} \frac{1}{T^3} \frac{1}{\sqrt{g_{\mathrm{eff}}}} \frac{K_1(x)}{K_2(x)} \Gamma(\sigma \to N_1 N_1)\, Y_\sigma \,,$$

where the superscript $\mathcal{D}$ serves as indication of the decay process.

$$\Gamma(\sigma \to N_1 N_1) = \frac{y_1^2}{16\pi} m_\sigma \left[1 - \frac{4 M_1^2}{m_\sigma^2} \right] .$$



$\Omega_{N_1}h^2$ within the 3σ range of $\Omega_{DM}h^2$
f=500 GeV
$\lambda=1.5\times10^{-8}$
f=1000 GeV
$\lambda=1.5\times10^{-8}$
f=500 GeV
$\lambda=2.0\times10^{-8}$
f=1000 GeV
$\lambda=2.0\times10^{-8}$
$M_{1,\max}$ for $\sin^2(2\theta)=5\times10^{-14}$
$M_{1,\max}$ for $\sin^2(2\theta)=1\times10^{-13}$
$M_{1,\max}$ for $\sin^2(2\theta)=1\times10^{-12}$
CDM region
(r_{FS}<0.01 Mpc)
HDM region (r_{FS}>0.1 Mpc)
Scalar lighter than SM Higgs
M_1[keV]
m_σ[GeV]