# Hadronization effects in $\tau \rightarrow \pi \gamma \nu_{\tau}$ decays

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Work done in collaboration with Z. H. Guo



Manchester, 14/09/10

## **SUMMARY:**

• Hadron decays of the  $\tau$  lepton

- Theoretical setting:  $\chi PT$ , Large  $N_c$ ,  $R\chi T$ 

•  $\tau^- \rightarrow \pi^- \gamma \nu_{\tau}$ 

Conclusions and Outlook

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$$\mathsf{M} = \frac{G_F}{\sqrt{2}} V_{CKM} \overline{u}(\nu_{\tau}) \gamma^{\mu} (1 - \gamma_5) u(\tau) T_{\mu}$$

 $T_{\mu} = \langle Hadrons | (V-A)_{\mu} e^{iS_{QCD}} | 0 \rangle = \Sigma_i (Lorentz Structure)^i F_i(Q^2, s_j)$ 

$$d\Gamma = \frac{G_F^2}{4M_\tau^2} |V_{CKM}|^2 d\Phi^{(n+1)} L_{\mu\nu} T^{\mu} T^{\nu*}$$

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Talks by M. Jamin and A.Pich

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$$\mathsf{M} = \frac{G_F}{\sqrt{2}} V_{CKM} \overline{u} (\nu_{\tau}) \gamma^{\mu} (1 - \gamma_5) u(\tau) T_{\mu}$$
  
Structure  
independent 
$$\begin{bmatrix} i\mathcal{M}_{\mathrm{IB}_{\tau+\pi}} = G_F V_{ud} e F_{\pi} m_{\tau} \epsilon^{\nu}(k) \overline{u}_{\nu_{\tau}}(q) (1 + \gamma_5) \left( \frac{s_{\nu}}{s \cdot k} - \frac{p_{\nu}}{p \cdot k} - \frac{k\gamma_{\nu}}{2s \cdot k} \right) u_{\tau}(s)$$

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$$\frac{d^2\Gamma}{dx\,dy} = \frac{m_\tau}{256\pi^3} \overline{|\mathcal{M}|^2}$$

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 $x := \frac{2s \cdot k}{m_{\tau}^2} \qquad \qquad y := \frac{2s \cdot p}{m_{\tau}^2} \qquad \qquad E_{\gamma} = \frac{m_{\tau}}{2}x \qquad \qquad E_{\pi} = \frac{m_{\tau}}{2}y$ Hadronization in  $\tau \to \pi \gamma v_{\tau}$  decays



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 $\tau^- \to \pi^- \gamma \nu_\tau$ 

#### Hadronic contributions

Axial form factor

Vector form factor

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## The program (for hadronic $\tau$ decays)

- After evaluating the matrix elements, we require the short-distance QCD constraints. This reduces the number of independent couplings and renders RχT predictive.
- Then we perform a phenomenological analysis using all the available information at hand.
- For the previous step a faithful description of the off-shell width of the broadest resonances is mandatory. (Phys.Rev.D62:054014,2000; Phys.Lett.B685:158-164,2010)

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## High-energy QCD constraints on $\tau^- \rightarrow \pi^- \gamma \nu_{\tau}$

(more details in backup slides)

• If one subtraction is assumed, no conditions on **axial** form factor.

(Decker, Finkemeier '93)

• If no subtraction is assumed in the **axial** form factor, the results are **consistent** with those in  $\tau^- \rightarrow (PPP)^- \nu_{\tau}$ 

(Phys.Rev.D81:034031,2010; Phys.Lett.B685:158-164,2010)

$$F_V^P(t 
ightarrow -\infty) = rac{F}{t}$$
 (Brodsky, Lepage '79, '81)

• In the VFF the results are **consistent** with those in  $\tau^- \rightarrow (PPP)^- \nu_{\tau}$ 

(Phys.Rev.D81:034031,2010)

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# The program (for $\tau^- \rightarrow \pi^- \gamma \nu_{\tau}$ )

- Short-distance QCD constraints required to the participating axial-vector and vector form-factors: 10 unknowns  $\rightarrow$  2 free couplings (isospin breaking).
- These 2 unknowns can be predicted using QCD highenergy conditions for the VVP Green Function (JHEP 0307:003,2003)
- Since this mode has not been measured yet there are no experimental constraints but we can give a parameter-free prediction to be tested with the discovery data.

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Model independent prediction: Only WZW for the VFF



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Model independent prediction: Only WZW for the VFF



For any reasonable cut on  $E_{\gamma}$ , this decay should have already been discovered by the heavy-flavour factories

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All contributions



Hadronization in  $\tau \to \pi \; \gamma \; \nu_\tau$  decays

All contributions



$$\Gamma(\tau^- \rightarrow \pi^- \nu_{\tau}) = 2.471 \cdot 10^{-13} \text{ GeV}$$

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# Use in data analysis

- τ decay dynamics is interesting in low-energy experiments (Eur. Phys. J.C66:585,2010).
- In order to obtain full benefit of precise data collected at τ-c factories, one should exploit the synergies of theory, and MCGen for bkg estimation and data analysis. For this purpose, TAUOLA (Z. Was talk, arXiv:1001.0070 hep/ph) is an essential tool at disposal of the experimental community that can be interfaced to their software (arXiv:0812.3215 hep/ph).
- There are as well interesting applications in high-energy Physics. In particular, in the Higgs discovery program at ATLAS (arXiv:0901.0512 hep/ex, arXiv:0903.4198 hep/ex)
- Close communication between experts in the theory and MC side and experimental Collaborations should be fostered (TAU10 conference and the satellite WG meeting are ideal arenas for that).

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# CONCLUSIONS

- Resonance Chiral Theory is a convenient framework to study hadron decays of the tau based on some properties of QCD: its chiral limit, its large-N<sub>c</sub> limit and its known asymptotic behaviour.
- We have applied to the study of the  $\tau^- \rightarrow \pi^- \gamma v_{\tau}$ , decays and checked the consistency of the whole procedure with previous results in other  $\tau^- \rightarrow (PPP)^- v_{\tau}$  processes.
- This rare decay is of great interest for the B- and τ-c-factories and should be discovered soon allowing for stringent tests of the SM through suitable ratios (Z.H. Guo and P. Roig, in progress).
- Our results are being implemented in TAUOLA (more details in the satellite meeting at Liverpool, 18th-19th of September) providing the experimental community a theory based tool to analyze these decays.

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# BACKUP SLIDES



Hadronization in  $\tau \rightarrow \pi \gamma \nu_{\tau}$  decays

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## Axial form factor and $a_1: \tau \rightarrow (3\pi)^- \nu_{\underline{\tau}}$ (Gómez-Dumm, Pich, Portolés '04) (Gómez-Dumm, Pich, Portolés, R. arXiv:0911.4436) 7 unknown couplings

Brodsky-Lepage behaviour demanded to the Form Factors (7-6 = 1 coupling).

We have **improved** the off-shell description of the  $a_1$  width by including all cuts corresponding to  $3\pi$  and KK $\pi$  intermediate states in the A-A correlator.

The value of this coupling that provides a pretty **accurate description of ALEPH data** is **consistent with** the prediction from **<VAP>** (Cirigliano, Ecker, Eidemüller, Pich, Portolés '04).

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## <u>Axial-FF and the $a_1$ : $\Gamma_a$ (in TAUOLA)</u>





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$$\underbrace{\mathsf{PT: The low-energy}}_{\mathbf{EFT of QCD}}$$
(Gasser & Leutwyler '84, '85)  
$$\underbrace{\mathsf{eft of QCD}}_{(x) = \begin{pmatrix} \frac{\pi^{0}}{\sqrt{2}} + \frac{\eta_{8}}{\sqrt{6}} & \pi^{+} & K^{+} \\ \pi^{-} & -\frac{\pi^{0}}{\sqrt{2}} + \frac{\eta_{8}}{\sqrt{6}} & K^{0} \\ \pi^{-} & -\frac{\pi^{0}}{\sqrt{2}} + \frac{\eta_{8}}{\sqrt{6}} & K^{0} \\ K^{-} & \overline{K}^{0} & -\frac{2\eta_{8}}{\sqrt{6}} \end{pmatrix}$$
(Goidstone Bosons)  
$$SU(3)_{L} \otimes SU(3)_{R} \rightarrow SU(3)_{V}$$
$$u(x) = \exp\left(\frac{i\phi(x)}{\sqrt{2F}}\right), \quad u_{\mu} = i\left[u^{\dagger}(\partial_{\mu} - ir_{\mu})u - u(\partial_{\mu} - il_{\mu})u^{\dagger}\right]$$
$$\chi = 2B_{0}(s + ip), \quad \chi_{\pm} = u^{\dagger}\chi u^{\dagger} \pm u\chi u$$
$$f_{\pm}^{\mu\nu} = uF_{L}^{\mu\nu}u^{\dagger} \pm u^{\dagger}F_{R}^{\mu\nu}u$$
$$\mathcal{L}^{(2)}_{\chi} = \frac{F^{2}}{4}\langle u_{\mu}u^{\mu} + \chi_{+} \rangle$$
$$\mathcal{L}^{(4)}_{\chi} = L_{1}\langle u_{\mu}u^{\mu}\rangle^{2} + \ldots + L_{4}\langle u_{\mu}u^{\mu}\rangle \langle \chi_{+}\rangle + \ldots + L_{7}\langle \chi_{-}\rangle^{2} + \ldots - iL_{9}\langle f_{+}^{\mu\nu}u_{\mu}u_{\nu}\rangle + \ldots$$
$$\mathcal{L}^{(4)}_{\chi, WZW} \text{ in the odd-intrinsic parity sector}$$

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## <u>RχT matching to the OPE allows it to</u> <u>reproduce QCD high-energy behaviour:</u>



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**Resonancest**  
**Goldstone**  
**Bosons**

$$\mathbf{TOOLS: R\chiT}$$
(Ecker, Gasser, Pich, De Rafael '89)  
(Ecker, Gasser, Leutwyler, Pich, De  
Rafael '89)  

$$\mathcal{L}^{(P_{1}=+)}_{R_{Z}T} = \mathcal{L}^{(2)}_{Z} + \mathcal{L}^{tin}_{V,A} + \mathcal{L}_{V} + \mathcal{L}_{A} + \mathcal{L}_{VAP};$$

$$\mathcal{L}_{V} = \frac{F_{V}}{2\sqrt{2}} \langle V_{\mu\nu} f_{+}^{\mu\nu} \rangle + \frac{iG_{V}}{\sqrt{2}} \langle V_{\mu\nu} u^{\mu} u^{\nu} \rangle$$
**Antisymmetric tensor formalism**

$$\begin{pmatrix} \rho^{0} \\ \sqrt{2} + \frac{\omega_{8}}{\sqrt{6}} \\ \rho^{-} \\ -\frac{\rho^{0}}{\sqrt{2}} + \frac{\omega_{8}}{\sqrt{6}} \\ \kappa^{*-} \\ \overline{\mathcal{K}^{*0}} \\ -\frac{2\omega_{8}}{\sqrt{6}} \\ \kappa^{*-} \\ \overline{\mathcal{K}^{*0}} \\ -\frac{2\omega_{8}}{\sqrt{6}} \\ \kappa^{*-} \\ \overline{\mathcal{K}^{*0}} \\ -\frac{2\omega_{8}}{\sqrt{6}} \\ \mu_{\nu} \\$$

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(Ecker, Gasser, Pich, De Rafael '89)

(Ecker, Gasser, Leutwyler, Pich, De Rafael '89) ,...



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Hadronization in  $\tau \rightarrow \pi \gamma \nu_{\tau}$  decays

1

0.01

0

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2

 $Q^2 (GeV^2)$ 

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3



## <u>The axial-form factor and the $a_1: \tau^- \rightarrow (3\pi)^- \nu_{\tau}$ </u>

(Gómez-Dumm, Pich, Portolés '00) (Gómez-Dumm, Pich, Portolés, R. arXiv:0911.4436)

$$\begin{split} &\Gamma_{\rho}(s) = \frac{M_{\rho}s}{96\pi F^{2}} \left[ \sigma_{\pi}^{3} \Theta(s - 4m_{\pi}^{2}) + \frac{1}{2} \sigma_{K}^{3} \Theta(s - 4m_{K}^{2}) \right] \\ &\Gamma_{a_{1}}(Q^{2}) = \Gamma_{a_{1}}^{3\pi}(Q^{2}) + \Gamma_{a_{1}}^{K\overline{K}\pi}(Q^{2}) + \Gamma_{a_{1}}^{(K\pi)^{0}K^{0}}(Q^{2}), \\ &\Gamma_{a_{1}}^{3\pi}(Q^{2}) = \frac{1}{48(2\pi)^{3}M_{a_{1}}} \left( \frac{Q^{2}}{M_{a_{1}}^{2}} \right) \iint ds dt \ F_{1}^{'}V_{1\mu} + F_{2}^{'}V_{2\mu} \\ &F_{1}^{'\dagger}V_{1\mu} + F_{2}^{'\dagger}V_{2\mu} \ , \ F_{i}^{'} = F_{i} \frac{M_{a_{1}}^{2} - Q^{2}}{\sqrt{2}F_{A}Q^{2}} \end{split}$$

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