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Bouncing Cosmological Solutions in Scalar-Tensor Gravity

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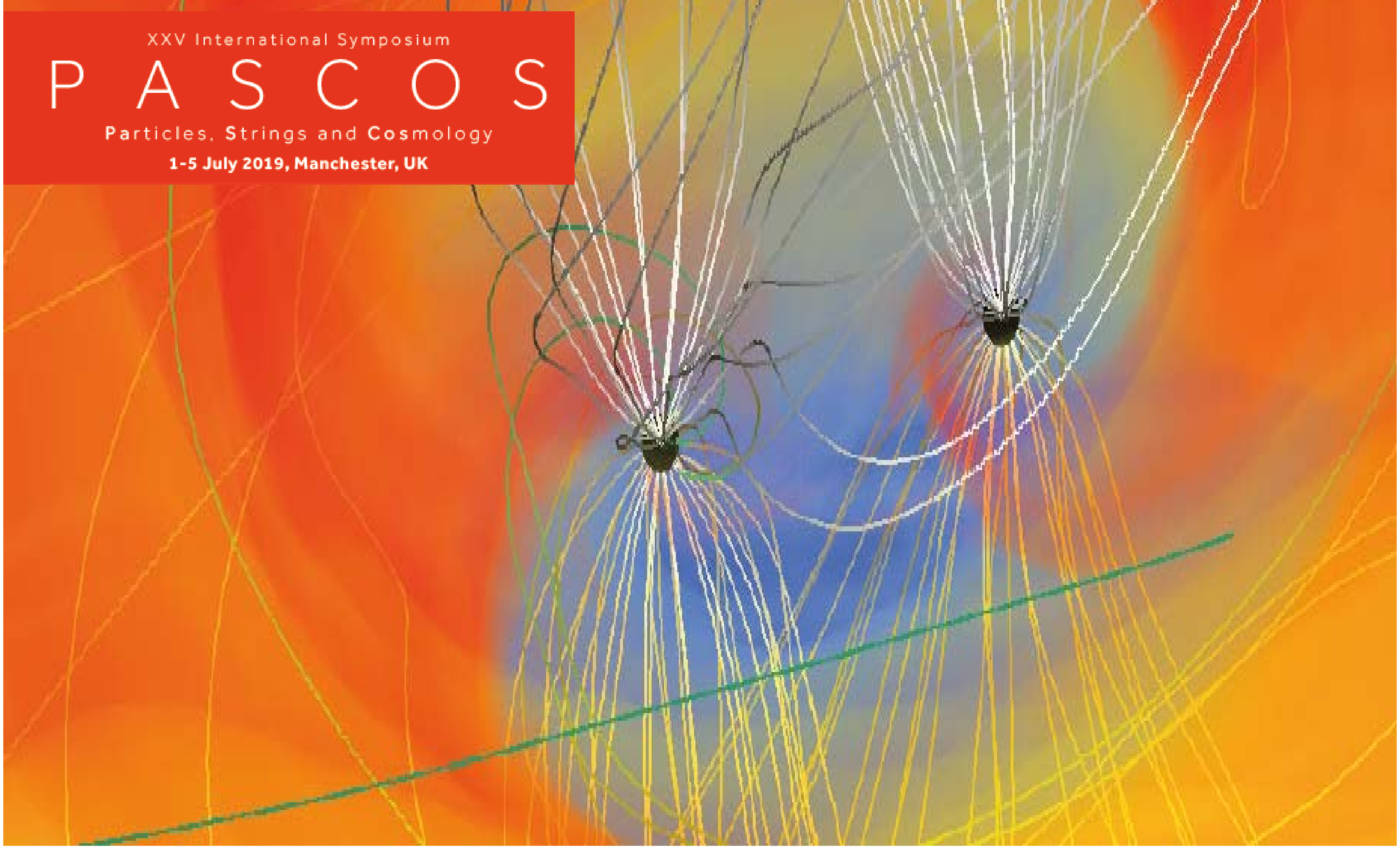


Scalar-Tensor non-minimal Coupling

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa}f(\Phi)R + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - U(\Phi)\right)$$

$\kappa = 8\pi G$

$$U(\Phi) = U_0 + \frac{m^2}{2}\Phi^2 + \frac{\lambda}{4}\Phi^4 + \frac{\nu}{6}\Phi^6$$

Effective (field-dependent) Newon constant: $G_{eff}(\Phi) = G / f(\Phi)$

Typical example: non-minimal coupling function:

$f(\Phi) = 1 - \xi\kappa\Phi^2$. $\xi = 1/6$ - conformal coupling.

signature = (+, - , - , -)

Inspiration: 3 papers by
Boisseau, Giacomini, Polarski & Starobinsky
(2015-2016)

General Field Equations:

$$\nabla_\mu \nabla^\mu \Phi + U'(\Phi) - \frac{R}{2\kappa} f'(\Phi) = 0$$

$$f(\Phi) G_{\mu\nu} + \kappa T^{(min)}_{\mu\nu} + \nabla_\mu \nabla_\nu f(\Phi) - g_{\mu\nu} \nabla_\rho \nabla^\rho f(\Phi) = 0$$

Einstein tensor

$$T^{(min)}_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - \left(\frac{1}{2} \partial_\rho \Phi \partial^\rho \Phi - U(\Phi) \right) g_{\mu\nu}$$

Tracing Einstein equations:

$$f(\Phi) R = 4\kappa U(\Phi) - \left(\kappa + 3 f''(\Phi)\right) \partial_\mu \Phi \partial^\mu \Phi - 3 f'(\Phi) \nabla_\mu \nabla^\mu \Phi$$

gives:

$$\left(2\kappa f(\Phi) + 3 f'(\Phi)^2\right) \nabla_\mu \nabla^\mu \Phi + 2\kappa f(\Phi) U'(\Phi) - f'(\Phi) \left(4\kappa U(\Phi) - \left(\kappa + 3 f''(\Phi)\right) \partial_\mu \Phi \partial^\mu \Phi\right) = 0$$

"effective force" $U'_{eff}(\Phi)$

Cosmology: equations for $a(t), \Phi(t)$

$ds^2 = dt^2 - a^2(t)(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2)$; *Hubble function* $H = \dot{a}/a$

$$\left(1 + \frac{3f'(\Phi)^2}{2\kappa f(\Phi)}\right)\left(\ddot{\Phi} + \frac{3\dot{a}}{a}\dot{\Phi}\right) + \frac{f'(\Phi)\left(\kappa + 3f''(\Phi)\right)}{2\kappa f(\Phi)}\dot{\Phi}^2 - \frac{2f'(\Phi)U(\Phi)}{f(\Phi)} + U'(\Phi) = 0$$

$$\left(2 + \frac{3f'(\Phi)^2}{\kappa f(\Phi)}\right)\frac{\ddot{a}}{a} + \left(1 + \frac{3f'(\Phi)^2}{\kappa f(\Phi)}\right)\frac{\dot{a}^2 + k}{a^2} - \frac{\dot{a}}{a}\frac{f'(\Phi)}{f(\Phi)}\dot{\Phi} + \frac{\kappa + 2f''(\Phi)}{2f(\Phi)}\dot{\Phi}^2 - \frac{f'(\Phi)U'(\Phi)}{f(\Phi)} - \frac{\kappa U(\Phi)}{f(\Phi)} = 0$$

1st order constraint:

$$\frac{\dot{a}^2 + k}{a^2} + \frac{\dot{a}}{a}\frac{f'(\Phi)}{f(\Phi)}\dot{\Phi} - \frac{\kappa}{3f(\Phi)}\left(\frac{\dot{\Phi}^2}{2} + U(\Phi)\right) = 0$$

Space curvature k is determined by energy density at a given time

Effective potential: $$U'_{eff}(\Phi) = \frac{f(\Phi)U'(\Phi) - 2f'(\Phi)U(\Phi)}{f(\Phi) + 3f'(\Phi)^2/(2\kappa)}$$

for $f(\Phi) = 1 - \xi\kappa\Phi^2$:
$$U'_{eff}(\Phi) = \frac{(1-\xi\kappa\Phi^2)U'(\Phi) + 4\kappa\xi\Phi U(\Phi)}{1-\kappa\xi(1-6\xi)\Phi^2}$$

and for a polynomial potential $U(\Phi) = U_0 + \frac{m^2}{2}\Phi^2 + \frac{\lambda}{4}\Phi^4 + \frac{\nu}{6}\Phi^6$ *:*

$$U_{eff}(\Phi) = \frac{\nu}{18(1-6\xi)}\Phi^6 - \frac{\nu(1-9\xi)}{6\kappa\xi(1-6\xi)^2}\Phi^4 - \left(\frac{\nu(1-9\xi)}{3(\kappa\xi)^2(1-6\xi)^3} + \frac{\lambda+\kappa\xi m^2}{2\kappa\xi(1-6\xi)}\right)\Phi^2 - \left(\frac{\nu(1-9\xi)}{3(\kappa\xi)^3(1-6\xi)^4} + \frac{\lambda}{2(\kappa\xi)^2(1-6\xi)^2} + \frac{(1-3\xi)m^2}{\kappa\xi(1-6\xi)^2} + \frac{2U_0}{1-6\xi}\right)\log\left(1-\kappa\xi(1-6\xi)\Phi^2\right)$$

$$\xi = 1/6 \Rightarrow \quad U_{eff}(\Phi) = \left(\frac{m^2}{2} + \frac{\kappa U_0}{3}\right)\Phi^2 + \frac{1}{4}\left(\lambda + \frac{\kappa m^2}{6}\right)\Phi^4 + \frac{\nu}{6}\Phi^6 - \frac{\kappa\nu}{144}\Phi^8$$

Use rescaled (dimensionless) quantities: $\sqrt{\xi\kappa}\,\Phi \to \Phi$, $\sqrt{\xi\kappa U_0}\,x^\mu \to x^\mu$ *and similarly for* m, λ *etc...*

"effectively": $U_0 = 1$, $\kappa = 1/\xi$ ...
$$\bar{U}_{eff}(\bar{\Phi}) = \left(\frac{\bar{m}^2}{2} + 2\right)\bar{\Phi}^2 + \frac{1}{4}\left(\bar{\lambda} + \bar{m}^2\right)\bar{\Phi}^4 + \frac{\bar{\nu}}{6}\bar{\Phi}^6 - \frac{\bar{\nu}}{24}\bar{\Phi}^8$$

Field Equations for Conformal Coupling:

$f(\Phi) = 1 - \xi\kappa\Phi^2 \;, \xi = 1/6$ Flat FLRW

$$\ddot{\bar{\Phi}} + \frac{3\dot{a}}{a}\dot{\bar{\Phi}} + \left(\bar{m}^2 + 4\right)\bar{\Phi} + \left(\bar{\lambda} + \bar{m}^2\right)\bar{\Phi}^3 + \bar{\nu}\bar{\Phi}^5 - \frac{\bar{\nu}}{3}\bar{\Phi}^7 = 0$$

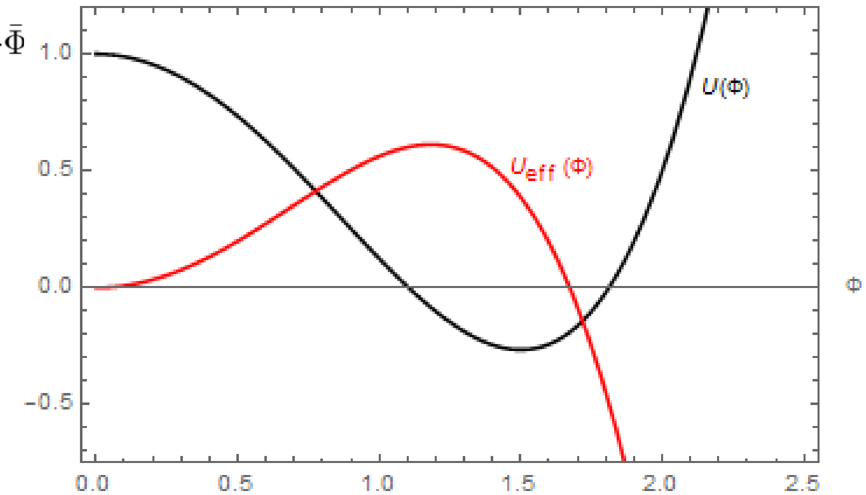
$$\left(1 - \bar{\Phi}^2\right)\frac{\dot{a}^2}{a^2} - 2\frac{\dot{a}}{a}\bar{\Phi}\dot{\bar{\Phi}} = 2\left(\frac{\dot{\bar{\Phi}}^2}{2} + 1 + \frac{\bar{m}^2}{2}\bar{\Phi}^2 + \frac{\bar{\lambda}}{4}\bar{\Phi}^4 + \frac{\bar{\nu}}{6}\bar{\Phi}^6\right)$$

Central tool: the effective potential:

$$\bar{U}_{eff}(\bar{\Phi}) = \left(\frac{\bar{m}^2}{2} + 2\right)\bar{\Phi}^2 + \frac{1}{4}\left(\bar{\lambda} + \bar{m}^2\right)\bar{\Phi}^4 + \frac{\bar{\nu}}{6}\bar{\Phi}^6 - \frac{\bar{\nu}}{24}\bar{\Phi}$$

Most obvious case for bounce solutions: $U_{eff}(\Phi)$ *has a maximum. If* $\nu = 0$,

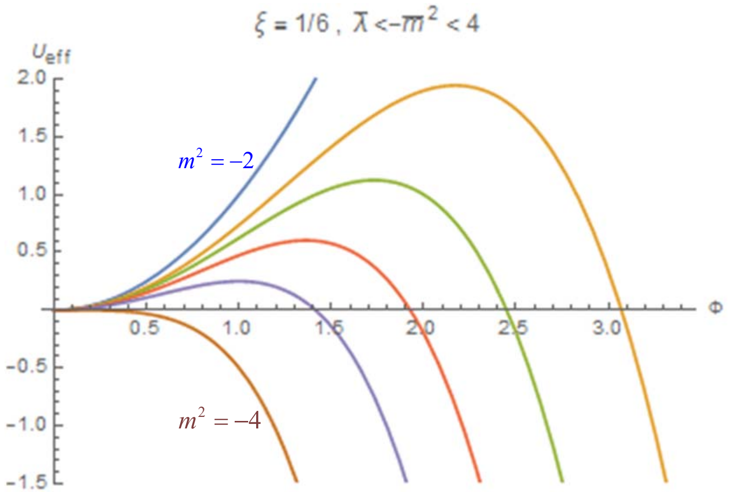
Condition: $-4 < \bar{m}^2 < -\bar{\lambda}$



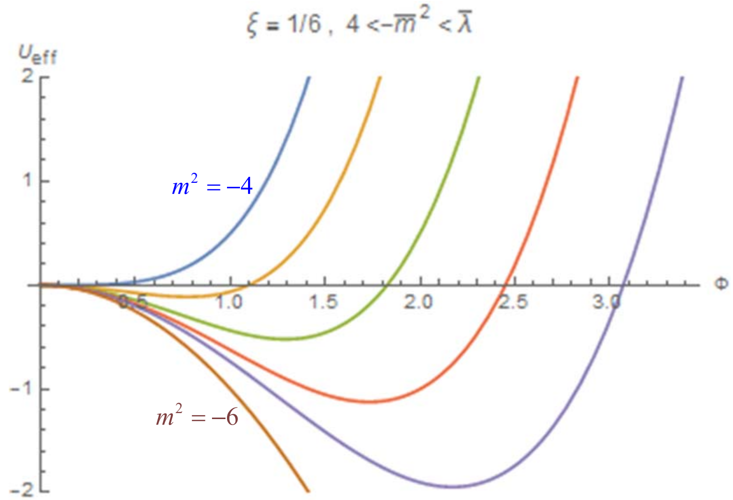
Effective potential $\xi = 1/6$

$U_{eff}(\Phi)$ may have 4 types of behavior: monotonically increasing/decreasing, or having one maximum or one minimum. Up to reflection symmetry.

$v = 0$

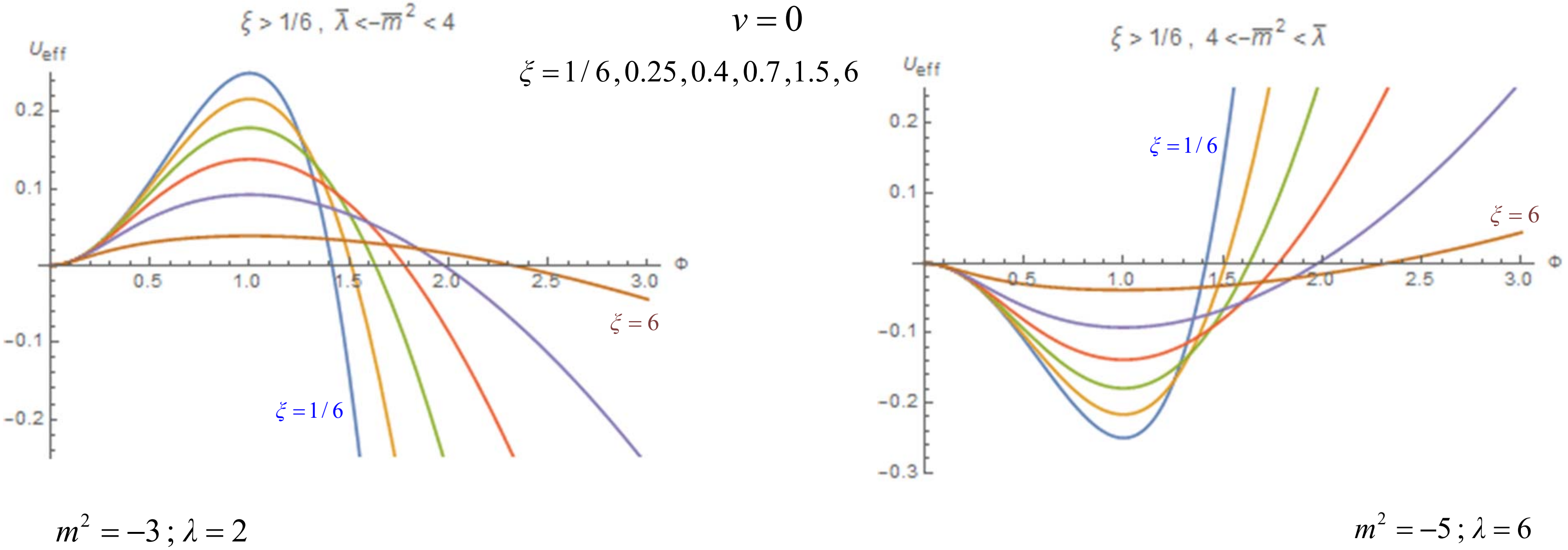


$\lambda = 2\,; -m^2 = 2, 2.35, 2.5, 2.7, 3, 4$



$\lambda = 6\,; -m^2 = 4, 4.75, 5.25, 5.5, 5.65, 6$

Effective potential $\xi > 1/6$:



Note common position (ξ-independent) of extremal points in dimensionless formulation

Analysis easier in terms of "force":

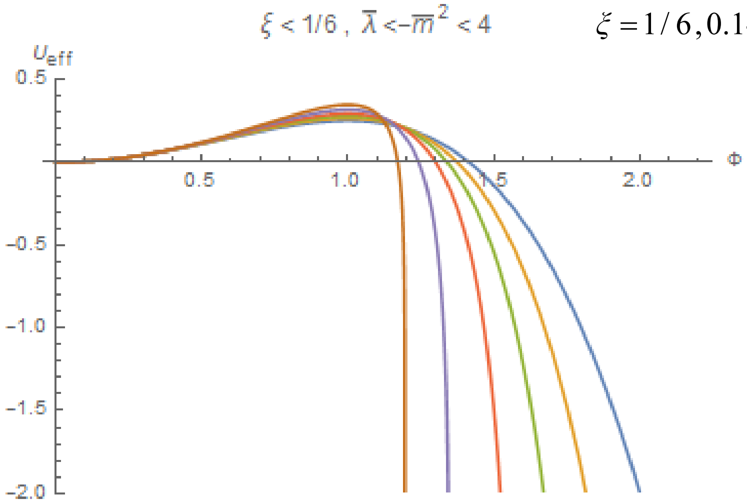
$$\bar{U'}_{eff}(\bar{\Phi}) = \frac{\left(\bar{m}^2 + 4\right)\bar{\Phi} + \left(\bar{\lambda} + \bar{m}^2\right)\bar{\Phi}^3 + \bar{\nu}\bar{\Phi}^5 - \frac{1}{3}\bar{\nu}\bar{\Phi}^7}{1 - (1 - 6\xi)\bar{\Phi}^2}$$

Effective potential $0 < \xi < 1/6$:

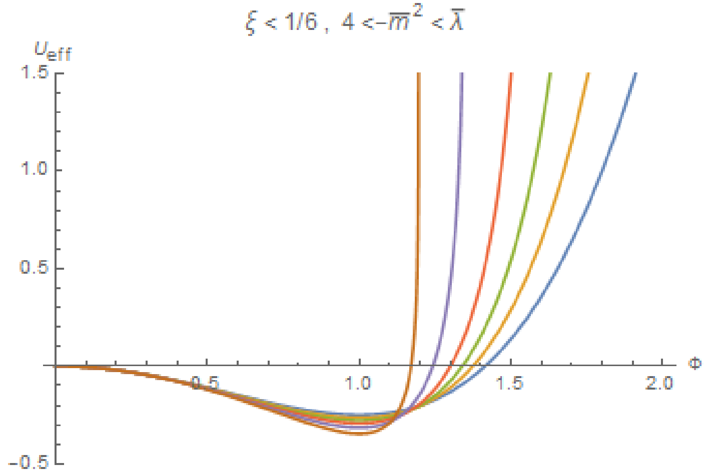
field bounded $\bar{\Phi}^2 < 1/(1-6\xi)$

$v = 0$

$\xi = 1/6, 0.14, 0.12, 0.1, 0.075, 0.05$



$m^2 = -3\,;\, \lambda = 2$



$m^2 = -5\,;\, \lambda = 6$

Note ξ-independence of extremal points

$$\bar{U}'_{eff}(\bar{\Phi}) = \frac{\left(\bar{m}^2 + 4\right)\bar{\Phi} + \left(\bar{\lambda} + \bar{m}^2\right)\bar{\Phi}^3 + \bar{\nu}\bar{\Phi}^5 - \frac{1}{3}\bar{\nu}\bar{\Phi}^7}{1 - (1-6\xi)\bar{\Phi}^2}$$

Back to $\xi = 1/6$: Asymptotic Behavior / Critical points

$$\ddot{\bar{\Phi}} + \frac{3\dot{a}}{a}\dot{\bar{\Phi}} + \left(\bar{m}^2 + 4\right)\bar{\Phi} + \left(\bar{\lambda} + \bar{m}^2\right)\bar{\Phi}^3 + \bar{\nu}\bar{\Phi}^5 - \frac{\bar{\nu}}{3}\bar{\Phi}^7 = 0$$

$$\left(1 - \bar{\Phi}^2\right)\frac{\dot{a}^2}{a^2} - 2\frac{\dot{a}}{a}\bar{\Phi}\dot{\bar{\Phi}} = 2\left(\frac{\dot{\bar{\Phi}}^2}{2} + 1 + \frac{\bar{m}^2}{2}\bar{\Phi}^2 + \frac{\bar{\lambda}}{4}\bar{\Phi}^4 + \frac{\bar{\nu}}{6}\bar{\Phi}^6\right)$$

$$\bar{\Phi}(t) \to 0 \;, \qquad \bar{H}^2(t) \to \bar{H}_0^2 = 2$$

$$\bar{\Phi}(t) \to \bar{\Phi}_* = \sqrt{-\frac{\bar{m}^2 + 4}{\bar{m}^2 + \bar{\lambda}}} \;, \qquad \bar{H}^2(t) \to \bar{H}_*^2 = \frac{4\bar{\lambda} - \bar{m}^4}{2(\bar{\lambda} + \bar{m}^2)}$$

Bounce Solutions: $\dot{a}(0) = 0 = H(0)\,, \quad \dot{H}(0) > 0$

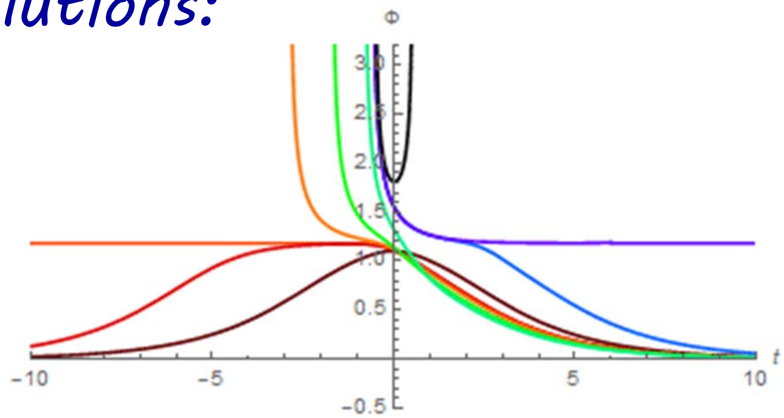
bounce point: t=0

Flat FLRW $f(\Phi) = 1 - \xi\kappa\Phi^2\,, \xi = 1/6$ ("conformal")

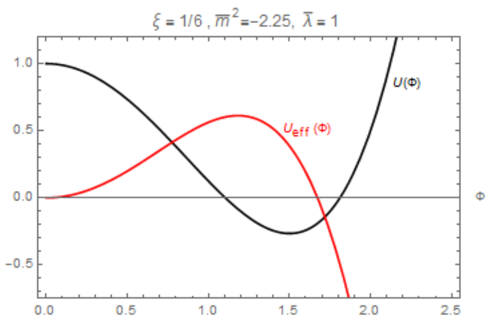
Typically, 2 kinds of bounce solutions:

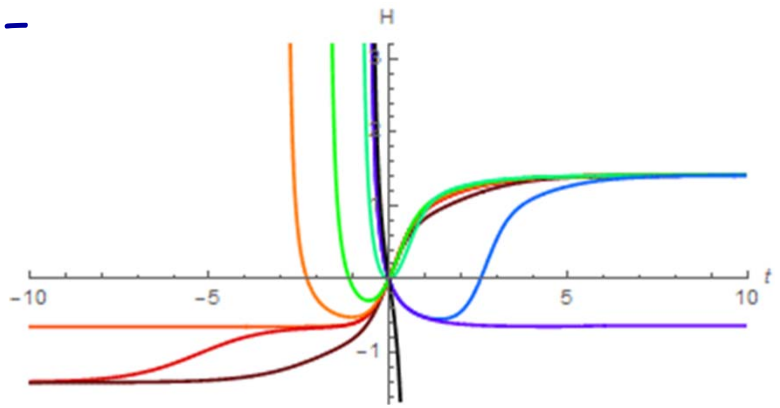
- *full bounce*
- *partial bounce*

Other solutions (BB, BB+BC…) exist as well



Solutions characterized by $\Phi(0)$ *which takes values in a continuous interval – no need for fine tuning*





Bounce Solutions: $\dot{a}(0) = 0 = H(0)\,, \quad \dot{H}(0) > 0$

bounce point: t=0

Flat FLRW $f(\Phi) = 1 - \xi\kappa\Phi^2$, $\xi = 1/6$ ("conformal")

Parameter Space: (λ, m^2) plane; *insist on:*

- $\lambda > 0$
- $m^2 < 0$

$U(\Phi)$ bounded from below

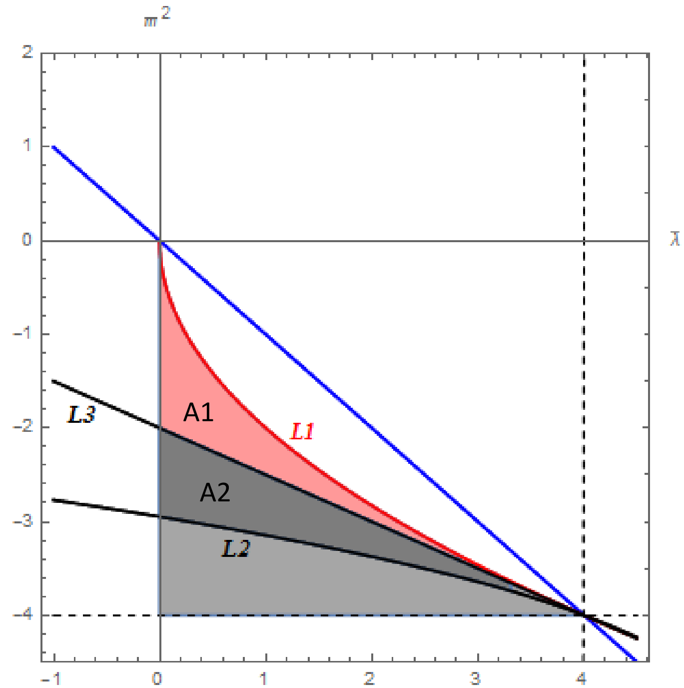
$U(\Phi)$ <0 at bounce point by constraint eq.

Region A: full and partial bounces exist between lines $L1$ and $L2$.

$L1$: $\bar{\lambda} = \bar{m}^4/4$

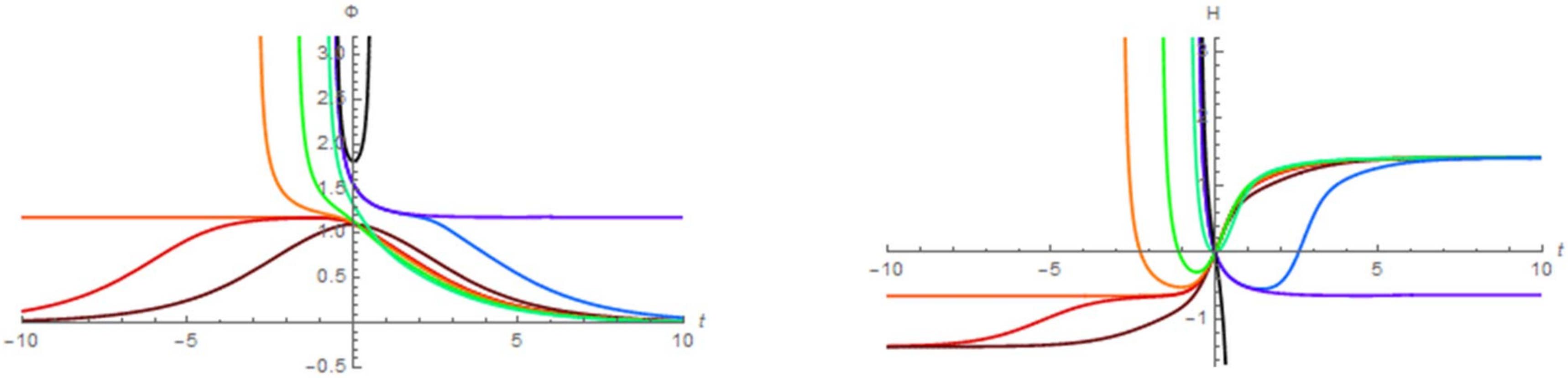
$L2$: $\bar{m}^2 = -4 + 0.42(4 - \bar{\lambda})^{2/3}$

$L3$: $\bar{m}^2 = -2 - \bar{\lambda}/2$



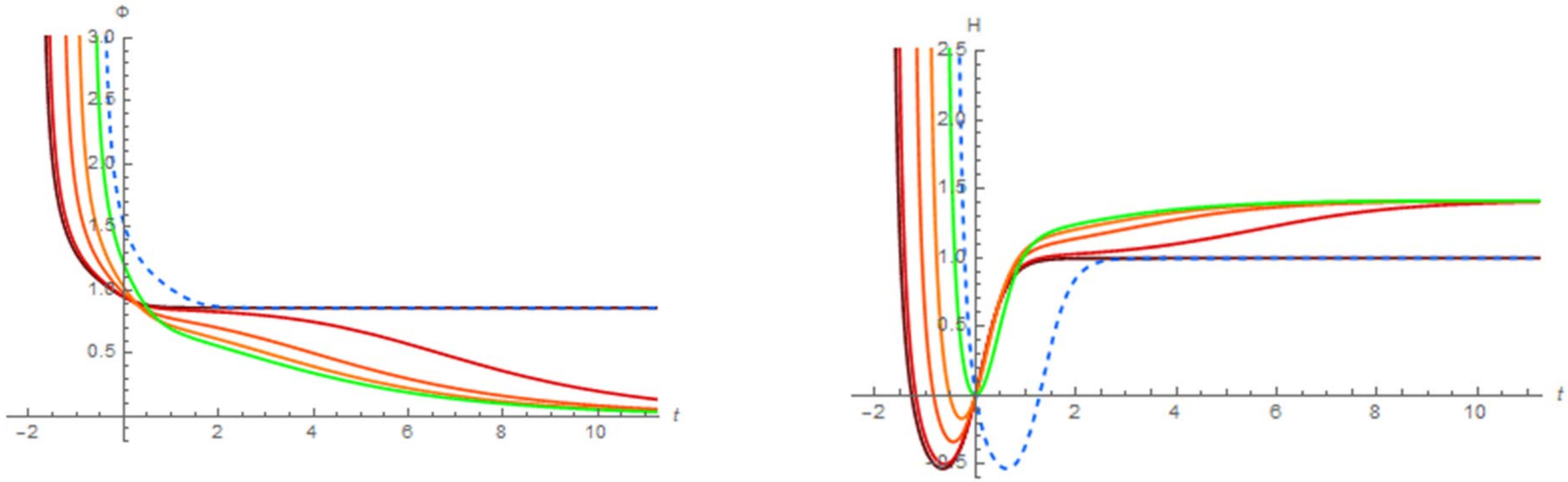
- $U_{eff}(\Phi)$ has a maximum: $\bar{\Phi}_* = \sqrt{-\dfrac{\bar{m}^2 + 4}{\bar{m}^2 + \bar{\lambda}}}$
- constraint equation gives additional condition: $0 < \bar{\lambda} < \bar{m}^4/4$ - line $L1$.
- no future singularities only above line $L2$.

- Above line $L3$ (region A1): full bounces and partial bounces:



$m^2 = -2.25\ ,\ \lambda = 1\ ;\ \Phi(0) = 1.104\ ,\ 1.114\ ,\ 1.11437\ ,\ 1.120\ ,\ 1.145\ ,\ 1.330\ ,\ 1.560\ ,\ 1.56025\ ,\ 1.810$

- Below line $L3$ (region A2): only partial bounces:

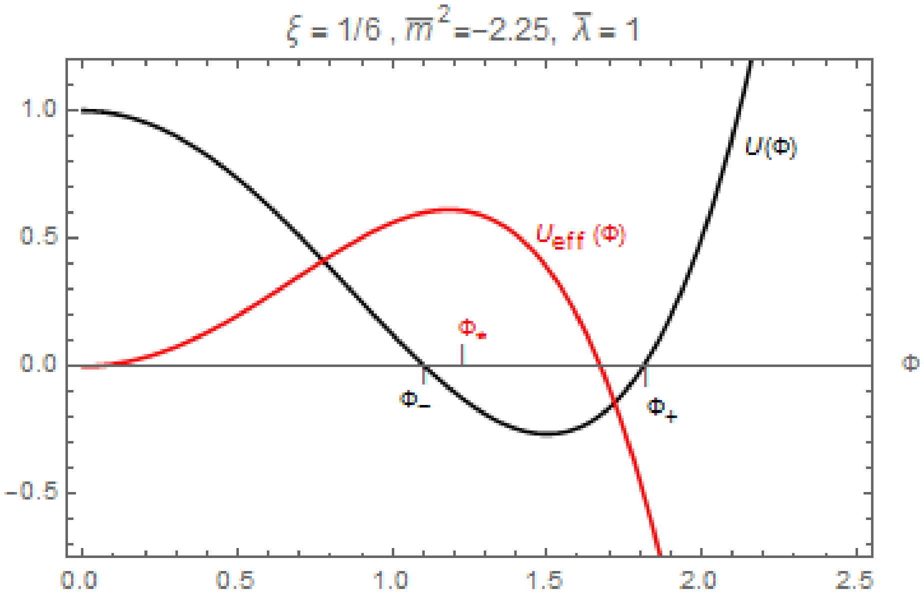


$m^2 = -2.7225\ ,\ \lambda = 1\ ;\ \Phi(0) = 0.94672\ ,\ 0.9500\ ,\ 0.9750\ ,\ 1.025\ ,\ 1.210\ ,\ 1.53142$

More insight from the constraint equation

For conformal coupling + k=0: $(1-\bar{\Phi}^2)H^2 - 2H\bar{\Phi}\dot{\bar{\Phi}} - \dot{\bar{\Phi}}^2 - 2U(\bar{\Phi}) = 0$

Taken at the bounce point (H=0): $U(\Phi(0)) < 0 \quad \Rightarrow \quad \bar{\Phi}_- < \bar{\Phi}(0) < \bar{\Phi}_+$



$$\bar{\Phi}_\pm^2 = -\frac{\bar{m}^2}{\bar{\lambda}} \pm \sqrt{\left(\frac{\bar{m}^2}{\bar{\lambda}}\right)^2 - \frac{4}{\bar{\lambda}}}$$

Recall: $\bar{\Phi}_* = \sqrt{-\frac{\bar{m}^2 + 4}{\bar{m}^2 + \bar{\lambda}}}$

Two options exist:

$\bar{\Phi}_- < \bar{\Phi}_* < \bar{\Phi}_+$ *: Region A1 (upper)*

$\bar{\Phi}_* < \bar{\Phi}_- < \bar{\Phi}_+$ *: Region A2 (lower)*

line L3 separates between the two options/regions

The Full Bounce Solutions Obtained Here are Unstable

all of them cross the point $\bar{\Phi} = 1$ *beyond which the effective gravitational constant becomes negative.*

Starobinsky (1981), Futamase&Maeda (1989), Futamase et al (1989), Caputa et al (2013):

Deviations from isotropy or homogeneity diverge as the effective gravitational constant changes sign.

$\xi > 0$ Bounce Solutions – Flat FLRW

$f(\Phi) = 1 - \xi\kappa\Phi^2$

Effect of $\xi \neq 1/6$ is quite weak on the field equations, especially for $\xi > 1/6$

$$\ddot{\bar{\Phi}} + \frac{3\dot{a}}{a}\dot{\bar{\Phi}} - \frac{(1-6\xi)\bar{\Phi}}{1-(1-6\xi)\bar{\Phi}^2}\dot{\bar{\Phi}}^2 + \frac{\left(\bar{m}^2+4\right)\bar{\Phi} + \left(\bar{\lambda}+\bar{m}^2\right)\bar{\Phi}^3 + \bar{\nu}\bar{\Phi}^5 - \frac{1}{3}\bar{\nu}\bar{\Phi}^7}{1-(1-6\xi)\bar{\Phi}^2} = 0$$

$$\left(1-\bar{\Phi}^2\right)\frac{\dot{a}^2}{a^2} - 2\frac{\dot{a}}{a}\bar{\Phi}\dot{\bar{\Phi}} = \frac{1}{3\xi}\left(\frac{\dot{\bar{\Phi}}^2}{2} + 1 + \frac{\bar{m}^2}{2}\bar{\Phi}^2 + \frac{\bar{\lambda}}{4}\bar{\Phi}^4 + \frac{\bar{\nu}}{6}\bar{\Phi}^6\right)$$

General pattern is similar: most of classifications of solutions carries over to $\xi > 1/6$ case: $\bar{\Phi}_*$, $\bar{\Phi}_+$, $\bar{\Phi}_-$ stay the same.

However, there are some new features:

Asymptotic Behavior / Critical points

$$\ddot{\bar{\Phi}} + \frac{3\dot{a}}{a}\dot{\bar{\Phi}} - \frac{(1-6\xi)\bar{\Phi}}{1-(1-6\xi)\bar{\Phi}^2}\dot{\bar{\Phi}}^2 + \frac{\left(\bar{m}^2+4\right)\bar{\Phi} + \left(\bar{\lambda}+\bar{m}^2\right)\bar{\Phi}^3 + \bar{\nu}\bar{\Phi}^5 - \frac{1}{3}\bar{\nu}\bar{\Phi}^7}{1-(1-6\xi)\bar{\Phi}^2} = 0$$

$$\left(1-\bar{\Phi}^2\right)\frac{\dot{a}^2}{a^2} - 2\frac{\dot{a}}{a}\bar{\Phi}\dot{\bar{\Phi}} = \frac{1}{3\xi}\left(\frac{\dot{\bar{\Phi}}^2}{2} + 1 + \frac{\bar{m}^2}{2}\bar{\Phi}^2 + \frac{\bar{\lambda}}{4}\bar{\Phi}^4 + \frac{\bar{\nu}}{6}\bar{\Phi}^6\right)$$

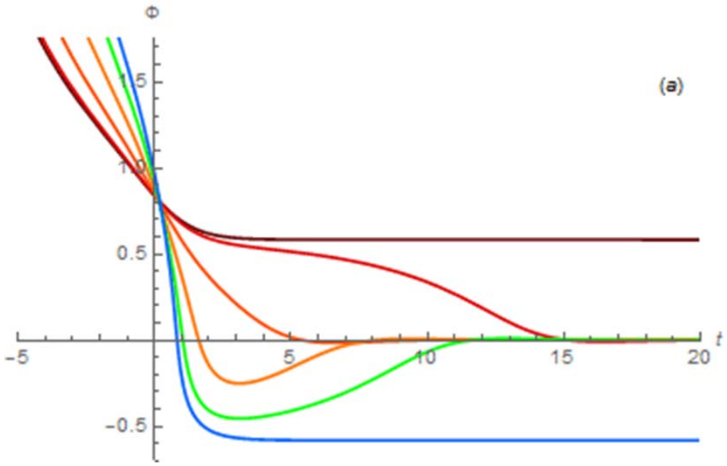
$$\bar{\Phi}(t) \to 0 \;\;, \quad \bar{H}^2(t) \to \bar{H}_0^2 = \frac{1}{3\xi}$$

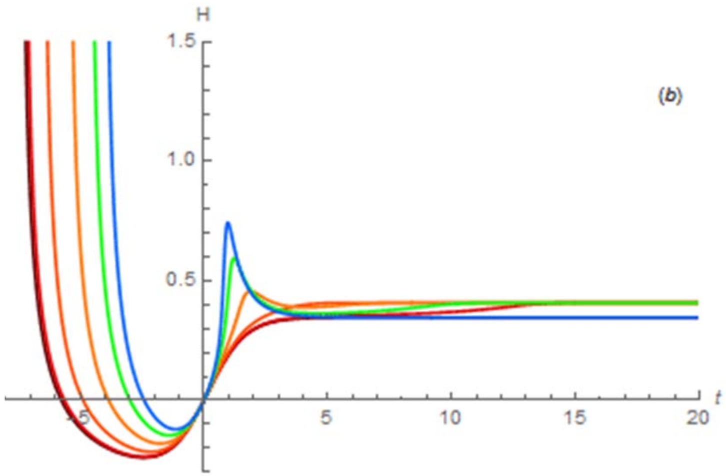
$$\bar{\Phi}(t) \to \bar{\Phi}_* = \sqrt{-\frac{\bar{m}^2+4}{\bar{m}^2+\bar{\lambda}}} \;\;, \quad \bar{H}^2(t) \to \bar{H}_*^2 = \frac{4\bar{\lambda}-\bar{m}^4}{12\xi(\bar{\lambda}+\bar{m}^2)}$$

Note ξ - dependence of asymptotic H

Bounce Solutions – Flat FLRW $f(\Phi)=1-\xi\kappa\Phi^2$

$\xi = 2$

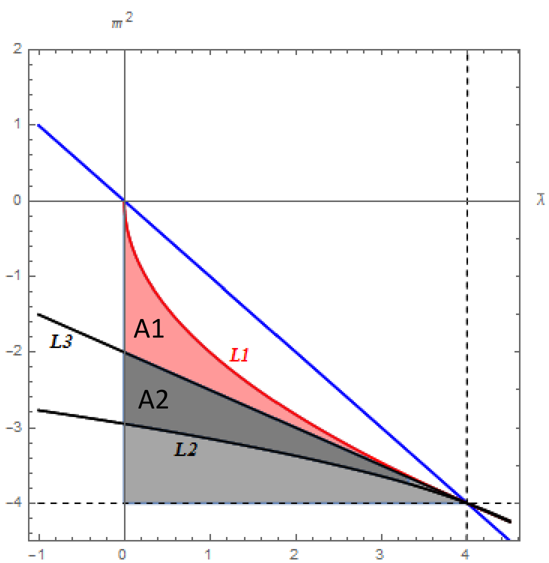




$m^2 = -3.24\,,\ \lambda = 1\,;\ \Phi(0) = 0.83888\,,\ 0.840\,,\ 0.850\,,\ 0.877\,,\ 0.921\,,\ 0.96956$

All solutions have $\Phi(0) < 1$ - stable in the future.
However, no solution to the past instabilities,
or those of the full bounce.

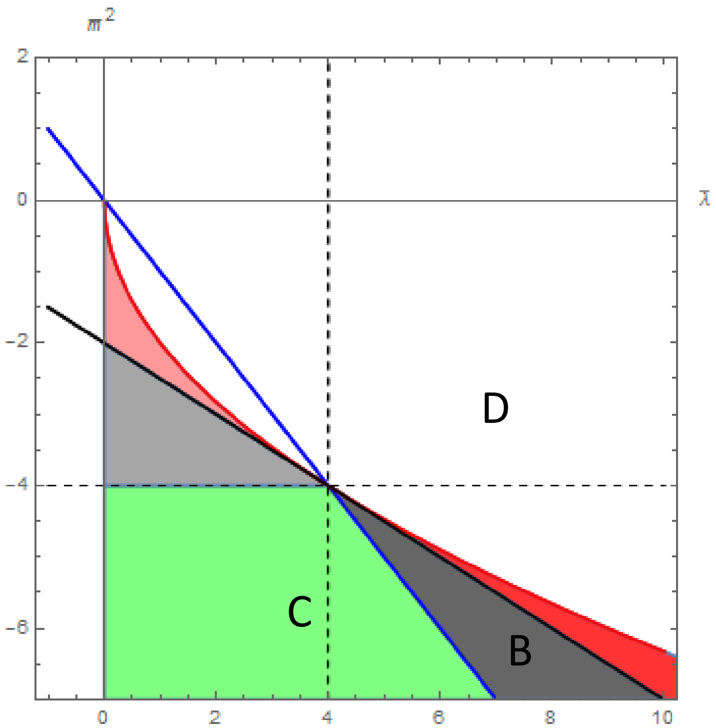
Parameter Space: (λ, m^2) plane



$$L1:\ \bar{\lambda} = \bar{m}^4/4$$

$$L2:\ \bar{m}^2 = -4 + 0.42(4-\bar{\lambda})^{2/3}$$

$$L3:\ \bar{m}^2 = -2 - \bar{\lambda}/2$$



Bounce solutions in regions B,C exist above a minimal ξ, but fine tuning required.
No bounce solutions in region D.

A: maximum to U_{eff}
B: minimum to U_{eff}
C: U_{eff} decreases monotonically
D: U_{eff} increases monotonically

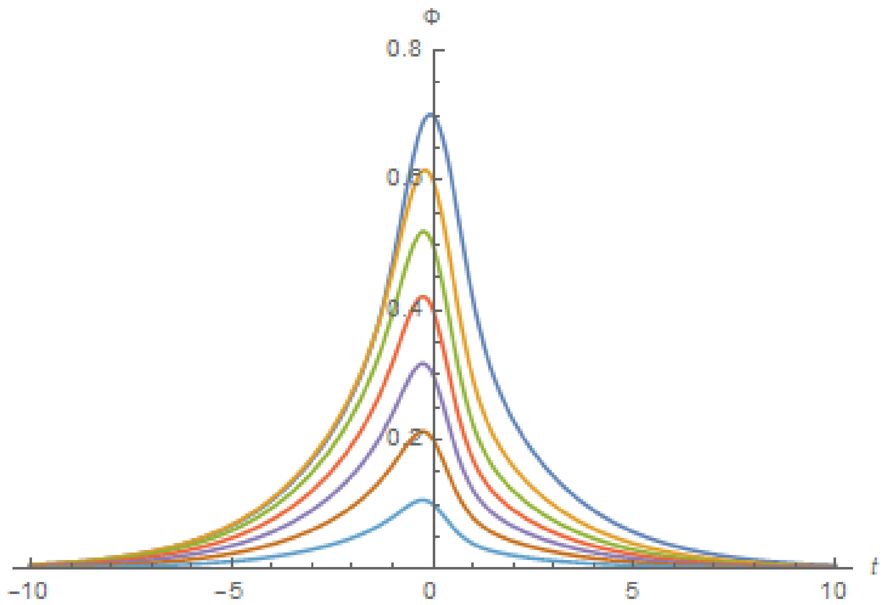
$k>0$ Bounce Solutions $f(\Phi) = 1 - \xi\kappa\Phi^2$

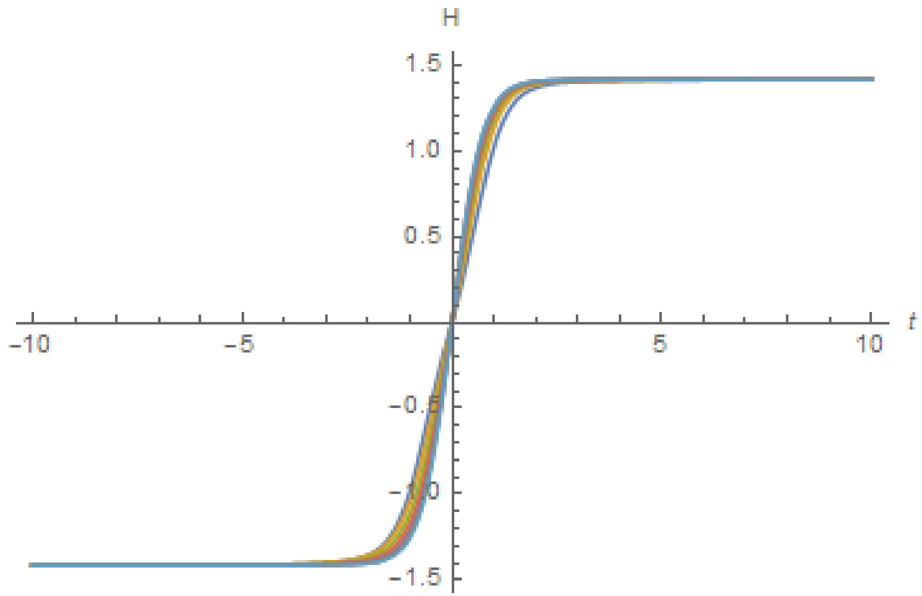
- *Effective potential stays the same - independent of k*
- *Full bounce solutions of the "old" kind exist*

HOWEVER,

- *Exists a new kind of full bounce solutions without suffering from the negative gravitational constant problem. Space curvature k should be large ebough.*

$k>0$ Bounce Solutions





$\xi = 1/6\,,\ m^2 = -2.25\,,\ \lambda = 1\,,\ k = 2\,;\ \Phi(0) = 0.1\,,\ 0.2\,,\ 0.3\,,\ 0.4\,,\ 0.5\,,\ 0.6\,,\ 0.7$


Thank you for your attention