Borromean rings and $^{22}\text{C}$

T. Miyamoto

Department of Physics, FEPS
university of Surrey, Guildford, UK

1st April 2015, IoP conference Manchester, Chemistry building
Outline of this talk

▶ A short introduction of $^{22}\text{C}$
▶ Bound states of $^{22}\text{C}$
▶ Calculations of the reaction observables of $^{22}\text{C}$
▶ Conclusion, discussion and future work
22C and Borromean rings

- Recent experimental data suggest that not only $^6$He, $^{11}$Li but also $^{22}$C have two valence nucleons. The finding that $^{21}$C is unbound and has no low-lying states (Mosby 2013) indicates that $^{22}$C has a Borromean structure.

- $^{22}$C has N=16 subshell closure, and the N=16 has been known as a hidden magic number (6,16,32,34) (Ozawa 2000). The rms radius $5.2 \pm 0.2$ of the nucleus is much larger than those $3.41 \pm 0.23, 3.19 \pm 0.13$ of the isotones $^{23}$N, $^{24}$O (Tanaka 2010).
Unbound $^{21}\text{C}$

$^{21}\text{C}$ single particle energy spectrum

- Use $^{20}\text{C} + n$ model
- Treat s-wave in a different way (Horiuchi 2006)
- $0d_{5/2}$ bound; its eigenenergy at least -2.93
- $1s_{1/2}$ continuum

$$U_{c1} = \frac{-V_0}{1 + \exp\left(\frac{r-R}{a}\right)} + (-4) \frac{1}{r} \frac{d}{dr} \left(\frac{-V_{so}}{1 + \exp\left(\frac{r-R}{a}\right)}\right) (1 \cdot s) \quad (1)$$
${}^{22}\text{C}$ and Borromean rings
Hyperspherical harmonics (HH) formalism

\[ \Psi_{JM,J}^T(x, y) = \frac{1}{\rho^{5/2}} \sum_{K\gamma} \chi_{K\gamma}(\rho) H_{K\gamma,JM,J}^T(\Omega_5, \sigma), \]  

(2)

A set of hyperradial equations involve:

▶ core-n potential
▶ GPT potential for nn potential
▶ and Hyperspherical 3B potential (if necessary)
▶ 3body Jacobi coordinates
▶ HH coordinates
▶ Expanding wave functions with regard to HH basis functions
▶ A set of quantum numbers \( \gamma = \{l_x, l_y, L, S\} \) of HH
▶ Solving a set of hyperradial equations

\[ Y_{KLM,L}^{l_x l_y} (\Omega_5) = Y_{l_x m_x}^{l_y m_y} (\theta_x, \phi_x) Y_{l_y m_y}^{l_x m_x} (\theta_y, \phi_y) N_{n_3}^{l_x + \frac{1}{2}, l_y + \frac{1}{2}} \]

\[ \times (\sin \alpha_2)^{l_x} (\cos \alpha_2)^{l_y} P_{\frac{K - l_x - l_y}{2}}^{l_x + \frac{1}{2}, l_y + \frac{1}{2}} (\cos (2\alpha_2)). \]  

(3)
Bound states of $^{22}$C

- HH and 3body model describe the Borromean property well
- Mean FT and many body approach do not work well
Reaction cross sections for $^{22}\text{C} + ^{12}\text{C}$ at 300 MeV/A

\[
S(R_b) = \langle \phi^{(3B)}_{IM'I} | e^{i(x_{ct} + x_{2t} + x_{1t})} | \phi^{(3B)}_{IMI} \rangle
\]  \hspace{1cm} (4)

where we put $(I, M_I) = (0, 0)$.

\[
\sigma_R = 2\pi \int_0^\infty (1 - |S(R_b)|^2) R_b dR_b.
\]  \hspace{1cm} (5)

- Glauber 4body approach
- Integration wrt hyperspherical coordinates (6dim)
- folding model for $^{20}\text{C} + ^{12}\text{C}$
- cf Jaros’s empirical value 1099.1 mb

<table>
<thead>
<tr>
<th>Method</th>
<th>Sharma</th>
<th>Kucuk</th>
<th>RI2v5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Glauber OLA</td>
<td>Glauber OLA</td>
<td>Glauber 4B</td>
</tr>
<tr>
<td>$\sigma_R$ (mb)</td>
<td>$\sim 1150$</td>
<td>1338.6</td>
<td>1352.9</td>
</tr>
</tbody>
</table>

MFT yields smaller values than the 3body HH.
E1 strength distribution. $^6\text{He}$, $^{22}\text{C}$

$$\hat{B}_\mu^{(E1)} = eZ_c\sqrt{\frac{2}{A_cA_{\text{proj}}}}|\mathbf{y}|Y_{1\mu}(\Omega_y).$$

(6)

$$\frac{dB}{dE_{\text{ex}}} = \frac{1}{2J+1}\rho^{(E1)}(\sum_{M_J\gamma\mu}^{}(2J' + 1)
\times \left( \begin{array}{ccc} L' & S' & J' \\ M_{L'} & M_{S'} & -M_{J'} \end{array} \right)
\times \left( \begin{array}{ccc} L' & S' & J' \\ M_{L'} & M_{S'} & -M_{J'} \end{array} \right)|M_\mu^\Xi|^2,$$

(7)

- $\hat{B}^{(E1)}$ in the hyperspherical coordinates
- phase shift factor 
  $$\rho^{(E1)} = \frac{4m^3_n}{\hbar^6} \cdot E_{\text{ex}}^2$$
- reduced matrix elements
- A set of quantum numbers:
  $$\gamma = \{K, L, L_x, L_y, S\}$$
- A set of quantum numbers: $\Xi$

$$\langle \phi^{(3B)}|\hat{B}_\mu^{(E1)}|\phi^{(3B)}\rangle = \sum_{\gamma'M_{L'}}^{}(-1)^{L'-S'+M_{J'}}\sqrt{2J'+1}
\times \left( \begin{array}{ccc} L' & S' & J' \\ M_{L'} & M_{S'} & -M_{J'} \end{array} \right) Y_{K' L' x L'} M_\mu^\Xi,$$

(8)
E1 strength distribution. $^6$He

HH formalism seems to describe these systems fairly well.
E1 strength distribution. $^{22}\text{C}$
\( ^6 \text{He} + ^{208} \text{Pb} \text{ at 70 MeV/A} \)

- Difference in the way of treating Coulomb interaction
- A smoothing factor for PW calculation
Conclusion, discussion and future work

- The HH method and the 3body model describe the Borromean nature of $^{22}$C, while MFT and many body approach do not
- This suggests that the cluster nature is the true nature of $^{22}$C
- Reaction cross sections of $^{22}$C + $^{12}$C and E1 strength distributions for $^6$He and $^{22}$C were presented. These results are compatible with the above idea.
- As future work, we will calculate the differential breakup cross sections for $^{22}$C with FSI.