Excluding a Generic Spin-2 Higgs Impostor

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SUSY 2014, Manchester

based on: A. Kobakhidze and J. Yue, *Phys. Lett. B* **727** (2013) 456
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Outline

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Spin of the 125-126 GeV resonance?

- Bosonic resonance of $m_h = 125$ - 126 GeV observed by ATLAS[a] and CMS[b]

- The determination of spin and parity (J^{CP}) is an important priority

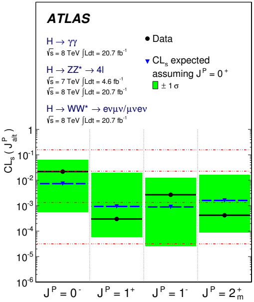
- *Landau-Yang Theorem*[c] — A vector ($J = 1$) particle cannot decay into $\gamma\gamma$

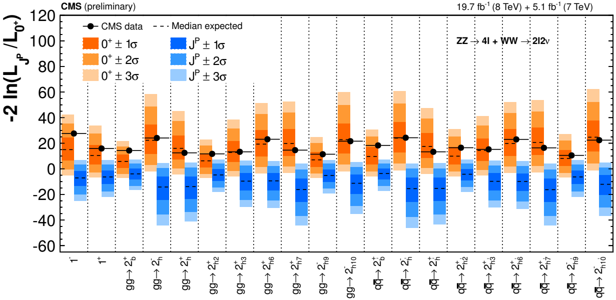
[E. Di Marco, ICHEP 2014]

[a] G. Aad *et al.* (ATLAS Collaboration), *Phys. Lett. B* **716**, 1-29 (2012)
[b] S. Chatrchyan *et al.* (CMS Collaboration), *Phys. Lett. B* **716**, 30-61 (2012)
[c] L.D. Landau, *Dokl. Akad. Nauk., USSR* **60**, 207 (1948); C.N. Yang, *Phys. Rev.* **77**, 242 (1950)

Spin of the 125-126 GeV resonance?

- Exclusion limits[1] on $J = 2^+$ (minimal graviton-like coupling):
  - ATLAS — 99.9% CL from WW, ZZ and $\gamma\gamma$ decay (ind. of gg or $q\bar{q}$ contr.)
  - CMS — 99.9% CL from WW, ZZ





$J^P = 2^{\pm}$ with non-minimal coupling?

[1] ATLAS-CONF-2013-040; CMS-PA-HIG-14-014

Outline

Mimicking Higgs

- Difficult to construct a self-consistent theory for massive spin-2
- Use an effective theory with non-universal couplings:

$$\mathcal{L}_{h-SM} = -\tfrac{\kappa_i}{2} h^{\mu\nu} T^i_{\mu\nu}$$

- Choose κ_i to mimic the SM Higgs decay rates

$$\Gamma(H \to VV^*) = \Gamma(h \to VV^*) \quad \Longrightarrow \quad \kappa_V^2 = 30 e^3 \frac{m_H}{m_h^3} \frac{m_V^2}{m_W^2} \frac{F_H(m_V/m_H)}{F_h(m_V/m_h)},$$

where $F_{h,H}(\epsilon)$ can be found in the references[2,3]

$$\kappa_W^2 \approx 3.61 \times 10^{-5}\ \text{GeV}^{-2} \ \text{and}^a\ \kappa_Z^2 \approx 4.42 \times 10^{-5}\ \text{GeV}^{-2}$$

[a] $G_F = 1.17 \times 10^{-5}\ \text{GeV}^{-2}$

- Energy scale at which perturbative unitarity $hZ \to hZ$ is broken indicates new physics

[2] J. Ellis, V. Sanz and T. Tou, arXiv:hep-ph/1211.3068 (2012)
[3] W.-Y. Keung and W. J. Marcino, *Phys.Rev. D* **30**, 248 (1984)

Outline

Unitarity

- Jacob-Wick expansion[4] for Lorentz invariant amplitude $\mathcal{M}$:

$$\mathcal{M}(s,\Omega) = 16\pi \sum_j (2j+1) a_j(s) P_j(\cos\theta)$$

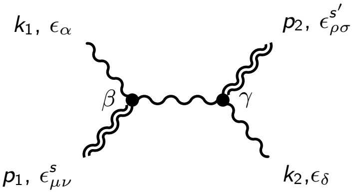
Unitarity bound

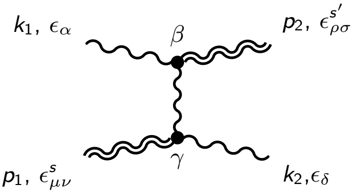
$$|\mathrm{Im}\ a_j| \geq |a_j|^2 \quad \Longrightarrow^{a} \quad |\mathrm{Re}\ a_j| \leq \frac{1}{2}$$

[a] W. Marciano, G. Valencia and S. Willenbrock *Phys. Rev. D* **40** 1725-1729 (1989)

[4] M. Jacob and G.C. Wick, *Ann. Phys.* **7**, 404-428 (1959)

$hZ \to hZ$





Longitudinal Polarisations

$J = 1: \qquad \epsilon^0_\mu(p) \sim \mathcal{O}(p)$

$J = 2: \qquad \epsilon^0_{\mu\nu}(p) \sim \mathcal{O}(p^2)$

$J = 1$ propagator

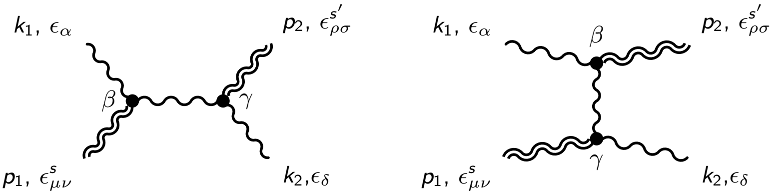
$$\frac{-\eta^{\beta\gamma} + q^\beta q^\gamma}{q^2 - m_V^2} \sim \mathcal{O}(p)$$

hZZ vertex

$$-i\frac{\kappa}{2}\Big((m^2 + k_1 \cdot k_2)C_{\mu\nu,\alpha\beta} + D_{\mu\nu,\alpha\beta}(k_1, k_2)\Big) \sim \mathcal{O}(p^2)$$

Expect scaling $\sim \mathcal{O}(p^{10})$

$hZ \to hZ$



- In the high energy amplitude $\sim \mathcal{O}(p^8)$:

$$\mathcal{M}_s \approx -\frac{\kappa_Z^2 s^3}{24 m_h^4}\left(1 - 4\frac{m_Z^2}{s}(1-\cos\theta)\right)$$

$$\mathcal{M}_t \approx \frac{\kappa_Z^2 s^4}{512 m_h^4 m_Z^2 (1-\cos\theta)}\left(\csc^2\frac{\theta}{2}\sin^6\theta + 8\frac{m_Z^2}{s}(1+\cos\theta)^4\right)$$

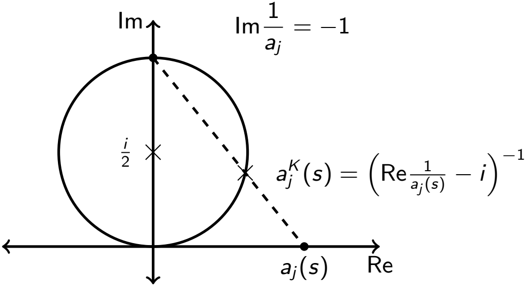
Unitarity Violation

New physics need to enter before $\Lambda \lesssim 600$ GeV

K-Matrix

- Scattering amplitude evaluated to all orders of perturbation theory should lie on the Argand circle

- Restores unitarity by projecting each partial wave onto the Argand-circle[5,6,7]:

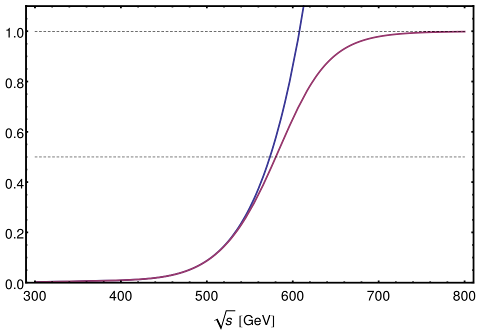


[5] A. Alboteanu, W. Kilian and J. Reuter, *JHEP* **0811** 010 (2008)

[6] V. Barger, K. Cheung and T. Han, *Phys. Rev. D* **42** 3052 (1990)

[7] M. S. Chanowitz, arXiv:hep-ph/9612240 (1996)

Additional resonance needed!



- New resonance(s) $J = 1, 2, 3$ required to rescue the theory from unitary violation!

- Difficult to write a gauge invariant, interacting theory for $J \geq 2$

- Scattering amplitude for $J \geq 2$ will be of even higher energy dependence

Adding a Z'

- Consider the simplest $J = 1$ case by introducing a new $U'(1)$ gauge field, Z' via a kinetic mixing term:

$$\mathcal{L} \supset -\frac{\sin\chi}{2} Z_{\mu\nu} Z'^{\mu\nu},$$

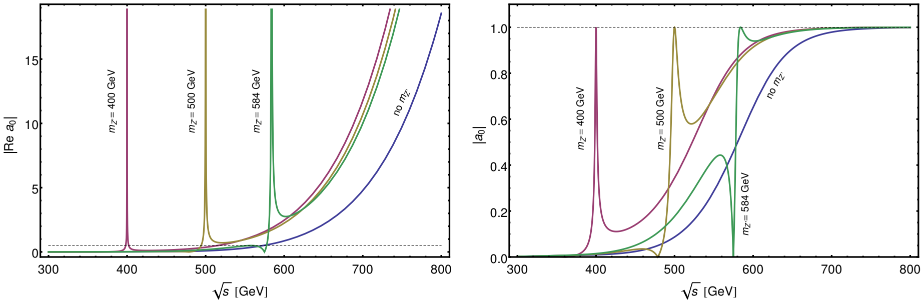
- The corresponding energy momentum tensor term is given by:

$$T_{\mu\nu}^{ZZ'} = \left(-\eta_{\mu\nu}\mathcal{L}^{ZZ'} + 2\frac{\delta\mathcal{L}}{\delta\hat{g}^{\mu\nu}} \right)\Bigg|_{\hat{g}=\eta}$$

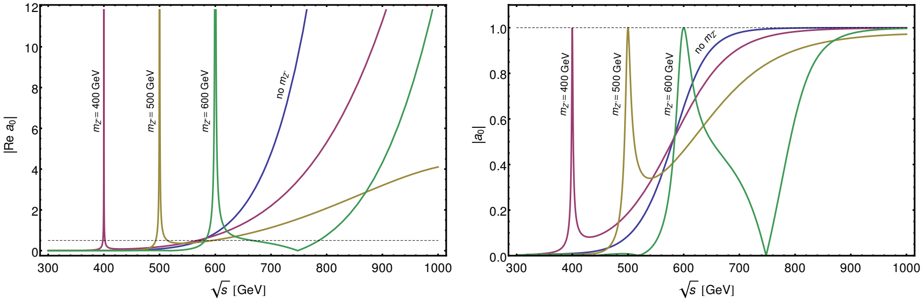
Feynman rule

$$hZZ' \text{ vertex: } -\tfrac{i\kappa'}{2}\Big[(k_1 \cdot k_2)\ C_{\mu\nu,\rho\sigma} + D_{\mu\nu,\rho\sigma}(k_1, k_2)\Big] \sim \mathcal{O}(p^2)$$

Unitarity ($\kappa' = \kappa$)



Unitarity — some tuning ($\kappa' = i\kappa$)



Outline

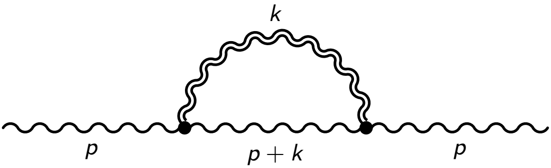
STU parameters

Definitions

$$
\begin{aligned}
\alpha S &:= 4 s_{\theta_W}^2 c_{\theta_W}^2 \frac{\Pi_Z(m_Z^2) - \Pi_Z(0)}{m_Z^2} \\
\alpha T &:= \frac{\Pi_W(0)}{m_W^2} - \frac{\Pi_Z(0)}{m_Z^2} \\
\alpha(S+U) &:= 4 s_{\theta_W}^2 \frac{\Pi_W(m_W^2) - \Pi_W(0)}{m_W^2}
\end{aligned}
$$

M. E. Peskin and T. Takeuchi, *Phys. Rev. D* **46** 381 (1992)

W and Z boson self energy



- Similar calculations for Kaluza-Klein (KK) gravitons have been already performed[8]

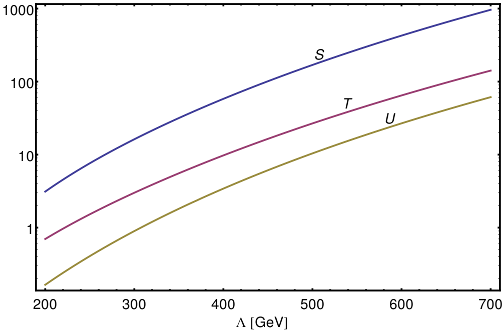
- So we simply remove summation over the KK tower:

$$
\begin{aligned}
\Pi_V(m_V^2) &= \frac{\kappa_V^2 m_V^2}{192\pi^2 m_h^4} \int_0^{\Lambda^2} dk_E^2 \int_0^1 dz \frac{k_E^2}{[k_E^2 + m_h^2(1-z) + m_V^2 z^2]^2} f_1(k_E^2, m_h^2, z, m_V^2) \\
\Pi_V(0) &= \frac{\kappa^2 m_V^2}{192\pi^2 m_h^4} \int_0^{\Lambda^2} dk_E^2 \frac{k_E^2}{(k_E^2 + m_V^2)(k_E^2 + m_h^2)} f_2(k_E^2, m_h^2, m_V^2)
\end{aligned}
$$

where $f_{1,2}$ are defined in the references[8]

[8] T.Han, D. Marfatia and R. -J. Zhang, *Phys. Rev. D* **62**, 125018 (2000)

Compatibility with STU parameters



- Even if we have heavier Higgs to solve the unitarity problem

$$
\begin{aligned}
\Delta S &\approx \frac{1}{6\pi}\ln\left(\frac{\Lambda}{m_h^{\mathrm{ref}}}\right) \\
&\approx 0.0245 \\
\Delta T &\approx -\frac{3}{8\pi c_{\theta_W}^2}\ln\left(\frac{\Lambda}{m_h^{\mathrm{ref}}}\right) \\
&\approx -0.0717 \\
\Delta U &\approx 0
\end{aligned}
$$

Parameter/Energy cutoff (Λ)	Exp*[9].	200 GeV	400 GeV	600 GeV
S	0.03 ± 0.01	-3.12	-58.0	-427.0
T	0.05 ± 0.12	-0.70	-9.75	-64.6
U	0.03 ± 0.10	-0.17	-3.42	-26.80

* $m_t^{\mathrm{ref}} = 173$ GeV, $m_h^{\mathrm{ref}} = 126$ GeV

[9] M. Baak *et. al.*, Eur. Phy. J. C **72** 2205 (2012)

Outline

Conclusions with generalisation

- Considered a spin-2 Higgs impostor with minimal non-universal couplings.
- Unitarity violation at around $\Lambda \approx 600$ GeV
- $J = 2$ ruled out by STU precision parameters
- For general coupling: expect $\mathcal{M} \sim E^6$ for even-parity and $\sim E^8$ for odd-parity

$\mathcal{J}^{\mathcal{P}}$	HZ^*Z coupling
2^+	$c_1(g^{\mu\beta_1}g^{\nu\beta_2} + g^{\mu\beta_2}g^{\nu\beta_1})$ $+ c_2 g^{\mu\nu}k^{\beta_1}k^{\beta_2}$ $+ c_3(g^{\mu\beta_1}k^{\beta_2} + g^{\mu\beta_2}k^{\beta_1})p^\nu$ $+ c_4(g^{\nu\beta_1}k^{\beta_2} + g^{\nu\beta_2}k^{\beta_1})p^\mu$ $+ c_5 p^\mu p^\nu k^{\beta_1}k^{\beta_2}$
2^-	$c_1\epsilon^{\mu\nu\beta_1\rho}p_\rho k^{\beta_2}$ $+ c_2\epsilon^{\mu\nu\beta_1\rho}k_\rho k^{\beta_2}$ $+ c_3(\epsilon^{\mu\beta_1\rho\sigma}p^\nu + \epsilon^{\nu\beta_1\rho\sigma}p^\mu)k^{\beta_2}$ $+ c_4\epsilon^{\mu\nu\rho\sigma}p_\rho k_\sigma k^{\beta_1}k^{\beta_2}$ $+ \beta_1 \leftrightarrow \beta_2$

$p = k(Z) + k(Z')$ and $k = k(Z) - k(Z')$

— S. Y. Choi *et. al.*, *Phys. Lett. B* **553** 61 (2003)

Violation of perturbative unitarity is expected to be at energies $\Lambda \sim 1$ TeV.

Thank You!

Visiting spin-1 (I)

- Constructing action corresponding to[10]:

$$\partial^2 A^\mu = 0$$

- Most general action with quadratic derivative

$$\mathcal{L} = a\ \partial_\mu A_\nu \partial^\mu A^\nu + b\ \partial_\mu A_\nu \partial^\nu A^\mu$$

- Eqn of motion:

$$\mathcal{E}^\mu(A) := \partial_\nu \frac{\partial \mathcal{L}}{\partial \partial_\nu A_\mu} = a\Box A^\mu + b\partial^\mu \partial^\nu A_\nu = 0$$

- If theory allowed to couple to a conserved source current ($\partial_\mu j^\mu = 0$):

$$\mathcal{L} \supset j_\mu A^\mu \implies \partial_\mu \mathcal{E}^\mu \Big|_{\text{on-shell}} \equiv 0 \qquad \implies a = -b$$

- Recover the Maxwell Lagrangian (choosing normalisation):

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

[10] C.-I. Chiang *General Relativity from Special Relativistic Field Theory*

Visiting spin-1 (II)

- Varying the action:

$$\delta S = \int_\Omega d^4x \, (\mathcal{E}^\mu \delta A_\mu - \partial_\mu (F^{\mu\nu} \delta A_\nu))$$

- Want to make $\partial_\mu \mathcal{E}^\mu = 0$ hold in the off-shell case $\Longrightarrow$ gauge invariance:

$$\delta A_\mu := \partial_\mu \Lambda(x) \quad \text{with} \quad \Lambda(x)\Big|_{\partial\Omega} = 0$$

- Stationary action:

$$0 = \delta S = \int_\Omega d^4x \left(-\partial_\mu \mathcal{E}^\mu \Lambda - \cancelto{0}{\partial_\mu(\mathcal{O}(\Lambda))} \right) \quad \Longrightarrow \quad \partial_\mu \mathcal{E}^\mu \Big|_{\text{off-shell}} = 0$$

- Now add mass term, then E.O.M coupled to conserved current recovers gauge condition:

$$\mathcal{E}^\mu + m^2 A^\mu = j^\mu \qquad \xRightarrow{\partial_\mu} \qquad \partial_\mu A^\mu = 0$$

Self consistency

Could have started with general Lagrangian respecting $\delta A^\mu = \partial^\mu \Lambda$.
This fixes the coefficients in such a way such that $\mathcal{L} \supset j^\mu A_\mu \Longrightarrow \partial_\mu j^\mu = 0$

Visiting spin-1 (III)

- $\partial_\mu A^\mu = 0$ removes 1 (A^0 ghost) of the 4 *dof* :

$$\begin{aligned}\mathcal{H} &= \frac{\partial \mathcal{L}}{\partial \partial_0 A_\alpha}\partial^0 A^\alpha - \mathcal{L} \\ &= -\frac{1}{2}(\partial_0 A_0^2 + (\nabla A_0)^2 + m^2 A_0^2) + \frac{1}{2}((\partial_0 \mathbf{A})^2 + (\partial_i A_j)^2 + m^2 \mathbf{A}^2)\end{aligned}$$

- What about gauge invariance of mass term?
- Stückelberg trick[11] — Starting with action:

$$S = \int d^4x \Big(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}m^2 A^\mu A_\mu + A^\mu j_\mu \Big)$$

- Introduce a new scalar field ϕ such that $A_\mu \to A_\mu + \partial_\mu \phi$

$$S \mapsto \int d^4x \Big(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}m^2 |A^\mu + \partial^\mu \phi|^2 + A^\mu j_\mu - \phi \partial_\mu j^\mu \Big)$$

- new action has gauge symmetry $\delta A_\mu = \partial_\mu \Lambda$ and $\delta\phi = -\Lambda$ but is still dynamically equivalent to the original action (which is a gauge fixed version of the new action with $\phi = 0$)

[11] see K. Hinterbichler, *Rev. Mod. Phys.* **84**, *671* (2012) for more detail

Spin-2 couplings

- In the linear approximation: $\mathcal{L}_{h-SM} \supset \kappa h_{\mu\nu} T^{\mu\nu}$ (like graviton)

- Consistency requirements:

Universal coupling

$$\boxed{\mathcal{L} \supset \kappa h_{\mu\nu} T^{\mu\nu}}$$

$$\boxed{\partial_\mu T^{\mu\nu} = 0} \qquad \boxed{\delta h_{\mu\nu} = 2\partial_{(\mu}\xi_{\nu)}}$$

Divergenceless
energy momentum tensor

Gauge
transformation

Massive spin-2

- Generic kinetic term for spin-2 [a,b,c]:

$$\mathcal{L} = a\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + b\partial_\mu h^{\mu\rho}\partial^\nu h_{\nu\rho} + c\partial_\mu h^{\mu\nu}\partial_\nu h + d\partial_\rho h\partial^\rho h$$

- We have:

$$\mathcal{E}^{\mu\nu}(h) := \partial_\rho \frac{\partial\mathcal{L}}{\partial\partial_\rho g_{\mu\nu}} = a\Box h^{\mu\nu} + 2b\partial_\rho\partial^{(\mu}h^{\nu)\rho} + c\partial^{(\mu}\partial^{\nu)}h + c\eta^{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + 2d\eta^{\mu\nu}\Box h$$

- If $\mathcal{L} \supset h_{\mu\nu}T^{\mu\nu}$ with $\partial_\mu T^{\mu\nu} = 0$:

$$\partial^\mu\mathcal{E}^{\mu\nu}\Big|_{\text{on shell}} \equiv 0 \quad\Longrightarrow\quad a = -\frac{1}{2}b = \frac{1}{2}c = -d$$

- Stationary variation to reveal gauge transformation:

$$0 = \delta S = \int_\Omega d^4x\mathcal{E}^{\mu\nu}\delta h_{\mu\nu} \quad\Longrightarrow\quad \delta h_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$$

- If $\mathcal{L} \supset m_h^2(h_{\mu\nu}h^{\mu\nu} - h^2)$, we can recover gauge condition (*De Donder*) from

$$\mathcal{E}^{\mu\nu} + m_h^2(h^{\mu\nu} - \eta^{\mu\nu}h^2) = T^{\mu\nu} \quad\xrightarrow{\partial_\mu T^{\mu\nu}=0}\quad \partial^\mu(h_{\mu\nu} - \eta_{\mu\nu}h) = 0$$

eliminating 4 (h_{00} and h_{0i}) out of 10 *dof* [d]

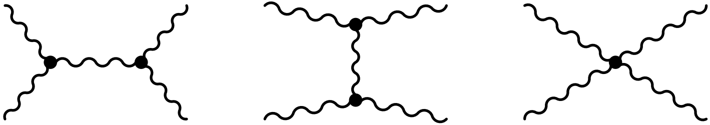
[a] M. Fierz and W.Pauli, *Proc. R. Soc. Lond. A* **173** 21 (1939)

[b] K. -I. Sato, arXiv:hep-th/0501042 (2005)

[c] C. de Rham, arXiv:hep-th/1401.4173 (2014)

[d] The traceless condition h^μ_μ eliminates another dof

Perturbative unitarity and Higgs



- Consider $W_L^+ W_L^- \to W_L^+ W_L^-$ scattering in a Higgsless theory[12,13,14]:

$$\mathcal{M}_{\text{gauge}} = \frac{1}{v^2}\left(\underbrace{-(s^2+4st+t^2)}_{Z/\gamma \text{ exchanges}} + \underbrace{(s^2+4st+t^2)}_{\text{direct } WWWW \text{ coupling}} +(s+t)+\mathcal{O}(E^0)\right)$$

- Unitarity violation:

$$a^{j=0}_{\text{gauge}} = \frac{s}{32\pi v^2} \leq 1 \quad \Longrightarrow \quad \sqrt{s} \lesssim 1.7 \text{ TeV}$$

[12] B. W. Lee, C. Quigg and H. B. Thacker, *Phys. Rev. D* **16** 1519 (1977)

[13] M. Kladiva, Diploma thesis, Charles University in Prague (2003)

[14] M. Gillioz, PhD thesis, Universität Zürich (2012)

Perturbative unitarity and Higgs

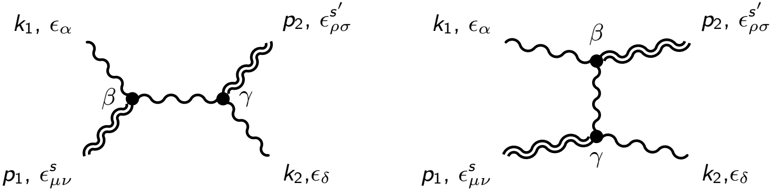
- Higgs enters with additional contribution:

$$\mathcal{M} \quad = \quad \frac{1}{v^2}\left(s + t - \frac{s^2}{s - m_H^2} - \frac{t^2}{t - m_H^2} - m_H^2\left(\frac{s}{s - m_H^2} + \frac{t}{t - m_H^2}\right)\right)$$

- The unitary constraint on the Higgs mass:

$$|a^{j=0}(W_L W_L \to W_L W_L)| = \frac{m_H^2}{8\pi v^2} \leq 1 \quad \stackrel{s \gg m_H}{\Longrightarrow} \quad m_H \leq 2\sqrt{\pi} v \approx 870 \text{ GeV}$$

$hZ \to hZ$



- Work in centre of momentum frame with $|\mathbf{p}_1| = |\mathbf{k}_1| = p$ define:

$$\cos\theta := \frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{|\mathbf{p}_1||\mathbf{p}_2|}$$

- Spin-1 propagator:

$$\frac{-\eta^{\beta\gamma} + q^\beta q^\gamma}{q^2 - m_V^2} \sim \mathcal{O}(1)$$

Polarisation for $J = 2$

- Polarisations for spin-2 are constructed from those of a spin-1

Spin-1

$$\epsilon^0_\mu = (\frac{|\mathbf{p}|}{m}, -\frac{\mathbf{p}}{|\mathbf{p}|}\frac{p_0}{m}) = \frac{p_\mu}{m} + \mathcal{O}\left(\frac{m}{p^0}\right), \qquad \epsilon^\pm_\mu = \frac{1}{\sqrt{2}}(\epsilon^1_\mu \pm i\epsilon^2_\mu)$$

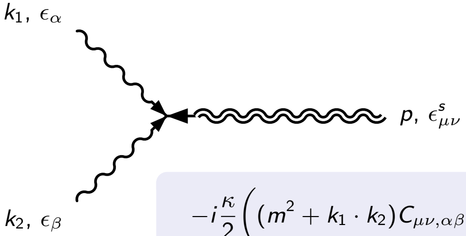
$$\epsilon^{\pm 2}_{\mu\nu} = \epsilon^\pm_\mu \epsilon^\pm_\nu, \sim \mathcal{O}(1)$$

$$\epsilon^{\pm}_{\mu\nu} = \frac{1}{\sqrt{2}}(\epsilon^\pm_\mu \epsilon^0_\nu + \epsilon^0_\mu \epsilon^\pm_\nu) \sim \mathcal{O}(p)$$

$$\epsilon^{0}_{\mu\nu} = \frac{1}{\sqrt{6}}(\epsilon^+_\mu \epsilon^-_\nu + \epsilon^-_\mu \epsilon^+_\nu - 2\epsilon^0_\mu \epsilon^0_\nu) \approx -\sqrt{\frac{2}{3}}\epsilon^0_\mu \epsilon^0_\nu \sim \mathcal{O}(p^2)$$

Highest energy dependence is given by longitudinal components

Spin-2 Feynman rule



$$-i\frac{\kappa}{2}\Big((m^2 + k_1 \cdot k_2)C_{\mu\nu,\alpha\beta} + D_{\mu\nu,\alpha\beta}(k_1, k_2)\Big) \sim \mathcal{O}(p^2)$$

where[15]:

$$C_{\mu\nu,\alpha\beta} := \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta},$$

$$D_{\mu\nu,\alpha\beta}(k_1, k_2) := \eta_{\mu\nu}k_{1\beta}k_{2\alpha} - \Big[\eta_{\mu\alpha}k_{1\beta}k_{2\nu} + \eta_{\mu\beta}k_{1\nu}k_{2\alpha} - \eta_{\alpha\beta}k_{1\mu}k_{2\nu} + (\mu \leftrightarrow \nu)\Big]$$

[15] T. Han, J. D. Lykken and R.-J. Zhang, *Phys. Rev. D* **59** 105006 (1999)

Landau Yang

- Also extended to *ZZ* decay[16]
- Proof below[17]:

5.5 Landau-Yang Theorem and its Generalization

In 1949 C.N. Yang wrote a paper called "Selection Rules for the Dematerialization of a Particle into Two Photons". It was motivated by an observation made by John Archibald Wheeler that positronium in the S-triplet state can not decay into two photons. Positronium is a bound state of an electron and a positron, the S refers to the fact that they have no relative orbital angular momentum and the triplet state means that they have their intrinsic spins aligned. It is therefore a state with total angular momentum $J = 1$. We will see that such a decay ($J = 1$ initial particle) is forbidden by selection rules. The selection rules are derived from principles of invariance under rotations in space and inversion (parity).

Yang starts the paper with saying "Consider two photons of equal wavelength λ_0 traveling in opposite directions along a z-axis". This is the same physical situation as if a particle X decayed into two photons. The different two-particle states of the system are denoted by $\Psi^{\lambda_1\lambda_2}$, where $\lambda_{1,2}$ takes the values R and L. The first index refers to the photon propagating in the $+z$ direction and the second to the photon propagating along the $-z$ direction. R means that the photon has polarization along its direction of motion, while L means polarization opposite the direction of motion. There are four different states to be considered, namely Ψ^{RR}, Ψ^{RL}, Ψ^{LR} and Ψ^{LL}. Figure 5.3 illustrates this. Yang goes on to see how these states are changed under the following operations:

1. R_θ, rotation by θ around z-axis
2. R_ϕ 180 degrees rotation around x-axis
3. P, reflection

Remembering that states are transformed by unitary operators under symmetry operations, we take a look at the result. It is summarized in table 5.1, which is reproduced from the original paper. In the C.O.M. frame the following arguments for the selection rules are made:

1. For an odd initial state (- under parity), the initial particle must decay to $\Psi^{RR} - \Psi^{LL}$, since it is the only odd final state

	$\Psi^{RR} + \Psi^{LL}$	Ψ^{RR} - Ψ^{LL}	Ψ^{RL}	Ψ^{LR}
R_θ rotation by θ around z-axis	1	1	$e^{2i\theta}$	$e^{-2i\theta}$
R_ϕ 180 degrees rotation around x-axis	1	1		
P, reflection	1	-1	1	1

Table 5.1: Eigenvalues of symmetry operators on two-photon helicity states

Parity(▽) J(▷)	0	1	2,4,6 ...	3,5,7 ...
even	$\Psi^{RR} + \Psi^{LL}$	forbidden	$\Psi^{RR} + \Psi^{LL}$, Ψ^{RL}, Ψ^{LR}	Ψ^{RL}, Ψ^{LR}
odd	$\Psi^{RR} - \Psi^{LL}$	forbidden	$\Psi^{RR} - \Psi^{LL}$	forbidden

Table 5.2: Selection rules for two-photon helicity states

2. An even initial state must go into one of the three final states Ψ^{RL}, Ψ^{LR} or $\Psi^{RR} + \Psi^{LL}$
3. For an initial state with angular momentum $J = 1, 3, 5...$, the only possible final states are Ψ^{RL} and Ψ^{LR}. We can rule out the other two because: $\Psi^{RR} + \Psi^{LL}$ and Ψ^{RR} - Ψ^{LL} are simultaneous eigenstates of R_θ and R_ϕ with eigenvalue 1, while an initial state with eigenvalue 1 under R_θ will have eigenvalue -1 under R_ϕ[1]
4. For an initial state with $J = 0, 1$ the only possible final states are $\Psi^{RR} + \Psi^{LL}$ and Ψ^{RR} - Ψ^{LL}. This is because the other two states have angular momentum $J = \pm 2\hbar$, which is to large for $J = 0$ or $J = 1$

Looking at the $J = 1$ case, argument 3 rules out $\Psi^{RR} + \Psi^{LL}$ and Ψ^{RR} - Ψ^{LL} as final states, while argument 4 rules out Ψ^{RL} and Ψ^{LR} as final states. Therefore no such decays should take place. These results are summarized here as table 5.2.

In the paper written in 2008, a similar analysis is made, but with two Z bosons instead of photons. The difference now is that massive particles have one extra degree

[1] Yang mentions that such a state has the rotational properties of the spherical harmonics. The spherical harmonics of odd powers of angular momentum, have a factor $e^{im\phi}$, where ϕ is the azimuthal angle and m is the projection of spin along the z-axis (odd number). Rotating about the x-axis means $\phi' \to \phi + \pi$, which gives a factor of -1

[16] W. -Y. Keung, I. Low and J. Shu, *Phys. Rev. Lett.* **101** 091802 (2008)

[17] A. Haarr, Master thesis The University of Bergen (2011)

Slide on Landau-Yang[18,19,20]

- In the cm-frame

$$|2\gamma\rangle_i = \int d^3p \chi_{ijk}(\vec{p}) a_j^\dagger(\vec{p}) a_k^\dagger(-\vec{p})|0\rangle$$

- Lorentz (rotational)

$$\chi_{ijk}(\vec{p}) \sim \epsilon_{ijk}, p_i\delta_{jk}, p_j\delta_{ik}, p_k\delta_{ij}, p_ip_jp_k$$
$$p_i\epsilon_{jkn}p_n, p_j\epsilon_{kin}p_n, p_k\epsilon_{ijn}p_n$$

- Gauge: $p_i a_i^\dagger(\vec{p}) = 0$, $\chi_{ijk}(\vec{p}) \sim \epsilon_{ijk}, p_i\delta_{jk}, p_i\epsilon_{jkn}p_n$

$$\chi_{ijk}(\vec{p}) = -\chi_{ikj}(-\vec{p})$$

- Bose

$$\begin{aligned} |2\gamma\rangle_i &= \frac{1}{2}\int d^3p \left[\chi_{ijk}(\vec{p}) + \chi_{ikj}(-\vec{p})\right] a_j^\dagger(\vec{p}) a_k^\dagger(-\vec{p})|0\rangle \\ &= 0 \end{aligned}$$

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[18] M. Luo, L. Wang and G. Zhu, talk, KITP (2008)

[19] based on L. Randall and M. B. Wise, arXiv:hep-ph/0807.1746 (2008)

[20] see also hep-ph/9406398v1;hep-ph/1102.5702