Excited meson spectroscopy from lattice QCD – resonances and scattering

Christopher Thomas, University of Cambridge

c.e.thomas@damtp.cam.ac.uk

IoP Meeting, Manchester, 1st Apr 2015

Hadron Spectrum Collaboration
Excited Meson Spectroscopy
Excited Meson Spectroscopy

$X(3872), Y(4260), Z^+(4430), Z_c^+(3900), Z_b^+, D_s(2317)$, light scalars, ... (also baryons)
Exotic $J^{PC}$?

Resonances or near threshold

BESIII  LHC  JLab @ 12 GeV
COMPASS  KLOE  CLAS12
Belle II
Excited Meson Spectroscopy

- $X(3872)$, $Y(4260)$, $Z^+(4430)$, $Z_c^+(3900)$, $Z_b^+$, $D_s(2317)$, light scalars, ... (also baryons)
- Exotic $J^{PC}$?

Resonances or near threshold

Can we understand these within QCD? $\rightarrow$ lattice QCD
Outline

• Introduction
• Resonances etc in lattice QCD
  • The $\rho$ in isospin-1 $\pi\pi$ scattering
  • $\pi K$, $\eta K$ coupled-channel scattering
• Summary
Lattice QCD

Discretise (spacing = $a$)
Finite volume
Imaginary (Euclidean) time, $t \rightarrow i t$
Lattice QCD

Discretise (spacing = $a$)
Finite volume
Imaginary (Euclidean) time, $t \rightarrow i t$

$$\langle f \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \; f e^{-\mathcal{S}[\psi, \bar{\psi}, U]}$$

- Finite $a$ and $L$ (and reduced sym.)
- Unphysical $m_\pi$

Quarks
Gluons

$$L$$

$$a$$

JLab
Spectroscopy on the lattice

Energy eigenstates from:

\[ C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}^+_j(0) | 0 \rangle \]

Interpolating operators

\[ C_{ij}(t) = \sum_n \frac{e^{-\frac{E_n}{2} t}}{2E_n} \langle 0 | \mathcal{O}_i(0) | n \rangle \langle n | \mathcal{O}^+_j(0) | 0 \rangle \]
Spectroscopy on the lattice

Energy eigenstates from:

\[ C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}^\dagger_j(0) | 0 \rangle \]

Interpolating operators

\[ C_{ij}(t) = \sum_n \frac{e^{-\frac{E_n}{2} t}}{2 E_n} \langle 0 | \mathcal{O}_i(0) | n \rangle \langle n | \mathcal{O}^\dagger_j(0) | 0 \rangle \]

Large basis of ops \( \Rightarrow \) matrix of corrs. – generalised eigenvalue problem
Light mesons (isospin = 0 and 1)

[Dudek, Edwards, Guo, CT, PR D88, 094505 (2013); update of PR D83, 111502 (2011)]
+ other volumes and $m_\pi$
Light mesons (isospin = 0 and 1)

[Dudek, Edwards, Guo, CT, PR D88, 094505 (2013); update of PR D83, 111502 (2011)] + other volumes and $m_\pi$
Scattering in LQCD

Scattering matrix elements cannot be extracted from infinite-volume Euclidean-space correlation functions (except at threshold) [Maiani & Testa, 1990]
Scattering in LQCD

Two hadrons: non-interacting

\[ E_{AB} = \sqrt{m_A^2 + \vec{k}_A^2} + \sqrt{m_B^2 + \vec{k}_B^2} \]

Infinite volume  Continuous spectrum
Scattering in LQCD

Two hadrons: **non-interacting**

Infinite volume

Continuous spectrum

Finite volume

Discrete spectrum

\[ E_{AB} = \sqrt{m_A^2 + \vec{k}_A^2} + \sqrt{m_B^2 + \vec{k}_B^2} \]

\[ \vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z) \]

periodic b.c.s (torus)

L
Scattering in LQCD

Two hadrons: **interacting**

- Infinite volume: Continuous spectrum
- Finite volume: Discrete spectrum

\[ \vec{k}_{A,B} \neq \frac{2\pi}{L}(n_x, n_y, n_z) \]

- periodic b.c.s (torus)
- c.f. 1-dim: \( k = \frac{2\pi}{L} n + \frac{2}{L} \delta(k) \)

scattering phase shift
Lüscher (elastic): \textbf{finite volume} energy levels \\
$\rightarrow \textbf{infinite volume}$ scattering phase shift at $E_{cm}$
Scattering – ‘Lüscher method’

Lüscher (elastic): **finite volume** energy levels
→ **infinite volume** scattering phase shift at $E_{cm}$

Map out phase shift (or scattering matrix) → resonance parameters etc

\[
\sigma_l(E') \propto \sin^2 \delta_l(E') = \frac{(\Gamma/2)^2}{(E - E_R)^2 + (\Gamma/2)^2}
\]
Scattering – ‘Lüscher method’

Lüscher (elastic): finite volume energy levels → infinite volume scattering phase shift at $E_{cm}$

Map out phase shift (or scattering matrix) → resonance parameters etc

$Lüscher$, $NP$ B354, 531 (1991); extended by many others

Need many (multi-hadron) energy levels

Note: reduced symmetry → mixing between partial waves

\[
\sigma_l(E') \propto \sin^2 \delta_l(E') = \frac{(\Gamma/2)^2}{(E - E_R)^2 + (\Gamma/2)^2}
\]
The $\rho$ resonance in $\pi \pi$ scattering

$(J^{PC} = 1^{--}, I = 1)$

$\text{BR}(\rho \rightarrow \pi \pi) \sim 100\%$
The $\rho$ resonance in $\pi\pi$ scattering

$C_{ij}(t) = \langle 0 | \Phi_i(t) \Phi_j^\dagger(0) | 0 \rangle$

$M_\pi \approx 400$ MeV, 3 volumes ($L \approx 2 - 3$ fm), $a_s \approx 0.12$ fm, $a_s/a_t \approx 3.5$

$J^{PC} = 1^{--}, I = 1$

$\text{BR}(\rho \to \pi\pi) \sim 100\%$

Dudek, Edwards, CT [PR D87, 034505 (2013)]
Assume $\delta_{l>3} \approx 0$ in this energy range – find no significant signal for $\delta_{l=3}$

$M_\pi \approx 400$ MeV, 3 volumes ($L \approx 2 - 3$ fm), $a_s \approx 0.12$ fm, $a_s/a_t \approx 3.5$

The $\rho$ resonance in $\pi\pi$ scattering

$(J^{PC} = 1^{--}, I = 1)$

$\text{BR}(\rho \rightarrow \pi\pi) \sim 100\%$

$C_{ij}(t) = \langle 0 | \Phi_i(t) \Phi_j^\dagger(0) | 0 \rangle$

single-meson

$\sim \overline{\psi} \Gamma D \ldots \psi$

and $\pi\pi$ ops.

Dudek, Edwards, CT [PR D87, 034505 (2013)]
The $\rho$ resonance in $\pi\pi$ scattering
The $\rho$ resonance in $\pi\pi$ scattering

$M_\rho = 854.1 \pm 1.1$ MeV
$\Gamma = 12.4 \pm 0.6$ MeV
$g = 5.80 \pm 0.11$

$[M_\pi \approx 400$ MeV$]$

$\Gamma(E_{\text{cm}}) = \frac{g^2 p_{\text{cm}}^3}{6\pi E_{\text{cm}}^2}$

c.f. experimentally

$M_\rho = 775.49 \pm 0.34$ MeV
$\Gamma = 149.1 \pm 0.8$ MeV
$g \approx 5.9$

Dudek, Edwards, CT [PR D87, 034505 (2013)]
\( \pi K, \eta K \ (I=1/2) \) coupled-channel scattering

<table>
<thead>
<tr>
<th>( J^P )</th>
<th>( K^0 (1430), \ldots )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0^+</td>
<td>( \kappa, K^0 (1430), \ldots )</td>
</tr>
<tr>
<td>1^-</td>
<td>( K^* (892), \ldots )</td>
</tr>
<tr>
<td>2^+</td>
<td>( K_2^* (1430), \ldots )</td>
</tr>
</tbody>
</table>

Isospin = 1/2  
Strangeness = 1
$\pi K$, $\eta K$ ($I=1/2$) coupled-channel scattering

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>$\kappa$, $K_0^*(1430)$, ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+$</td>
<td>$K_0^<em>$, $K^</em>$, $K_2^*$, ...</td>
</tr>
<tr>
<td>$1^-$</td>
<td>$K^*(892)$, ...</td>
</tr>
<tr>
<td>$2^+$</td>
<td>$K_2^*(1430)$, ...</td>
</tr>
</tbody>
</table>

Isospin $= 1/2$
Strangeness $= 1$

$$C_{ij}(t) = \langle 0 | \phi_i(t) \phi_j^+(0) | 0 \rangle$$

Single-meson

$$\sim \bar{\psi} \Gamma D \ldots \psi$$

$\pi K + \eta K$ ops.

$M_\pi = 391$ MeV, $M_K = 549$ MeV, $M_\eta = 589$ MeV; 3 volumes as before

David Wilson, Jo Dudek, Robert Edwards, CT [PRL 113, 182001; PR D91, 054008]
Scattering – ‘Lüscher method’

Lüscher, NP B354, 531 (1991); extended by many others

Extension **to inelastic scattering:**

relate finite vol. energy levels to infinite vol. $t$-matrix.

**Underdetermined problem**

$\rightarrow$ parameterize $E_{cm}$ dependence of $t$-matrix and fit $E_{\text{lat}}$ to $E_{\text{param}}$
Scattering – ‘Lüscher method’

Extension to inelastic scattering:
relate finite vol. energy levels to infinite vol. t-matrix.

Underdetermined problem
→ parameterize $E_{cm}$ dependence of t-matrix and fit $E_{\text{lat}}$ to $E_{\text{param}}$

$L$-matrix param. – respects unitarity (conserve prob.) and flexible

$$t_{ij}^{-1}(s) = \frac{1}{(2k_i)^\ell} K_{ij}^{-1}(s) \frac{1}{(2k_j)^\ell} + I_{ij}(s)$$

$$\text{Im}I_{ij} = -\delta_{ij}\rho_i(s)$$
πK, ηK (I=1/2) spectra

Neglect ℓ ≥ 4: only ℓ = 0 contributes

P = [0,0,0] A₁⁺
$\pi K, \eta K$ (I=1/2) spectra

$P = [0,0,0] A_1^+$

Neglect $\ell \geq 4$: only $\ell = 0$ contributes

$\chi^2 / N_{dof} = \frac{6.40}{15-6} = 0.71$
$\pi K, \eta K \ (l=1/2)$ spectra

> 100 energy levels in total
πK (I=1/2): P-wave near threshold

$$k^3 \cot \delta_1 = (m_R^2 - s) \frac{6\pi \sqrt{s}}{g_R^2}$$

$$\chi^2/N_{dof} = \frac{9.23}{19-2} = 0.54$$
$\pi K (I=1/2)$: P-wave near threshold (well below $\eta K$ threshold)

\[ k^3 \cot \delta_1 = (m_R^2 - s) \frac{6\pi \sqrt{s}}{g_R^2} \]

$g_R = 5.93(26)$  
c.f. exp. = 5.52(16)  
$M_R = 933(1)$ MeV  
c.f. exp $\approx 892$ MeV

\[ \chi^2 / N_{\text{dof}} = \frac{9.23}{19-2} = 0.54 \]
\( \pi K, \eta K (J=1/2): S & P\)-waves

(73 energy levels)

\[ \chi^2/N_{dof} = 49.1/(61 - 6) = 0.89 \]

\[ \chi^2/N_{dof} = 15.0/(19 - 5) = 1.0 \]
$\pi K, \eta K (I=1/2)$: t-matrix poles
$\pi K$, $\eta K$ ($I=1/2$): t-matrix poles

$T = 2 \cdot \text{Im}\sqrt{s_0} / \text{MeV}$

- Broad resonance
  - c.f. $K_0^*(1430)$
  - (exp. B.R. to $\pi\pi K \sim 50\%$)

- Narrow resonance
  - c.f. $K_2^*(1430)$
  - (exp. B.R. to $\pi\pi K \sim 50\%$)
\( \pi K, \eta K (I=1/2) \): t-matrix poles

- **Broad resonance c.f. \( K_0^* (1430) \)**
- **Narrow resonance c.f. \( K_2^* (1430) \)**
  (exp. B.R. to \( \pi \pi K \sim 50\% \))
- **Bound state just below threshold c.f. \( K^* (892) \)**
\(\pi K, \eta K \ (I=1/2): \) t-matrix poles

- **Bound state just below threshold c.f. \(K^*(892)\)**

- **Virtual bound state**
  - [pole below threshold on unphysical sheet(s)]
  - c.f. unitarised \(\chi pt\)
  - [Nebreda & Pelaez, PR D81, 034035 (2010)]

- **Narrow resonance c.f. \(K_2^*(1430)\)**
  - (exp. B.R. to \(\pi\pi K \sim 50\%\))

- **Broad resonance c.f. \(K_0^*(1430)\)**
Summary

- Many (multi-hadron) energy levels → map out energy dependence of scattering amps. in detail
  - \( \pi\pi \) I=1 – \( \rho \) resonance
  - \( \pi K, \eta K \) I=1/2 – first coupled-channel scattering from LQCD → broad & narrow resonances, bound state, v.b.s.
  - (also \( \pi\pi \) I=2 in S and D-wave and \( \pi K \) I=3/2)

Summary and outlook
Summary

• Many (multi-hadron) energy levels
  \( \rightarrow \text{map out} \) energy dependence of scattering amps. in detail
• \( \pi\pi \) I=1 – \( \rho \) resonance
• \( \pi K, \eta K \) I=1/2 – first coupled-channel scattering from LQCD
  \( \rightarrow \) broad & narrow resonances, bound state, v.b.s.
• (also \( \pi\pi \) I=2 in S and D-wave and \( \pi K \) I=3/2)

Outlook

• Many other interesting channels (e.g. charm, charmonium)
• \( >2 \) hadrons is challenge. Lighter \( \pi \) \( \rightarrow \) lower 3-hadron thresh.
πK, ηK (I=1/2): D-wave

Assume ℓ ≥ 3 negligible
Up to πππK threshold; neglect coupling to ππK