Higher-order scalar interactions and SM vacuum stability

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based on:

Z. Lalak, P. Olszewski and ML, JHEP05(2014)119

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SM Effective potential

Standard Model Effective potential

$$V_{SM}(\mu) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \sum_i \frac{n_i}{64\pi^2} M_i^4 \left[\ln\left(\frac{M_i^2}{\mu^2}\right) - C_i\right]$$

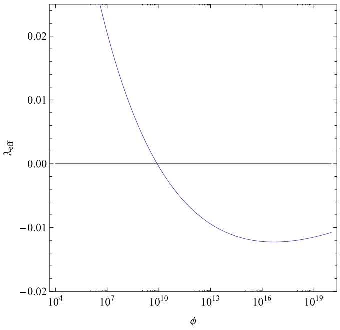
For large field values $m^2 << \phi^2$ and $\mu = \phi$ the potential is very well approximated by

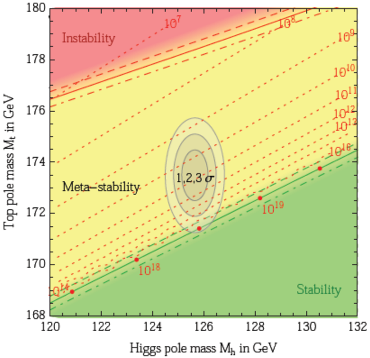
$$V_{SM}(\phi) \approx \phi^4 \left\{ \frac{\lambda}{4} + \frac{1}{64\pi^2} \left[6\left(\frac{g_2^2}{4}\right)^2 \left(\ln\left(\frac{g_2^2}{4}\right) - \frac{5}{6}\right) + 3\left(\frac{g_1^2+g_2^2}{4}\right)^2 \left(\ln\left(\frac{g_1^2+g_2^2}{4}\right) - \frac{5}{6}\right) \right.\right.$$
$$\left.\left. -12\left(\frac{y_t^2}{2}\right)^2 \left(\ln\left(\frac{y_t^2}{2}\right) - \frac{3}{2}\right) + \left(\frac{3\lambda}{2}\right)^2 \left(\ln\left(\frac{3\lambda}{2}\right) - \frac{3}{2}\right) + 3\left(\frac{\lambda}{2}\right)^2 \left(\ln\left(\frac{\lambda}{2}\right) - \frac{3}{2}\right) \right]\right\}$$

$$V_{SM}(\phi) \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4$$

SM Metastability

$$\lambda_{eff} < 0 \Longrightarrow \text{Metastability}$$





D. Buttazzo, et al. [arXiv:1307.3536].
G. Degrassi, et al. [arXiv:1205.6497].

Tunneling

Standard semiclassical formalism

S. R. Coleman, Phys. Rev. D **15** (1977) 2929.
C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D **16** (1977) 1762.

$O(4)$ symmetric solution to euclidean equation of motion

$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{\partial V(\phi)}{\partial \phi},$$

$$s = \sqrt{\vec{x}^2 + x_4^2}.$$

with

- $\dot{\phi}(s = 0) = 0$ at the true vacuum
- $\phi(s = \infty) = \phi_{min}$ at the false vacuum

Tunneling

Action of the bounce solution

$$
\begin{aligned}
S_E &= \int d^4x \left\{ \frac{1}{2} \sum_{\alpha=1}^{4} \left(\frac{\partial \phi(\mathbf{x})}{\partial x^\alpha} \right)^2 + V(\phi(\mathbf{x})) \right\} \\
&= 2\pi^2 \int ds s^3 \left(\frac{1}{2} \dot{\phi}^2(s) + V(\phi(s)) \right),
\end{aligned}
$$

allows us to calculate decay probability dp of a volume d^3x

$$
dp = dt d^3x \frac{S_E^2}{4\pi^2} \left| \frac{det'[-\partial^2 + V''(\phi)]}{det[-\partial^2 + V''(\phi_0)]} \right|^{-1/2} e^{-S_E}.
$$

Simplifying

- normalisation factor replaced with width of the barrier $\propto \phi_0$
- size of the universe is $T_U = 10^{10}$yr

we can calculate the lifetime of the false vacuum ($p(\tau) = 1$)

$$
\frac{\tau}{T_U} = \frac{1}{\phi_0^4 T_U^4} e^{S_E}.
$$

Analytical solution

K. M. Lee and E. J. Weinberg, Nucl. Phys. B **267** (1986) 181.

Quartic potential :

$$V(\phi) = \tfrac{\lambda}{4}\phi^4 \qquad \Longrightarrow \qquad S_E = \tfrac{8\pi^2}{3|\lambda|}$$

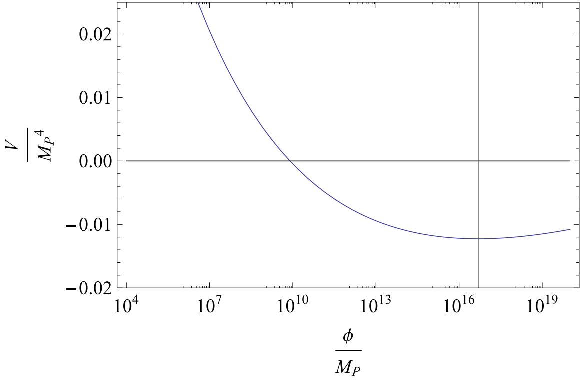
for $\lambda < 0$.

Standard Model

Approximating by a quartic potential:

$$\frac{\tau}{T_U} = \frac{1}{\phi^4(\lambda_{min})\,T_U^4}\, e^{\frac{8\pi^2}{3|\lambda_{min}|}} \approx 10^{540}.$$

lifetime is minimal for ϕ that minimizes $\lambda_{eff}(\phi)$.



Effective potential with nonrenormalisable interactions

We add new nonrenormalisable couplings
(similar to V. Branchina and E. Messina, [arXiv:1307.5193].)

$$V \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4 + \frac{\lambda_6}{6!}\frac{\phi^6}{M_p^2} + \frac{\lambda_8}{8!}\frac{\phi^8}{M_p^4}.$$

That modify the potential around the Planck scale:

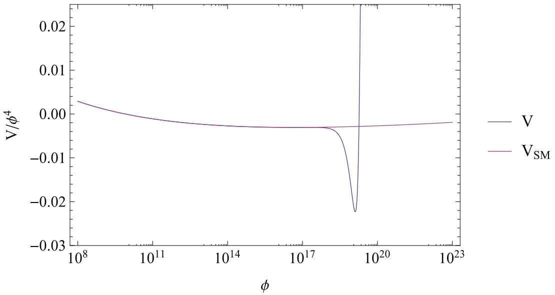


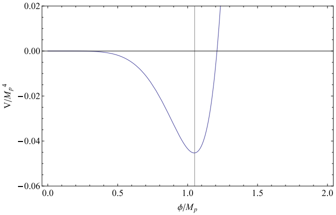
Figure: effective potential with $\lambda_6 = -1$ and $\lambda_8 = 1$.

Standard Model with nonrenormalisable interactions

Using simple quartic potential approximation:

We minimize

$$4\frac{V}{\phi^4} = \lambda_{eff}^{SM}(\phi) + 4\frac{\lambda_6}{6!}\frac{\phi^2}{M_p^2} + 4\frac{\lambda_8}{8!}\frac{\phi^4}{M_p^4}.$$

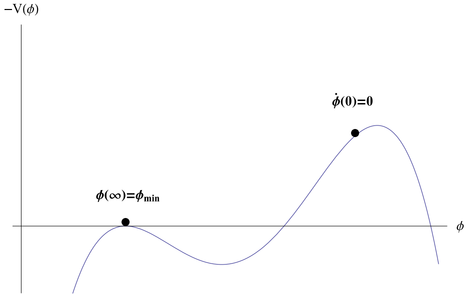


Numerical calculations

Equation we need to solve

$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{\partial V(\phi)}{\partial \phi},$$

is an equation of motion of a particle in potential $-V(\phi)$ with a "time" dependent friction $\frac{3}{s}\dot{\phi}$.



We used a simple Overshot Undershot algorithm

Numerical vs Analytical

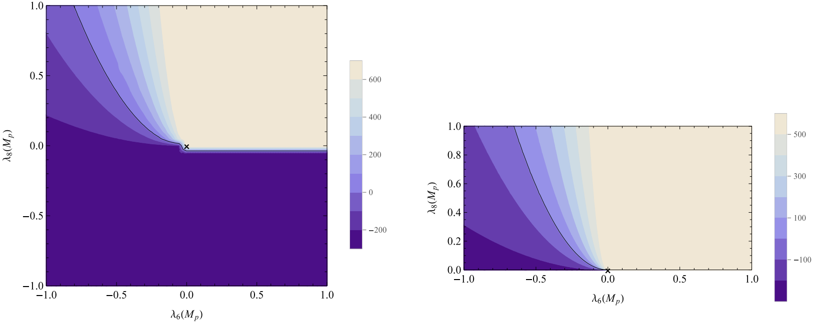


Figure: Decimal logatihm of lifetime of the universe in units of T_U as a function of the nonrenormalisable λ_6 and λ_8 couplings, calculated numerically (left panel) and analytically (right panel).

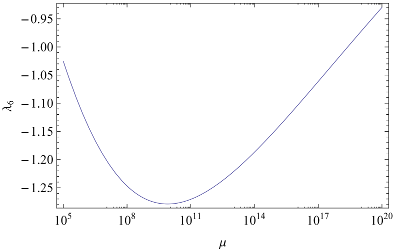
RG improvement

The correction to the running of the quatric Higgs coupling is of the form

$$\Delta\beta_\lambda = \frac{\lambda_6}{16\pi^2}\frac{m^2}{M_p^2}.$$

One-loop beta functions of new couplings take the form

$$\begin{aligned}16\pi^2\beta_{\lambda_6} &= \frac{10}{7}\lambda_8\frac{m^2}{M^2} + 18\lambda_6 6\lambda - 6\lambda_6\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2\right),\\ 16\pi^2\beta_{\lambda_8} &= \frac{7}{5}28\lambda_6^2 + 30\lambda_8 6\lambda - 8\lambda_8\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2\right),\end{aligned}$$



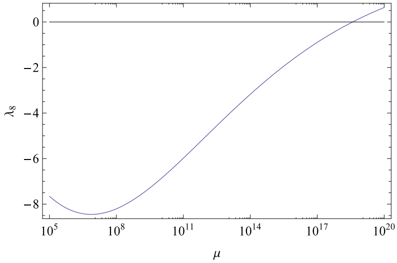


Figure: Example solution with $\lambda_6(M_p) = -1$ and $\lambda_8(M_p) = -0.1$.

Numerical vs Analytical again

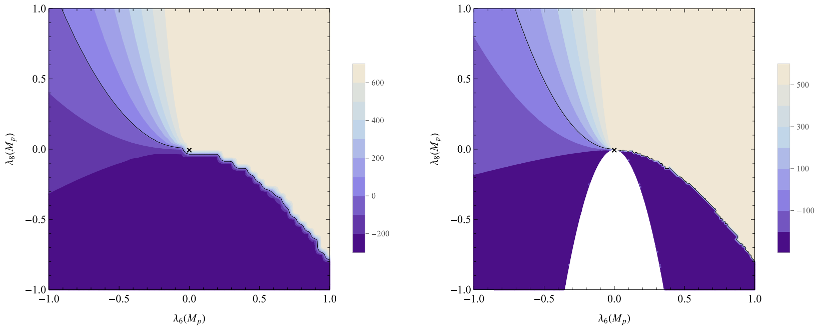


Figure: Decimal logatihm of lifetime of the universe in units of T_U as a function of the nonrenormalisable $\lambda_6(M_p)$ and $\lambda_8(M_p)$ couplings, calculated numerically (left panel) and analytically (right panel).

Comparison

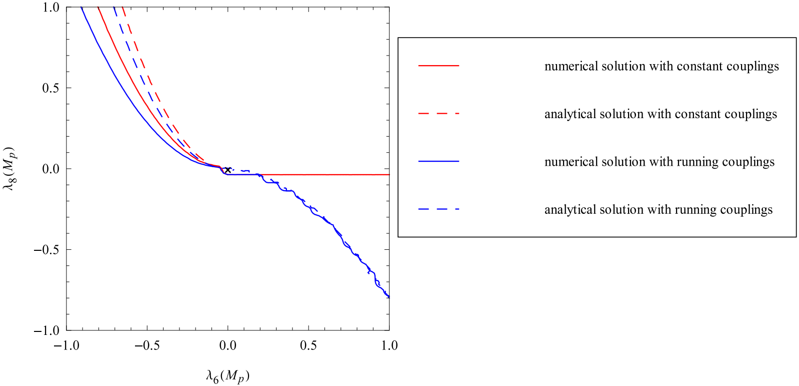
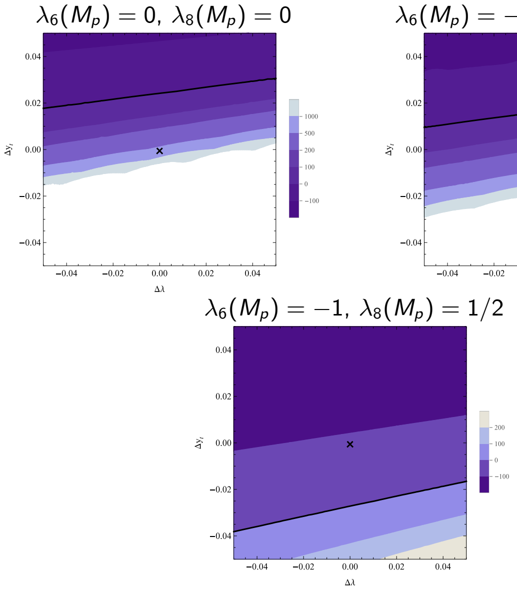


Figure: Contours corresponding to metastability boundary ($\tau = T_u$) obtained using four different methods.

SM phase diagram



Conclusions

- Analytical approximation of vacuum lifetime is fairly accurate
- RG improvement stabilizes significant parts of the parameter space
- Standard Model vacuum lifetime can be significantly changed by high energy new physics

Analytical solutions for simple potentials

K. M. Lee and E. J. Weinberg, Nucl. Phys. B **267** (1986) 181.

Quartic potential:

$$V(\phi) = \frac{\lambda}{4}\phi^4 \qquad \Longrightarrow \qquad S_E = \frac{8\pi^2}{3|\lambda|}$$

for $\lambda < 0$.

Quartic and linear potential :

$$V_\eta(\phi) = \begin{cases} \frac{\lambda}{4}\phi^4 \,, & \phi \leqslant \eta \\ \frac{\lambda}{4}\eta^4 - K\,(\phi - \eta)\,, & \phi \; > \eta \end{cases}, \qquad \Longrightarrow \qquad \begin{gathered} S_E = \frac{8\pi^2}{3|\lambda|}\left(1 - (\gamma + 1)^4\right) \\ \gamma = \frac{|\lambda|\eta^3}{K} \end{gathered}$$

for $\lambda < 0$ and $-1 < \gamma < 0$

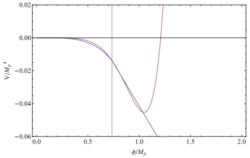
Standard Model with nonrenormalisable interactions

Approximating by quartic and linear potential

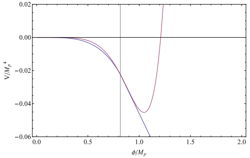
$$\frac{\tau}{T_U} = \frac{1}{\eta^4 T_U^4} e^{\frac{8\pi^2}{3|\lambda(\eta)|}(1-(\gamma+1)^4)},$$

with: $\lambda(\eta) = 4\frac{V(\eta)}{\phi^4} = \lambda_{eff}^{SM}(\eta) + 4\frac{\lambda_6}{6!}\frac{\eta^2}{M_p^2} + 4\frac{\lambda_8}{8!}\frac{\eta^4}{M_p^4}.$

We still have to chose η:



$(1-(\gamma+1)^4) = 0.994027$
$\log_{10}(\frac{\tau}{T_U}) = -178.4$



$(1-(\gamma+1)^4) = 0.999999$
$\log_{10}(\frac{\tau}{T_U}) = -181.4$

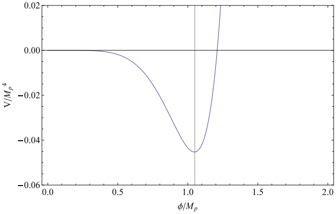
The difference comes from our arbitrary choice of η, the factor $(1-(\gamma+1)^4)$ is always negligible .

Standard Model with nonrenormalisable interactions

Using simpler quartic potential approximation:

We minimize

$$4\frac{V}{\phi^4} = \lambda_{eff}^{SM}(\phi) + 4\frac{\lambda_6}{6!}\frac{\phi^2}{M_p^2} + 4\frac{\lambda_8}{8!}\frac{\phi^4}{M_p^4}.$$



$$\log_{10}\left(\frac{\tau}{T_U}\right) = -189.6$$

Magnitude of the suppression scale

Approximate lifetime:

$$\frac{\tau}{T_U} = \frac{1}{\mu^4(\lambda_{min})\, T_U^4} e^{\frac{8\pi^2}{3|\lambda_{min}|}}.$$

Positive λ_6 and λ_8 $\rightarrow$ stabilizing the potential

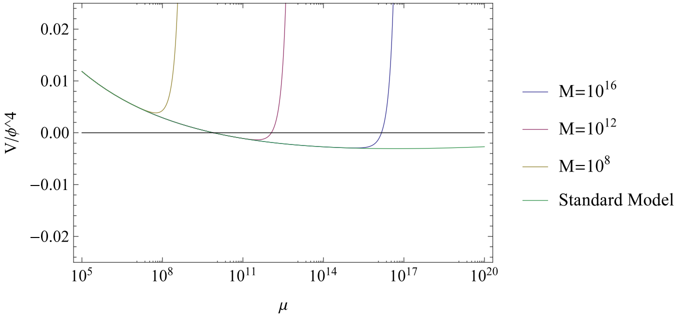


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = \lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$ are, respectively, $\log_{10}(\frac{\tau}{T_U}) = \infty, 1302, 581$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

Magnitude of the suppression scale

Positive λ_8 and negative λ_6 $\rightarrow$ New Minimum

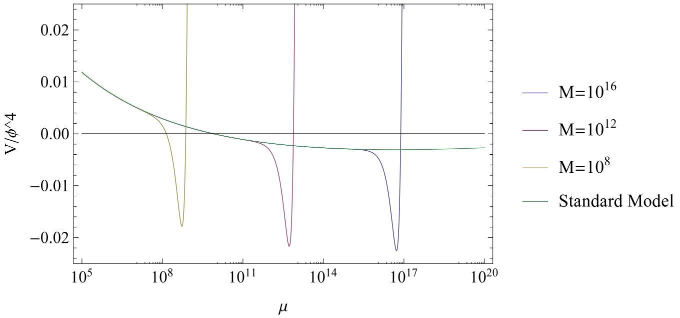


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = -1$ and $\lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$, are, respectively, $\log_{10}(\frac{\tau}{T_U}) = -45, -90, -110$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.