Alpha Cluster Breaking in $^9\text{Be}$: Experimental and Theoretical Developments

Robin Smith

M. Freer, C. Wheldon, M. A. Caprio, P. Maris, J. Vary
Agenda

• Introduction: Molecular structure of $^9\text{Be}$

• Experimental method and setup

• Key experimental results

• Comparison with *ab initio* no-core shell model (NCSM) nuclear theory

• Summary
α:n:α Molecular Structure of $^{9}\text{Be}$

- Large quadrupole moment (+53 mb) [1]
- Agreement with cluster and molecular models
- Well-developed rotational band structure

$$E(J) = E_0 + A[J(J + 1)]$$

Experimental Hurdles

- “Missing” key structural indicators
- Exotic structures – unbound resonances
- Scarcity of electromagnetic moment and transition data
Ab initio approaches: α:n:α Structure of $^9\text{Be}$.

No-Core Shell Model (NCSM)

Antisymmetrized Molecular Dynamics (AMD)

Experiment Overview

- **Notre Dame tandem**
- Inelastic scattering 22 and 26 MeV alpha particles
- 1 mg/cm² beryllium-9 target

Excited nucleus then breaks up:

\[ ^9\text{Be}^* \rightarrow ^8\text{Be}_{\text{gs}} + n \]

Final state:

\[ 2\alpha + n + \alpha \]
Reaction Chamber

-69°  -30°  33°  70°
$^9$Be Excitation Spectra: $^8$Be$_{gs}$ Breakups
$^9$Be Excitation Spectra: $^8$Be$_{gs}$ Breakups

No known levels between 8 and 11 MeV
Peak Fitting: Known Spectrum

- Fit of *known* levels shown by blue dashed line [4]

- Voigt profiles (≈ 600 keV res)

- Centroids and widths defined by previous experimental studies
  - Except 8 MeV state

- Poor agreement with experimental data

Peak Fitting: New Levels

- New levels introduced

- Single new level in 22 MeV beam data
  - \( \chi^2/\text{DOF} = 3.18 \rightarrow 1.74 \)

- Two new levels 26 MeV beam data fit
  - \( \chi^2/\text{DOF} = 3.78 \rightarrow 0.78 \)

- Consistent fit parameters

- Features are not an unaccounted background
# Fit Summary

- Summary of fit parameters across each data set

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<th>22 MeV</th>
<th>26 MeV</th>
<th>Weighted Mean</th>
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<td>E (MeV)</td>
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<td>8.07(2)</td>
<td>825(78)</td>
<td>8.11(6)</td>
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<td><strong>New state 1</strong></td>
<td>9.25(6)</td>
<td>280(97)</td>
<td>9.34(3)</td>
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<td><strong>New state 2</strong></td>
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Ab initio No-Core Shell Model (NCSM)

- Large dimension eigenvalue problem
- JISP16 inter-nucleon interaction
- Basis – Antisymmetrised products of harmonic oscillator wave functions
- Eigenvalues $\rightarrow$ binding energies
- Eigenvectors $\rightarrow$ wavefunctions of the A-body system

Results of Calculations: Energy Eigenvalues

Ex (MeV) vs J

- K = 3/2 band
- K = 1/2 band
- New quartet
- Experiment
Results of Calculations: Energy Eigenvalues

Energy Plot

- GS band
- Excited band
- New quartet
- Experiment

K = 3/2 band
K = 1/2 band
New quartet
Experiment
Results of Calculations: Energy Eigenvalues

\[ \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2} \]

Ex (MeV)

\[ 0, 2, 4, 6, 8, 10, 12, 14, 16 \]

\[ J \]

\[ K = 1/2 \text{ rotational band?} \]

\[ \alpha \alpha \approx 2.2 \text{ fm} \]

\[ \alpha \approx 5 \text{ fm} \]
Deformation, Clustering and Collectivity?

• Nuclear density projections? – Non-trivial!

• Quadrupole moments? – Negative

• Transition Matrix Elements?
  – No collective enhancement
  – Do not follow rotational patterns

\[ B(E2; J_i \rightarrow J_f) = \frac{5}{16\pi} (J_i K 20 | J_f K)^2 (eQ_0)^2 \]
Shell Model Decomposition

Orbit (n,l,j) 

< N >

Neutron Occupancies

J = 1/2
J = 3/2
J = 5/2
J = 7/2

Neutrons
Protons

(0,0,1/2) (0,1,1/2) (0,1,3/2) (1,0,1/2) (0,2,3/2)
M-Scheme J coupling

- Construct a set of states: 1/2-, 3/2-, 5/2-, 7/2- from simple angular momentum coupling

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0+ 2+  1/2- 3/2- 5/2- 7/2-
Summary

- Analysis has been developed to cleanly measure beryllium-9 levels populated by inelastic scattering

- Two new states at 9.32(3) and 10.2(1) MeV have been measured – consistency between different beam runs

- Improvements in quantities for known levels.

- Predicted by \textit{ab initio} no core configuration interaction (NCCI) calculations

- Structural transition away from molecular model towards a mean field description
That's all Folks!

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- Web: http://www.np.ph.bham.ac.uk/postgrads/smithr/
- Twitter: @UnclearPhysics
NCSM Formulation

• Construct translationally-invariant Hamiltonian for A-body system:

\[ H_A = T_{\text{rel}} + V = \frac{1}{A} \sum_{i<j} \frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j} V_{ij} \quad \text{(JISP16)} \quad (+ 3\text{-body ... etc}) \]

• Add a COM HO Hamiltonian

\[
\begin{align*}
H_{cm} &= T_{cm} + U_{cm} \\
U_{cm} &= \frac{1}{2} Am \Omega \vec{R}^2 \\
\vec{R} &= \frac{1}{A} \sum_{i=1}^{A} \vec{r}_i.
\end{align*}
\]

\[
H_A^\Omega = \sum_{i=1}^{A} T_i + \sum_{i<j} V_{ij} + \sum_{i=1}^{A} \frac{1}{2} Am \Omega^2 \vec{r}_i^2
\]

\[
= \sum_{i=1}^{A} h_i + \sum_{i<j} \left[ V_{ij} - \frac{m \Omega^2}{2A} (\vec{r}_i - \vec{r}_j)^2 \right]
\]

• Relevance of oscillator parameter: 
  \(- \Omega\)

\[
h_i = -\frac{\hbar^2}{2m} \nabla_i^2 + \frac{1}{2} m \Omega \vec{r}_i^2.
\]
The Many Particle Hilbert Space

• Write the Hamiltonian matrix in a basis of antisymmetrised products of single particle HO wavefunctions

\[ \hat{H} \Psi_i(r_1, r_2, \ldots, r_A) = E_i(r_1, r_2, \ldots, r_A) \]

\[ \Psi(r_1, r_2, \ldots, r_A) = \sum_k a_k \psi_k(r_1, r_2, \ldots, r_A) \]

\[ \psi_k(r_1, \ldots, r_A) = A[\phi_{n_1l_1j_1m_1}(r_1), \phi_{n_2l_2j_2m_2}(r_2), \ldots \phi_{n_Al_Aj_Am_A}(r_A)] \]

• Reduces problem to eigenvalue problem of a sparse matrix

• N_{\text{MAX}} Truncation
(Un)Convergence

- Eigenvalues are sensitive to basis truncation and the oscillator parameter, $\Omega$ [2]

- Basis extrapolation [2]

Necessary Checks

• Did the beam interact with 9Be or a (12C or 16O) contaminant target?

• Ensure that we are observing inelastic scattering (rather than an alpha or neutron transfer reaction)?

• What channel did the excited 9Be break up through?
Transition Matrix Elements: Enhancements

`Network Diagram`

Transition Matrix Elements: Rotational Patterns

Transition Matrix Elements: New Levels

Transition matrix elements are a factor of 5 – 10 lower than for known intra-band transitions.